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## Coulomb and Phonon Scattering Processes in Metal–Oxide–Semiconductor Inversion Layers: Beyond Matthiessen’s Rule

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The separability of Coulomb and phonon scattering processes in inversion layers of metal–oxide–semiconductor field-effect-transistors (MOSFETs) was studied. The effect of finite collisional duration due to phonon scattering was considered in the evaluation of Coulomb scattering-limited mobility to investigate the relationship between the separability of Coulomb and phonon scattering processes and the long-range nature of Coulomb potential. It was found that the condition under which Coulomb scattering is separated from phonon scattering is determined by the relationship between the screening length due to free carriers in the inversion layers and the phonon mean free path. It was also found that the long-range component of the Coulomb potential is effectively cut off by phonon scattering. [DOI: 10.1143/JJAP.44.1682]

KEYWORDS: Matthiessen’s rule, Coulomb scattering, phonon scattering, finite collisional duration

### 1. Introduction

The importance of understanding the carrier transport properties in inversion layers of metal–oxide–semiconductor field-effect-transistors (MOSFETs) is increasing due to the aggressive scaling down of device sizes. Generally, the scattering mechanisms determining the carrier transport properties in MOS inversion layers are characterized on the basis of three scattering components, namely, Coulomb scattering, phonon scattering, and surface roughness scattering,<sup>1)</sup> which are characteristic of the low-, intermediate-, and high-effective-electric-field regions, respectively. Conventionally, this type of classification of scattering components has been performed on the basis of Matthiessen’s rule,<sup>1,2)</sup> which is the basis for the understanding of carrier transport properties in inversion layers of MOSFETs.

Matthiessen’s rule is founded on the assumption that total scattering rate is expressed as the sum of the scattering rates of individual scattering components. This assumption holds under the condition that scattering events occur locally and are short-ranged. However, when a scattering event is long-ranged; the effect of finite collisional duration cannot be neglected. In such a case, each scattering component cannot be treated separately and is affected by other scattering components. Therefore, the validity of Matthiessen’s rule must be re-examined from the viewpoint of the role played by the finite collisional duration in scattering processes. In this study, we investigate the physical basis of the separability of the scattering components in the inversion layers of MOSFETs beyond Matthiessen’s rule, by considering the Coulomb and phonon scattering as examples. By considering the effect of the finite collisional duration due to phonon scattering in the evaluation of Coulomb scattering-limited mobility, the separability of Coulomb and phonon scattering processes is examined on the basis of the relationship between the long-range nature of Coulomb potential and the finite collisional duration.

The paper is organized as follows. Section 2 shows how the finite collisional duration is considered in the evaluation of Coulomb scattering through the phonon mean free path,

while §3 explains the formulation of Coulomb scattering in the inversion layers of MOSFETs. Section 4 describes calculation results, and finally, §5 presents the conclusions.

### 2. Effect of Finite Collisional Duration on Scattering Rate

The consideration of the finite collisional duration due to phonon scattering in the evaluation of Coulomb scattering is the key in the study of the separability of Coulomb and phonon scattering processes. In this section, the formulation for considering the phonon lifetime (phonon mean free path) in the mobility limited by Coulomb scattering ( $\mu_{\text{rcs}}$ ) is briefly explained. We start with the equation of motion for the density matrix<sup>3,4)</sup>

$$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar} [H, \rho] \equiv iL\rho. \quad (1)$$

$H$  is the Hamiltonian of the system and is expressed as

$$H = H_0 + H_1, \quad (2)$$

where  $H_0$  and  $H_1$  are the unperturbed and perturbed Hamiltonians, respectively. In addition, the projection operator  $P$  for operator  $A$  is defined as

$$\langle l|PA|m\rangle = \langle l|A|m\rangle \delta_{lm}, \quad (3)$$

$$\langle l|P'A|m\rangle = \langle l|A|m\rangle (1 - \delta_{lm}). \quad (4)$$

Using the projection operator  $P$  thus defined, eq. (1) is transformed into the following equation when  $P\rho = \rho_d$

$$\begin{aligned} \frac{\partial \rho_d}{\partial t} = & PiL\rho + PiL \int_0^t d\tau e^{(t-\tau)P'iL} P'iLP\rho(\tau) \\ & + PiLe^{iP'iL} P'\rho_0. \end{aligned} \quad (5)$$

