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Inverse bremsstrahlung in strong radiation fields—the Born approximation re-examined

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Abstract. The calculation of the energy absorption in a strong radiation field where the electron quiver energy is much larger than the thermal or photon energies is performed using the Born approximation for the collision. The multiphoton sums are evaluated with the aid of asymptotic forms to yield analytic expressions. Comparison with earlier results from the impact (or classical) model shows disagreements. The origin to this difference is ascribed to the breakdown of perturbation theory in this limit, and it is concluded that the impact approximation is more reliable at very high fields.

1. Introduction

The advent of ultra-short pulse high-power lasers has opened the possibility of generating plasma with very low residual energy through ATI multiphoton ionization. The excess heating through collisions at high densities has re-awakened interest in the well worked field of high-field inverse bremsstrahlung absorption.

This is a topic which has been addressed by a large number of workers since Silin (1965) first identified the well known reduction in the absorption rate associated with the change in the Coulomb cross section due to the electron quiver velocity during electron–ion collisions. Since the problem was, at that time, associated with relatively long-pulse lasers it was difficult to envisage situations in which the quiver velocity greatly exceeded the thermal speed, and it was eventually pointed out by Langdon (1980) that under these conditions, the principal effect was a modification of the electron distribution function away from Maxwellian to give a reduction in absorption, typically by a factor $\sim$0.5.

With the advent of ultra-short pulse lasers and cold plasma, the situation is changed and the early work again becomes relevant. Since the pulse lengths are typically less than an electron–ion collision time, the electrons remain cold throughout the pulse, and the distribution function changes are unimportant. The strong-field limits on the calculation of the inverse bremsstrahlung rate have been examined in two recent papers (Polishchuk and Meyer-ter-Vehn 1994, Pert 1995). Since there is a slight difference in the results (although for practical purposes probably not serious) we will examine the inherent validity of the calculations, and reproduce and extend Polishchuk and Meyer-ter-Vehn's (1994) result.

The calculations in this paper are performed for the standard inverse bremsstrahlung problem in which a single electron is scattered by a pure Coulomb field in the presence of an electromagnetic field. We do not, therefore, consider the role of any remaining bound states, which will be dressed by the field; this is probably not a severe restriction since only tightly bound electrons will survive multiphoton ionization. The effect of distant

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perturbers in a dilute plasma, when correlations can be neglected, was considered earlier (Pert 1995) and is not appropriate to the present study. Finally, we will treat only the local absorption; the heating in an experimental situation involving the temporal and spatial structure of the laser pulse must be dealt with by a fuller simulation (Janulewicz *et al* 1996). The calculations are all performed within the non-relativistic dipole approximation. At the highest fields these conditions may no longer be valid.

2. Inverse bremsstrahlung approximations

The general theory of collision absorption in arbitrary fields has not yet been developed, although a basic formalism was identified by Kroll and Watson (1973). Rather, the problem has been tackled by various approximations, valid under appropriate conditions as a set of overlapping limit solutions which span nearly the entire parameter space. These are identified by limits defined by three characteristic energies: $\hbar\omega$, the photon energy; $\frac{1}{2}mv^2$, the 'thermal' energy; and $\frac{1}{2}mu^2$, the quiver energy. There exist essentially three distinct approximations, within which many different calculations fall.

(i) *Low field.* The general single-photon quantum mechanical theory of Sommerfeld (1939), developed by Elwert (1948) gives a complete description in the case $\frac{1}{2}mu^2 \ll \hbar\omega$. The classical description of bremsstrahlung emission may be extended to absorption by the use of the principle of detailed balance. Although valid provided $\frac{1}{2}mu^2 \ll \frac{1}{2}mv^2$, and therefore for multiphoton absorption, it yields the same result as the single-photon theory in the classical limit. A useful review of this case is given by Oster (1961). The results are usually expressed in terms of a correction, the Gaunt factor, similar to the collisional Coulomb logarithm.

(ii) *Straight line path.* The general straight line path approximation requires that the trajectory of the electron associated with the thermal motion be essential linear, i.e. that the perturbation induced by the collision including absorption makes only a small change to the thermal velocity. In this case also there are two approaches depending on whether the electron is treated quantum mechanically or classically. The quantum mechanical analysis is the Born approximation, and was originally formulated by Bunkin and Federov (1966). The classical formulation can be treated via a collective (plasma) description (Dawson and Oberman 1962, Silin 1965) or single-particle picture (Pert 1979). The model is valid if $\frac{1}{2}mu^2 \lesssim \frac{1}{2}mv^2$.

(iii) *Impact (classical) approximation.* In this approximation the photon energy is small, so that the electron motion may be treated as a vector sum of the thermal velocity $\boldsymbol{v}$ and the instantaneous quiver velocity $\boldsymbol{u}$ with a precisely defined phase. Only collisions completed within a short time compared to the period of the field are considered to give rise to a change in the thermal velocity (impact approximation) (Pert 1972, Bunkin *et al* 1973). The collision is controlled by the total electron velocity $(\boldsymbol{u} + \boldsymbol{v})$, which limits the impact time, and also establishes whether the collision itself can be treated by the classical or Born limits for elastic scattering. This case is valid if $\hbar\omega \ll \frac{1}{2}mu^2$ or $\frac{1}{2}mv^2$.

Calculations in the high-field limit, $\frac{1}{2}mu^2 \gg \frac{1}{2}mv^2$, have nearly all been performed using one of the straight line path approximations, the Born approximation being particularly popular. The classical approach has the disadvantage of requiring an imposed cut-off to take account of large angle scatter with impact parameter approximately equal to the Landau parameter. However, our previous discussion indicates that these results may be in error.

Calculations with the impact approximation are limited, but some general results have been obtained by the author (Pert 1995). In this paper we will examine their relationship with straight line path calculations.

3. The Born approximation

The Born approximation for multiphoton inverse bremsstrahlung was developed by Bunkin and Federov (1966) and subsequently by numerous workers. The Born approximation for electron scattering in an electromagnetic field (multiphoton inverse bremsstrahlung) was developed by Bunkin and Federov (1966) and subsequently by numerous workers. These calculations are performed in direct analogy with the standard Born approximation for zero EM field elastic scattering by replacing the plane-wave free-electron wavefunction by the equivalent exact solution in the electromagnetic field (Volkov wavefunction). The free electron is therefore fully dressed by the field. The wavefunction is parametrized by its 'thermal' momentum, and the validity of the perturbation approximation requires that the change in this term be small. The essential result for the photon cross section for absorption (+) or stimulated emission (−) of n photons in an electron–ion collision may be written as

$$\sigma_{\pm n} = \frac{2[4\pi\hbar^2 Ze^2]^2}{\pi\hbar^4 cE^2}\rho n\hbar\omega \int \mathrm{d}\boldsymbol{q}\, \frac{1}{q^4} J_n^2\left[\frac{e\boldsymbol{E}}{m\hbar\omega^2}\cdot\boldsymbol{q}\right]\delta(\varepsilon' - \varepsilon \mp n\hbar\omega) \tag{1}$$

where e and m are the electronic charge and mass, respectively, Z the ionic charge number, c the velocity of light, E the peak electric field intensity in the wave and ω the angular frequency of the wave: ε and ε' are the electron energies before and after collision, $\boldsymbol{q}$ the momentum transfer vector and ρ is the ion number density.

For large values of the parameter $\gamma = eEv/\hbar\omega^2$, where v is the incoming electron speed, the Bessel function $J_n(z)$ is dominated by its asymptotic form, which gives a prescription first used correctly by Elyutin (1974):

$$J_n(z) \simeq \begin{cases} \sqrt{2/\pi z}\cos\left[z - \left(n + \frac{1}{2}\right)\pi\right] & z > n \\ 0 & z < n \end{cases} \tag{2}$$

for handling this term. Hence averaging over the direction of the incoming electron motion for an isotropic distribution, or equivalently over the direction of the field, we obtain

$$\sigma_{\pm n} = \frac{2^6\pi Z^2 e^3\hbar^2\omega^3\rho n}{cE^3m^2v^4}\int \mathrm{d}Q\,\frac{1}{Q^4}\ln[\gamma Q/n]\,. \tag{3}$$