Using the relations

$$iL_0P = 0, \quad (6)$$

$$PiL_0 = 0, \quad (7)$$

$$PiL_1P = 0, \quad (8)$$

$$PiLP = 0, \quad (9)$$

for operators  $L_0$  and  $L_1$  defined as

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$$iL_0\rho = \frac{1}{i\hbar} [H_0, \rho], \tag{10}$$

$$iL_1\rho = \frac{1}{i\hbar} [H_1, \rho], \tag{11}$$

we obtain

$$\frac{\partial P\rho}{\partial t} = PiL_1 \int_0^t d\tau e^{(t-\tau)P'} iL_1 P\rho. \tag{12}$$

In deriving eq. (12), we assume

$$P'\rho_0 = 0. \tag{13}$$

This equation indicates that the nondiagonal part of the distribution function is neglected. If we treat  $H_1$  as a small perturbation, we obtain

$$\frac{\partial \rho_d}{\partial t} = PiL_1 \int_0^t d\tau e^{(t-\tau)iL_0} iL_1 \rho_d(\tau) \tag{14}$$

in the lowest order approximation. The right-hand side of eq. (14) can be rewritten as

$$-\frac{1}{\hbar^2} \int_{-t}^0 ds P[H_1(0)H_1(s)\rho_d(t+s) + \rho_d(t+s)H_1(s)H_1(0) - H_1(0)\rho_d(t+s)H_1(s) + H_1(s)\rho_d(t+s)H_1(0)]. \tag{15}$$

When we assume the Markov process in which the change in the form of distribution function during collision is neglected, the time dependence of  $\rho_d$  is assumed to be weaker than that of  $H_1(s)$ , and the following substitution holds:

$$\rho_d(t+s) \rightarrow \rho_d(t). \tag{16}$$

Moreover, if we express the Schrödinger equation for the unperturbed state as

$$H_0\phi_l = E_l\phi_l, \tag{17}$$

the matrix representation of eq. (15) becomes

$$-\sum_m \frac{1}{\hbar^2} \int_{-t}^0 ds (e^{-s(E_l-E_m)/i\hbar} + e^{s(E_l-E_m)/i\hbar}) | \langle l|H_1|m \rangle |^2 \times [ \langle l|\rho(t)|l \rangle - \langle m|\rho(t)|m \rangle ]. \tag{18}$$

When eq. (18) is integrated with respect to time, the well-known expression

$$\frac{2\hbar}{(E_l - E_m)} \sin\left(\frac{(E_l - E_m)t}{\hbar}\right) \tag{19}$$

is obtained. When the collision is assumed to decay faster in time, the lower limit in the integration range in eq. (18) can be made  $-\infty$  and the following substitution holds:

$$\frac{2}{\Delta E} \sin\left(\frac{\Delta E t}{\hbar}\right) \rightarrow 2\pi\delta(\Delta E). \tag{20}$$

Thus, the usual collision term of the Boltzmann equation is obtained.

However, the assumption such as eq. (20) does not hold anymore for a long-range interaction such as Coulomb interaction. For such a case, the finite lifetime during the collision is not neglected and we include the finite lifetime by introducing the decaying function in the uncertainty in the energy of the transition rate as

$$E_m - E_l \rightarrow E_m - E_l \pm i\Gamma, \tag{21}$$

where  $\Gamma$  is the decaying function. It should be noted here that the decay of eigenstates cannot be considered by the first principle based on eq. (12). To consider the decay of the eigenstates in the present level of transport theory, there is no way except for including the decaying function manually as in eq. (21). In this study, we define

$$\Gamma = \frac{\hbar}{\tau_{\text{phonon}}}, \tag{22}$$

since we consider the finite collisional duration due to phonon scattering, where  $\tau_{\text{phonon}}$  is the relaxation time due to phonon scattering. Thus, eq. (18) can be rewritten as

$$-\sum_m \frac{1}{\hbar^2} \int_{-t}^0 ds (e^{-s(E_l-E_m+i\Gamma)/i\hbar} + e^{s(E_l-E_m-i\Gamma)/i\hbar}) | \langle l|H_1|m \rangle |^2 \times [ \langle l|\rho(t)|l \rangle - \langle m|\rho(t)|m \rangle ]. \tag{23}$$