The range of the integral over $Q = q/mv$ is determined by the range allowed by the δ function during the angular integration consistent with $\gamma Q > n$ from (2). Thus we have:

$$\begin{aligned} &\text{absorption}: \quad \lambda = \sqrt{1 + 2n\hbar\omega/mv^2} > 1 \qquad (\lambda - 1) \leqslant Q \leqslant (\lambda + 1) \\ &\text{emission}: \quad \lambda = \sqrt{1 + 2n\hbar\omega/mv^2} < 1 \qquad (1 - \lambda) \leqslant Q \leqslant (1 + \lambda)\,. \end{aligned} \tag{4}$$

In absorption the range of n, over which contributions to the total energy gain are generated, is limited by the condition above to $n \lesssim 2e^2E^2/m\omega^2\hbar\omega = 4\frac{1}{2}mu^2/\hbar\omega$, i.e. the quiver energy limits the total energy gain. In emission $n < \frac{1}{2}mv^2/\hbar\omega$, i.e. the kinetic energy limits the maximum energy loss.

The integral (3) may be simply evaluated, and the total energy absorbed and emitted calculated by evaluating the corresponding cross section sums numerically. The approximations for the absorption and emission cross section, respectively, are

$$A_n = \frac{2^5\pi Z^2 e^3 \rho}{cE^3}\frac{1}{2n^2\xi}\begin{cases}\left(\lambda^2+\frac{1}{3}\right)\left[\frac{1}{3}+\ln(\gamma\sqrt{2\xi/n})\right]-1/2\lambda(\lambda^2/3+1)\ln\left[\frac{(\lambda+1)}{(\lambda-1)}\right] \\ \qquad\qquad 1\leqslant n\leqslant 2\gamma(\gamma\xi-1) \\ (\lambda-1)^3/18[\{[\gamma/n(\lambda+1)^3]-1\}-3\ln\{\gamma(\lambda+1)/n\}] \\ \qquad\qquad 2\gamma(\gamma\xi-1)\leqslant n\leqslant 2\gamma(\gamma\xi+1) \\ 0 \qquad\qquad n>2\gamma(\gamma\xi+1)\end{cases} \tag{5}$$

and

$$E_n = \frac{2^5\pi Z^2 e^3 p}{cE^3}\frac{1}{2n^3\xi}\begin{cases}(\lambda^2/3+1)\left[\frac{1}{3}+\ln(\gamma\sqrt{2\xi/n})\right]-1/2\lambda(\lambda^2+1/3)\ln\left[\frac{(1+\lambda)}{(1-\lambda)}\right] \\ \qquad\qquad 1\leqslant n\leqslant n' \\ (1-\lambda)^3/18[\{[\gamma/n(1+\lambda)\}^3-1]-3\ln\{\gamma(1+\lambda)/n\}] \\ \qquad\qquad n'\leqslant n\leqslant n'' \\ 0 \qquad\qquad n>n''\end{cases} \tag{6}$$

where

$$\begin{aligned}&n' = (1 \text{ if } \gamma > 1/\xi) \text{ or } (2\gamma(1-\gamma\xi) \text{ if } \xi/2\leqslant\gamma\leqslant 1/\xi) \quad\text{or}\quad (\xi/2 \text{ otherwise}).\\ &n'' = \left(2\gamma(1-\gamma\xi) \text{ if } \gamma\xi<\tfrac{1}{2}\right) \quad\text{or}\quad (\xi/2 \text{ otherwise})\end{aligned} \tag{7}$$

and $\xi = \hbar\omega/mv^2$.

We note the upper absorption cut-off on n is at $\sim 2\gamma^2\xi \sim 2mu_0^2/\hbar\omega$, i.e. the classical cut-off.

If $\hbar\omega$ is small the number of terms involved is large. In this case we may approximately evaluate the sums by replacing them by integrals which we may, with some labour, calculate. In doing this we observe that the terms A_n and E_n are largest for small n where the error associated with the replacement is greatest. However, since the net absorption is formed by the difference of $(A_n - E_n)$, which are nearly equal when n is small, this procedure is quite accurate for the net term. Thus the total absorption cross section:

$$A = \frac{2^6\pi Z^2 e^3\hbar^2\omega^3\rho}{cE^3m^2v^4}\int_0^\infty \frac{\mathrm{d}Q}{Q^4}\int_{\max[(Q^2-2Q)/2\xi,\frac{1}{2})}^{\min[(Q^2+2Q)/2\xi,\gamma Q]} \mathrm{d}n\, n\ln(\gamma Q/n) \tag{8}$$

with the emission cross section

$$E = \frac{2^6\pi Z^2 e^3\hbar^2\omega^3\rho}{cE^3m^2v^4}\int_0^\infty \frac{\mathrm{d}Q}{Q^4}\int_{\frac{1}{2}}^{\min[(2Q-Q^2)/2\xi,\frac{1}{2}\xi]} \mathrm{d}n\, n\ln(\gamma Q/n)\,. \tag{9}$$

These integrals can be evaluated analytically within different regimes depending on the relationships between the terms in the limits. Thus for absorption the high-field regime, $\gamma > [\sqrt{(1+\xi)}+1]/2\xi$:

$$\begin{aligned}A = \frac{2^4\pi Z^2 e^3\omega\rho}{cE^3}\Bigg\{&2\ln(\gamma\xi)\ln[4(\gamma\xi-1)/\xi] - 2[\mathrm{dilog}(\gamma\xi+1)-\mathrm{dilog}(\gamma\xi)]\\ &+2\ln\left[\tfrac{1}{2}\{\sqrt{(1+\xi)+1}\}\right]\ln\left[\tfrac{1}{2}\{\sqrt{(1+\xi)-1}\}\right]+\tfrac{1}{2}(1+4/\xi)\ln(4\gamma^2\xi)\\ &+\tfrac{1}{2}(1-4/\xi)\{\sqrt{(1+\xi)}\ln\left\{\frac{[\sqrt{(1+\xi)}+1]}{[\sqrt{(1+\xi)}-1]}\right\}-1\right\}\\ &-\frac{1}{6\xi}\left\{(4+3\xi)\left[\ln(4\gamma^2\xi)+\tfrac{5}{3}\right]-\sqrt{(1+\xi)(4+\xi)}\ln\left[\frac{\{\sqrt{(1+\xi)}+1\}}{\{\sqrt{(1+\xi)}-1\}}\right]\right\}.\end{aligned} \tag{10}$$

At lower fields $[\sqrt{(1+\xi)-1}]/2\xi < \gamma < [\sqrt{(1+\zeta)+1}]/2\xi$

$$A = \frac{2^4\pi Z^2 e^3\omega\rho}{cE^3}\left\{\frac{8}{9}\gamma^3\xi^2 - 2\gamma\xi + \frac{1}{4}\left[\left(3-\frac{4}{\xi}\right)\sqrt{(1+\xi)} - \left(1-\frac{4}{\xi}\right)\right]\right.$$
$$-2\big[\mathrm{dilog}(\gamma\xi+1) - \mathrm{dilog}\big\{\tfrac{1}{2}[\sqrt{(1+\xi)+1}]\big\}\big]$$
$$-2\ln\big\{\tfrac{1}{2}[\sqrt{(1+\xi)+1}]\big\}\ln\big[\gamma\xi\big\{\tfrac{1}{2}[\sqrt{(1+\xi)-1}]\big\}\big]$$
$$+\tfrac{1}{2}[(1+4/\xi)+(1-4/\xi)\sqrt{(1+\xi)}]\ln\big[\gamma\xi/\big\{\tfrac{1}{2}[\sqrt{(1+\xi)-1}]\big\}$$
$$\left.+\frac{1}{6\xi}[\{(4+\xi)\sqrt{(1+\xi)}-(4+3\xi)\}\{\ln[2\xi/\sqrt{(1+\xi)+1}\}]+\frac{5}{6}\right\}. \tag{11}$$

This result is prone to increasing error towards its lower bound as $\gamma \to 1$ as the use of the asymptotic form (2), and the omitted terms become important.