By integrating eq. (23) with respect to time and considering the limit  $t \rightarrow \infty$ , we obtain

$$\frac{\partial}{\partial t} \langle l|\rho(t)|l \rangle = -\sum_m \frac{2\Gamma}{\hbar} \frac{1}{(E_m - E_l)^2 + \Gamma^2} | \langle l|H_1|m \rangle |^2 \times [ \langle l|\rho(t)|l \rangle - \langle m|\rho(t)|m \rangle ]. \tag{24}$$

Thus, the effect of the finite collisional duration due to phonon scattering has been formulated in the form of spectral function.

It should be noted here that the actual correlation between Coulomb scattering and phonon scattering is more complex than that formulated above. Although we have considered only the effect of the finite collision duration due to phonon scattering on Coulomb scattering, the finite collision duration due to Coulomb scattering also affects the Coulomb scattering rate. Therefore, the effect of the finite collision duration due to Coulomb scattering as well as the finite collision duration due to phonon scattering must also be considered in the formulation. By considering these facts, the total scattering rate is considered to be determined by certain self-consistent relations. On the other hand, the present analysis corresponds to the first-order iterated calculation that considers only the effect of the finite collision duration due to phonon scattering on Coulomb scattering and ignores the complex correlations between Coulomb scattering and phonon scattering beyond this first-order calculation. Since our first-order iterated calculation is not formulated to consider self-consistently the effect of the finite collision duration due to both the phonon and Coulomb scattering processes on the same ground, our first-order iterated calculation may overestimate the effect of the finite collision duration when the effect of the finite collision duration due to Coulomb scattering is considered in the evaluation of the Coulomb scattering rate. A unified formulation, which enables us to consider the finite collision duration due to both the phonon and Coulomb scattering processes self-consistently, is required in confirming the validity of our first-order iterated calculation. Since we do not have such a unified theory, we have not considered the effect of the finite collision duration due to Coulomb scattering.

### 3. Formulation of Mobility Limited by Coulomb Scattering

In this section, the formulation of the mobility limited by Coulomb scattering,  $\mu_{rcs}$ , for the inversion layer of MOS-FETs is briefly explained.<sup>5)</sup> For substrate impurities with density  $N_A$ , Poisson's equation, which determines the potential distribution responsible for Coulomb scattering,  $\psi_{rcs}(\mathbf{r}, z)$ , is expressed as

$$\nabla[\epsilon(z)\nabla\psi_{rcs}(\mathbf{r}, z)] = e \left\{ \sum_i \delta(\mathbf{R} - \mathbf{R}_i)|_{Si} - N_A \right\} + \frac{2\epsilon_{si}}{e} \sum_{i,k} s_i^k \bar{\psi}_{rcs}^{ik}(\mathbf{r}) g_i^k(z), \quad (25)$$

where

$$\bar{\psi}_{rcs}^{ik}(\mathbf{r}) = \int dz \psi_{rcs}(\mathbf{r}, z) g_i^k(z), \quad (26)$$

and  $g_i^k(z)$  is the carrier density in the substrate of subband  $i$  at valley  $k$ .  $s_i^k$  is the screening parameter for the subband  $i$  in the valley  $k$  and is given as

$$s_i^k = q_d = \frac{e^2 N_i^k}{2\epsilon_{si} E_d^{ik}}, \quad (27)$$

where  $E_d^{ik} = kT(1 + e^{-x}) \ln(1 + e^{-x})$ ,  $x = (E_F - E_i^k)/kT$ , and  $N_{i,k}$  is the two-dimensional surface carrier concentration on the surface of the substrate and is expressed as

$$N_{i,k}(\mathbf{r}) = \frac{n_v^{ik} m_d^k}{\pi \hbar^2} F_0 \left( \frac{E_F - E_i^k - e\bar{\psi}_{rcs}^{ik}(\mathbf{r})}{k_B T} \right), \quad (28)$$