The emission integrals are similarly evaluated. Thus at high fields $\gamma > [1 + \sqrt{(1+\xi)}]/2\xi$:

$$E = \frac{2^4\pi Z^2 e^3\omega\rho}{cE^3}\left\{\frac{1}{2}\sqrt{(1-\xi)}\left[\left(1+\frac{4}{\xi}\right)\{\ln(4\gamma^2\xi)+1\}+2\right]\right.$$
$$+2\big[\mathrm{dilog}\big\{\tfrac{1}{2}\big[1+\sqrt{(1+\xi)}\big]\big\} - \mathrm{dilog}\big\{\tfrac{1}{2}\big[1-\sqrt{(1-\xi)}\big]\big\}\big]$$
$$-\left[\ln(\gamma^2\xi^2)-\frac{1}{2}+\frac{2}{\xi}\right]\ln\left[\frac{\{1+\sqrt{(1-\xi)}\}}{\{1-\sqrt{(1-\xi)}\}}\right]$$
$$\left.-\frac{1}{6\xi}\left\{(4-\xi)\sqrt{(1-\xi)}\big[\ln(4\gamma^2\xi)+\tfrac{5}{3}\big]-(4-3\xi)\ln\left[\frac{\{1+\sqrt{(1-\xi)}\}}{\{1-\sqrt{(1-\xi)}\}}\right]\right\}\right\} \tag{12}$$

and at lower fields: $[1-\sqrt{(1-\xi)}]/2\xi < \gamma < [1+\sqrt{(1-\xi)}]/2\xi$

$$E = \frac{2^4\pi Z^2 e^3\omega\rho}{cE^3}\left\{\frac{8}{9}\gamma^3\xi^2 + 2\gamma\xi + \frac{1}{4}\left[\left(3+\frac{4}{\xi}\right)\sqrt{(1-\xi)} - \left(1+\frac{4}{\xi}\right)\right]\right.$$
$$+2\big[\mathrm{dilog}(\gamma\xi) - \mathrm{dilog}\big\{\tfrac{1}{2}[1-\sqrt{(1-\xi)}]\big\}\big]$$
$$+2\ln(\gamma\xi)\ln\big\{(1-\gamma\xi)/\big[\tfrac{1}{2}\{1+\sqrt{(1-\xi)}\}\big]\big\}$$
$$+\frac{1}{2}\left[\left(1-\frac{4}{\xi}\right)+\left(1+\frac{4}{\xi}\right)\sqrt{(1-\xi)}\right]\ln\big[\gamma\xi/\big\{\tfrac{1}{2}[1-\sqrt{(1-\xi)}]\big\}\big]$$
$$\left.+\frac{1}{6\xi}[(4-3\xi)-(4-\xi)\sqrt{(1-\xi)}]\ln\big[2\gamma\big\{1+\sqrt{(1-\xi)}\big\}\big]+\frac{5}{6}\right\}. \tag{13}$$

This result is subject to error as $\gamma \to 1$. We note that emission can only take place if $\xi < 1$ (more accurately $\xi < \frac{1}{2}$, but we used $n = \frac{1}{2}$ as the lower limit to the integral).

4. High-field limit

The net overall absorption is given by the difference of these two terms. Thus if we consider the case of small photon energy $\xi \ll 1$ in a high field so that we retain only the leading terms in γ and $(\gamma\xi)$ we obtain for the net absorption rate per electron:

$$R = \frac{2^4\pi Z^2 e^3\omega\rho}{cE^3}\frac{cE^2}{8\pi}\{2\ln(\gamma\xi)\ln(4\gamma) - \ln(\gamma^2\xi^2)\ln(\xi/4)\}\,. \tag{14}$$

Similarly, for large photon energy $\xi \gg 1$, $E = 0$ and

$$R = \frac{2^4 \pi Z^2 e^3 \omega \rho}{c E^3} \frac{c E^2}{8\pi} 2\left\{\ln(\gamma\xi)\ln(4\gamma) + \left[\ln\left(\tfrac{1}{2}\sqrt{\xi}\right)\right]^2\right\}. \tag{15}$$

To compare these results with previous work it is convenient to introduce some new terms:

$$x = \frac{mu^2}{mv^2} = \gamma^2\xi^2 \qquad y = \frac{mv^2}{\hbar\omega} = \xi^{-1} \qquad z = \frac{mu^2}{\hbar\omega} = \gamma^2\xi \tag{16}$$

and

$$R = \frac{4Z^2 e^3 \omega\rho}{E} S\,. \tag{17}$$

Thus

$$S = \begin{cases} \ln(4y)\ln(x) + \frac{1}{4}[\ln(x)]^2 & y \gg 1 \\ \frac{1}{4}[\ln(4z)]^2 & y \ll 1\,. \end{cases} \tag{18}$$

These results are identical to those obtained by Polishchuk and Meyer-ter-Vehn if the thermal velocity is taken to be $\sqrt{2kT/m}$.

However, if we use the impact approximation we obtain (Pert 1995)

$$S = \begin{cases} \ln(2y)\ln(4x) + \frac{1}{2}[\ln(4x)]^2 & y \gg 1 \\ \frac{1}{2}[\ln(2z)]^2 & y \ll 1 \end{cases} \tag{19}$$

and we note the difference in multiplying factor from $\frac{1}{4}$ to $\frac{1}{2}$.

In order to understand this difference more clearly we must re-examine the relationship between the Born approximation and the impact approximation in detail. In an earlier paper (Pert 1975) we showed that the impact approximation was an asymptotic limit of the Born approximation in the limit that the number of photons absorbed (n) was large. In that work we argued that a cut-off had to be imposed determined by the total velocity. If this is done the analysis given in Pert (1976) shows that the result is (19). However, a more careful re-analysis (see the appendix) shows that the cut-off should be determined by the thermal velocity alone, in which case (18) results, demonstrating consistency of the methods.

So which result is correct? Since the Born approximation is a straight line path approximation, this regime lies outside its range of validity. The nature of the discrepancy arising from the use of the thermal, rather than the total, velocity is easily understood, since within its range the two are nearly identical. Thus in the case of $y \gg 1$, small photon energy, the impact approximation is correct.

The question now arises as to whether we may also use the impact approximation in the opposite limit of low temperature ($y \ll 1$). In our earlier work (Pert 1995) we recommended the use of the Born result in this limit. As we have now argued this is incorrect. In fact the impact approximation requires that the phase be well defined, or equivalently that there is an energy uncertainty $\sim \hbar\omega$. Thus if the number of photons absorbed is large, i.e. $n \gg 1$, the impact approximation should still give an accurate result. Thus equation (19) is valid throughout its range.

Similar considerations arise with the classical straight line path analysis. Thus if we carefully re-examine the outer cut-off limit in Pert (1979) we again find it should be determined by the thermal velocity, not the total as we earlier inferred.

5. Medium power absorption

We now question whether these results have any useful value. Clearly the perturbation approach can only be used if the absorbed energy is sufficiently small not to greatly perturb the thermal motion. In the limit of small photon energy (n large) this requires the quiver energy to be less than the thermal, i.e. $x = \gamma^2\xi^2 \lesssim 1$. Thus the results (5), (6) and (11), (13) may be quite widely used in the regime $x \lesssim 1$, $y \gg 1$ when the electron motion is essentially quantum mechanical. This is a quite useful practical result, as accurate values in this regime are not obtainable from the other methods, but the condition is important for some practical applications.