where  $E_F$  is the Fermi level and  $F_0(x) = \ln(1 + e^x)$ .  $N_s = \sum_{i,k} N_{i,k}$  is the surface carrier concentration and  $N_A$  is the impurity density in the substrate.  $\delta(\mathbf{R} - \mathbf{R}_i)|_{Si}$  indicates the  $i$ -th position of the impurities in the substrate. It should be noted here the reason of the use of the expression of  $\delta(\mathbf{R} - \mathbf{R}_i)|_{Si} - N_A$  instead of  $\delta(\mathbf{R} - \mathbf{R}_i)|_{Si}$ . The reason that we have expressed the charge density, which is responsible for Coulomb scattering, as  $\delta(\mathbf{R} - \mathbf{R}_i)|_{Si} - N_A$ , is that what actually scatters the electrons is the spatial variation of the charged centers in a plane parallel to the interface.<sup>6)</sup> In an ideal model in which charged centers are uniform charge sheets, the electrons in the inversion layer would not be scattered by charged centers.

We evaluate  $\mu_{rcs}$  on the basis of relaxation time approximation. By introducing the notation

$$\rho_{ext}^{Si}(\mathbf{R}) = \sum_i \delta(\mathbf{R} - \mathbf{R}_i)|_{Si} - N_A, \quad (29)$$

eq. (25) is rewritten as

$$\nabla[\epsilon(z)\nabla\psi_{rcs}(\mathbf{r}, z)] = e\rho_{ext}^{Si}(\mathbf{R}) + 2\epsilon_{si} \sum_{i,k} s_i^k \bar{\psi}_{rcs}^{ik}(\mathbf{r}) g_i^k(z). \quad (30)$$

By multiplying both sides of eq. (30) by  $\exp(i\mathbf{q} \cdot \mathbf{r})$  and integrating over  $\mathbf{r}$ , we obtain

$$\left( \frac{\partial}{\partial z} \epsilon(z) \frac{\partial}{\partial z} - \epsilon(z) q^2 \right) A(\mathbf{q}, z) = e\rho_{ext}^{Si}(\mathbf{q}, z) + 2\epsilon_{si} \sum_{i,k} s_i^k g_i^k(z) \int dz' A(\mathbf{q}, z') g_i^k(z'). \quad (31)$$

in which we have introduced the Fourier transformation

$$\rho_{ext}^{Si}(\mathbf{q}, z) = \int d\mathbf{r} \exp(i\mathbf{q} \cdot \mathbf{r}) \rho_{ext}^{Si}(\mathbf{r}, z),$$

$$A(\mathbf{q}, z) = \int d\mathbf{r} \exp(i\mathbf{q} \cdot \mathbf{r}) \psi_{rcs}(\mathbf{r}, z). \quad (32)$$

To solve eq. (31), we introduce the Green function  $G_q(z, z')$  that satisfies

$$\left( \frac{\partial}{\partial z} \epsilon(z) \frac{\partial}{\partial z} - \epsilon(z) q^2 \right) G_q(z, z') = \delta(z - z'). \quad (33)$$

Expressions for  $G_q(z, z')$  have been given by Fischetti.<sup>7)</sup> By multiplying both sides of eq. (31) by  $G_q(z, z')$  and integrating over  $z$ , we obtain

$$A(\mathbf{q}, z) = e \int dz' (\rho_{ext}^{Si}(\mathbf{q}, z')) G_q(z, z') + 2\epsilon_{si} \sum_{i,k} s_i^k A_i^k(\mathbf{q}) \int dz' G_q(z, z') g_i^k(z'), \quad (34)$$

where

$$A_i^k(\mathbf{q}) = \int dz A(\mathbf{q}, z) g_i^k(z). \quad (35)$$

By multiplying both sides of eq. (34) by  $g_j^l(z)$  and integrating over  $z$ , we obtain

$$A_j^l(\mathbf{q}) = e \int dz' \rho_{ext}^{Si}(\mathbf{q}, z') G_j^l(\mathbf{q}, z') + 2\epsilon_{si} \sum_{i,k} s_i^k A_i^k(\mathbf{q}) G_{ji}^{lk}(\mathbf{q}), \quad (36)$$

where

$$G_j^l(\mathbf{q}, z') = \int dz G_q(z, z') g_j^l(z),$$

$$G_{ji}^{lk}(\mathbf{q}) = \int dz g_j^l(z) \int dz' g_i^k(z') G_q(z, z'). \quad (37)$$

Using  $A_j^l(\mathbf{q})$ , the momentum relaxation time  $\tau_{rcs}^{jl}$  is expressed as

$$\frac{1}{\tau_{rcs}^{jl}(\epsilon, \theta)} = \frac{1}{2\pi\hbar} \int d\mathbf{k}' (1 - \cos\theta) \Gamma(\epsilon' - \epsilon) |eA_j^l(\mathbf{q})|, \quad (38)$$

where  $q = |\mathbf{k}' - \mathbf{k}|$  and  $\Gamma(\epsilon)$  is usually given by

$$\Gamma(\epsilon' - \epsilon) = \delta(\epsilon' - \epsilon). \quad (39)$$

Since the aim of the present study is to investigate the separability of Coulomb and phonon scattering processes beyond Matthiesen's rule, the effect of the finite collisional duration due to phonon scattering must be considered in the evaluation of  $\mu_{rcs}$ . In this study, we include the finite collisional duration due to phonon scattering in the evaluation of Coulomb scattering rate through the following spectral density function derived in the previous section

$$\Gamma(\epsilon' - \epsilon) = \frac{1}{\pi} \cdot \frac{\hbar}{\tau_{phonon}(\epsilon')} \cdot \frac{1}{(\epsilon' - \epsilon)^2 + \frac{\hbar^2}{\tau_{phonon}^2(\epsilon')}}. \quad (40)$$

Here,  $\tau_{phonon}$  is the relaxation time due to phonon scattering.<sup>8-10)</sup>

Under this assumption,  $\mu_{rcs}$  is separated into the contribution from subband 0,  $\mu_{rcs}^0$ , and the contribution from

subband  $0'$ ,  $\mu_{rcs}^{0'}$ , and is expressed as

$$\mu_{rcs} = \sum_{i=0,0'} N_i \mu_{rcs}^i, \quad (41)$$

where  $N_i$  is the occupancy for each subband  $i = 0, 0'$ .  $\mu_{rcs}^i$  is calculated from the relaxation time  $\tau_{rcs}^i$  as

$$\mu_{rcs}^i = \frac{e \langle \tau_{rcs}^i \rangle}{m_c^i}, \quad (42)$$

where  $m_c^i$  is the conductivity mass of subband  $i$  ( $= 0, 0'$ ). The average relaxation time  $\langle \tau_{rcs}^i \rangle$  is given by

$$\langle \tau_{rcs}^i \rangle = \frac{\int_{E_i}^{\infty} \tau_{rcs}^i(E) \frac{\partial f_0}{\partial E} (E - E_i) dE}{\int_{E_i}^{\infty} f_0 dE}, \quad (43)$$

where  $E_0$  and  $E_{0'}$  denote the subband energies for  $i = 0$  and  $i = 0'$ , respectively, and  $f_0$  denotes the Fermi distribution function. The momentum relaxation time  $\tau_{rcs}^i$  for subband  $i$  is given as:

$$\frac{1}{\tau_{rcs}^i(\epsilon)} = \int_0^{2\pi} \frac{1 - \cos \theta}{\tau_{rcs}^i(\epsilon, \theta)} d\theta, \quad (44)$$

where  $q = 2k \sin(\theta/2)$  and  $\tau_{rcs}^i(\epsilon, \theta)$  is obtained by solving eq. (38).  $\mu_{rcs}$  for charges located on the substrate surface is formulated in the same manner.

In this study, we assume only 2-subband( $0, 0'$ ) occupation, because the integration in eq. (38) is time-consuming. We assume the bulk MOS structure in the analysis. The phonon mean free path is calculated using the relaxation time of phonon scattering ( $\tau_{\text{phonon}}$ ), using the results of the subband calculation, such as the subband energy, the electron occupancy and the eigenfunctions in each subband. Twenty subbands have been considered in the subband calculation.