In the important case where $\xi \ll 1 (y \gg 1)$ equations (11) and (13) simplify to

$$A = \frac{2^4\pi Z^2 e^3 \omega\rho}{cE^3}\left\{\tfrac{8}{9}\gamma^3\xi^2 - 2[\mathrm{dilog}(1+\gamma\xi) + \gamma\xi]\right\} \tag{20}$$

and

$$E = \frac{2^4\pi Z^2 e^3 \omega\rho}{cE^3}\left\{\tfrac{8}{9}\gamma^3\xi^2 - 2[\mathrm{dilog}(1-\gamma\xi) - \gamma\xi]\right\} \tag{21}$$

and the net absorption rate:

$$R \simeq \frac{2^5\pi Z^2 e^3 \omega\rho}{cE^3}\frac{cE^2}{8\pi}\left[\mathrm{dilog}(1-\gamma\xi) - \mathrm{dilog}(1+\gamma\xi) - 2\gamma\xi\right] \tag{22}$$

$$= \frac{2^6\pi Z^2 e^6 \rho}{\mathrm{cm}^3 v^3\omega^2}\frac{cE^2}{8\pi}\sum_{k=0}\frac{x^k}{(2k+3)^2}\,. \tag{23}$$

This result is applicable if $\gamma \gg 1, \xi \ll 1$ in the region $\xi/4 \ll \gamma\xi < 1$, so $y \gg 1$, $y^{-1/2} \ll x < 1$. Although the result (23) is expressed as a power series in terms of the field intensity, it should not be interpreted in terms of a multiphoton expansion, as is easily seen by comparison with equations (5) and (6).

The result is applicable to the case where the electron scattering is non-classical. An equivalent result for classical electrons can be derived from the analysis in Pert (1979), but the impact approximation (Pert 1995) is satisfactory in this regime.

6. Conclusion

In this paper we have re-examined the use of the Born approximation for calculating the collisional absorption coefficient in high-radiation fields. It is found that there is a small, but non-negligible, difference between the results obtained from this approximation, and those from the impact model. It is believed that this stems from the use of perturbation theory under conditions in which the collision can no longer be considered to be a small effect. A similar problem arises in the use of the classical straight line path approaches.

In detail the difference can be tied down to the cut-offs used in the impact model. In earlier work (Pert 1975, 1979) these were identified in perturbation calculations to be determined by the total velocity. A more careful analysis shows that this is unsatisfactory, and that the limit is determined by the thermal speed alone. In this case the Born calculation is consistent with the impact model.

This raises the question as to whether the cut-off determined by the total velocity is actually correct in the impact model. In the past the argument based on the asymptotic limit of the straight line path theories has been used as a support. However, as we point out above, this is no longer tenable. We must therefore re-examine the question from the consistency of the model itself. The familiar cut-off at an impact parameter v/ω is determined by the fact that within the kinematics of an elastic collision, an overall change in the thermal velocity can only occur if the collision is completed without significant change in the phase of the electromagnetic wave. If a substantial phase change occurs during the collision, the quiver oscillation is averaged out and no net energy occurs. The limiting parameter is clearly set by an individual collision at a well defined phase. In this classical picture the collision is controlled by the total velocity, not the average. The results of our earlier study (Pert 1995) remain valid.

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Appendix

The asymptotic form of the Born approximation was investigated using saddle-point methods to generate the impact approximation (Pert 1975). In that paper it was concluded that a logarithmic cut-off was required when the momentum transfer $\boldsymbol{q}$ satisfied two limits, whereas in fact the following is correct:

$$\boldsymbol{M}\cdot\boldsymbol{q} > 2\boldsymbol{p}\cdot\boldsymbol{q} + q^2 \gg m\hbar\omega$$

where $\boldsymbol{p} = m\boldsymbol{v}$ and $\boldsymbol{M} = m\boldsymbol{u}_0$. On this basis it was suggested that the cut-off should be $q' = \gamma m\hbar\omega/\Pi$ where $\boldsymbol{\Pi} = \boldsymbol{p} + \boldsymbol{M}$, i.e. determined by the 'total' momentum, whereas the correct value is $q' = \gamma m\hbar\omega/p$.

A similar analysis also applies in the classical case (Pert 1979).

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