#### 4. Results

Figure 1 shows a comparison of the calculated  $\mu_{rcs}$ 's due to the scattering by charges located on the substrate surface with and without the phonon lifetime. The figure indicates that  $\mu_{rcs}$  is larger with the phonon lifetime than without the phonon lifetime in the low- $N_s$  region, whereas the difference

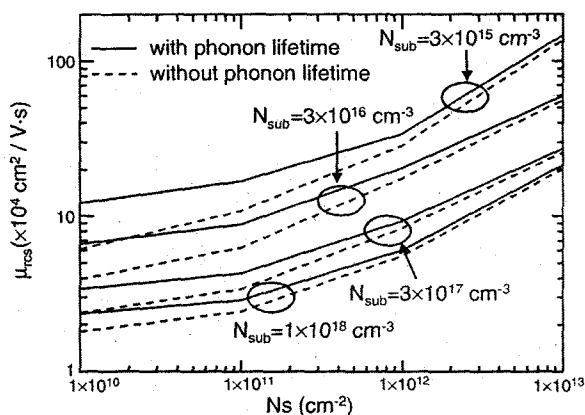


Fig. 1. Comparison of the calculated  $\mu_{rcs}$ 's for charges at the interface between substrate and gate oxide with and without the phonon lifetime as a function of surface carrier concentration,  $N_s$ .

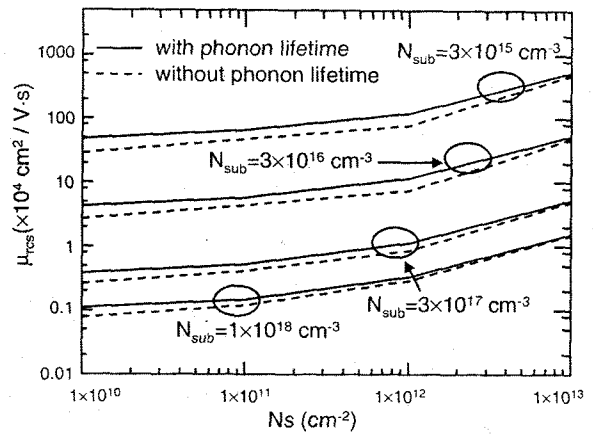


Fig. 2. Comparison of the calculated  $\mu_{rcs}$ 's for substrate impurities with and without the phonon lifetime as a function of surface carrier concentration,  $N_s$ .

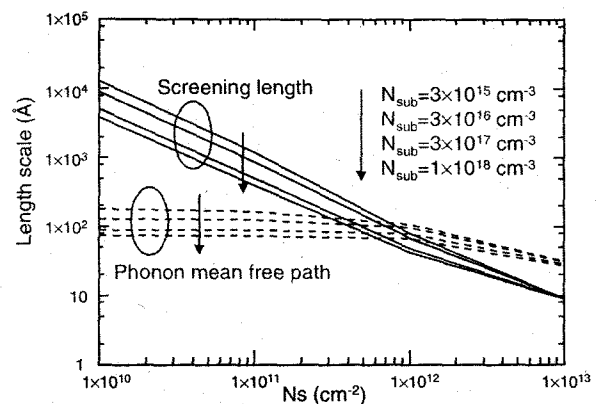


Fig. 3. Comparison of the screening length and the phonon mean free path for the lowest subband 0 as a parameter of the substrate impurity concentration.

between the two values is small in the high- $N_s$  region. The same behavior is also observed in Fig. 2, where  $\mu_{rcs}$  values due to scattering by the substrate impurities with and without the phonon lifetime are compared. To understand the behavior observed in Figs. 1 and 2, the two length scales in the system must be considered: the screening length due to carriers in the inversion layer and the phonon mean free path. The interrelationship between these two length scales determines the magnitude of  $\mu_{rcs}$ . Thus, the behavior observed in Figs. 1 and 2 is closely related to the  $N_s$  dependence of the screening length and the phonon mean free path and is explained as follows.

Figure 3 shows a comparison of the  $N_s$  dependences of the screening length due to carriers and the phonon mean free path for subband 0. This figure indicates that the screening length due to the free carriers in the inversion layer in the low- $N_s$  region is greater than the phonon mean free path. It should be noted here that a large screening length indicates a weak screening effect. From Fig. 3, it is found that the Coulomb potential in the low- $N_s$  region is long-ranged due to a large screening length. In this case, the effect of finite collisional duration becomes more significant, and as a result, the Coulomb potential in the low- $N_s$  region is

effectively cut off by phonon scattering. Therefore, the magnitude of  $\mu_{rcs}$  becomes larger than that without the finite collisional duration. This finding indicates that Coulomb scattering cannot be separated from phonon scattering in the low- $N_s$  region.

On the other hand, for the high- $N_s$  region, the screening length due to carriers becomes smaller than the phonon mean free path, and the effect of the finite collisional duration due to phonon scattering becomes weak. As a result, the effect of phonon scattering on Coulomb scattering becomes almost negligible, and the Coulomb and phonon scattering processes are separated properly in the high- $N_s$  region.

The  $N_{sub}$  dependence of the difference between  $\mu_{rcs}$  with and without the finite collisional duration by phonon scattering is also attributed to the dependence of the screening length due to carriers on  $N_{sub}$ . The screening length for the low- $N_{sub}$  region is greater than that for the high- $N_{sub}$  region. This finding indicates that the Coulomb potential has a greater interaction range in the low- $N_{sub}$  region than in the high- $N_{sub}$  region. Therefore, the effect of the finite collisional duration due to phonon scattering is greater in the low- $N_{sub}$  region than in the high- $N_{sub}$  region, and the effect of phonon scattering on Coulomb scattering is also enhanced in the low- $N_{sub}$  region.

On the basis of these observations, the condition that ensures proper separation between Coulomb and phonon scattering is expressed as

$$\frac{l_{scr}}{l_{mfp}} \leq 1, \quad (45)$$

where  $l_{scr}$  is the screening length due to the carriers in the inversion layer and  $l_{mfp}$  is the phonon mean free path. However, for the low- $N_s$  and low- $N_{sub}$  regions, eq. (45) does not hold because of the large screening length due to carriers. Thus, the separability of Coulomb and phonon scattering processes has been related to the two characteristic length scales: the screening length due to free carriers in the inversion layers and the phonon mean free path. Because the phonons effectively cut off the long-range component of the Coulomb potential, this component of the Coulomb potential does not affect the transport properties in the inversion layers through the collisional term in the Boltzmann equation. This behavior suggests that it is more appropriate to treat the long-range component of the Coulomb potential in the drift term than in the collisional term of the Boltzmann equation.<sup>11,12)</sup>

It should be noted here that in the region where a proper separation between the Coulomb and phonon scattering processes is not obtained (at the lower ends of the low- $N_{sub}$  and low- $N_s$  regions), the extraction of only  $\mu_{rcs}$  is not possible in the strict sense. Since actual correlations between

the characteristic length scales may be more complex than those indicated in this study, a unified theory is required in formulating a scattering model under the condition in which a proper separation between the Coulomb and phonon scattering processes is not obtained. Although the details of the interrelationships between the separability of scattering components and the characteristic length scales in the system require further investigation,<sup>13)</sup> the present study provides certain insight into this issue.

## 5. Conclusions

The physical basis of the separability of Coulomb and phonon scattering processes was quantitatively examined by investigating the effect of the finite collisional duration due to phonon scattering on Coulomb scattering in MOS inversion layers. It was found that the condition under which Coulomb scattering is separated from phonon scattering is determined using the relationship between the screening length due to the free carriers in the inversion layer and the phonon mean free path. In the region where the screening length due to the free carriers in the inversion layer is greater than the phonon mean free path, the phonons effectively cut off the long-range component of the Coulomb potential. On the basis of this finding, it was also found that a proper separation between the Coulomb and phonon scattering processes is not obtained at the lower ends of the low- $N_{sub}$  and low- $N_s$  regions.

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