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30 January 1995

PHYSICS LETTERS A

Physics Letters A 197 (1995) 365,366

Comments on "Stochastic resonance in a periodic potential system under a constant driving force"

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Received 28 February 1994; revised manuscript received 5 July 1994; accepted for publication 14 November 1994
Communicated by A.R. Bishop

Abstract

We discuss the recently suggested possibility of observing stochastic resonance (SR) effects in the long time response of a stochastic system moving in a periodic potential under the action of a weak constant force [G. Hu, Phys. Lett. A 174 (1993) 247] and we show that SR effects do not exist within the Langevin description of the dynamics.

The term stochastic resonance (SR) was coined some years ago in order to describe a phenomenology associated with the cooperative effects associated with weak time-sinusoidal forcing terms and noise for a variety of model systems involving motion on bistable potentials [1]. Recently Hu has proposed a new class of systems that seem to be good candidates to exhibit SR effects without resorting to time-dependent forcing terms [2]. In those systems a single variable moves along a periodic potential and is subject both to a constant force and to a white noise term. In his analysis the Langevin description of the dynamics is replaced under certain limiting conditions by a master equation (ME) approach.

The main purpose of this comment is to point out that within the Langevin description of the model, the stationary response has a monotonic behaviour with the noise strength, so that the effect predicted by Hu does not exist.

Let us consider the Langevin equation,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\sin(x) + b + \xi(t) , \qquad (1)$$

where $\xi(t)$ is a Gaussian white noise with zero mean and correlation given by $\langle \xi(t)\xi(s) \rangle = 2\theta\delta(t-s)$. The noise averaged velocity in the stationary regime is given by Risken [3]. In the limit $b \ll \theta$, this exact expression simplifies to

$$\langle v \rangle = b[I_0(1/\theta)]^{-2} , \qquad (2)$$

where $I_0(y)$ is the zeroth order modified Bessel function of the first kind. For very small θ, the asymptotic form of $I_0(y)$ allows us to obtain the expression

$$\langle v \rangle = \frac{2\pi b}{\theta} \mathrm{e}^{-2/\theta} , \qquad (3)$$

valid for $b \to 0$, $\theta \to 0$ and $b \ll \theta$. In Fig. 1, we plot the behavior of $\langle v \rangle / b$ with θ as given by Eqs. (2) and (3). Clearly, both curves coincide in the small θ region. On the other hand, the exact (for $b \to 0$) formula, Eq. (2), shows a monotonic behavior with θ, thus negating any stochastic resonance effect in the mobility when the system is described in terms of a Langevin dynamics. The approximate expression, Eq. (3), predicts a maximum of the mobility at $\theta = 2$, but

SSDI 0375-9601(94)00948-1

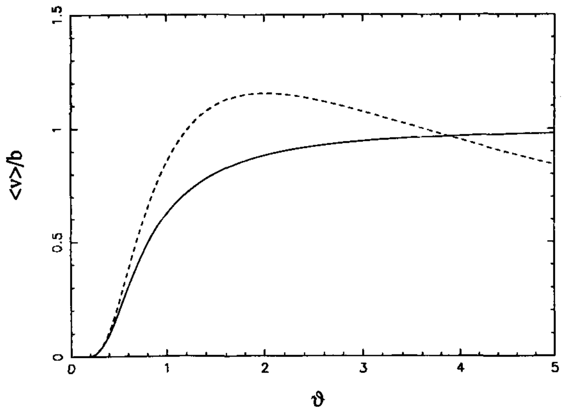

Fig. 1. Comparison between the exact result for the stationary drift velocity in the Langevin description in the $b\to 0$ limit and its asymptotic evaluation in the small-θ region. Full line: exact expression in this limit as a function of θ; broken line: asymptotic small-θ expression.

this value is much too large for the asymptotic formula to be valid.

Hu replaces the Langevin equation by a ME (Eq. (6) of Ref. [2]) describing an asymmetric random walk on an infinite lattice. The drift velocity obtained from this ME is given by

$$\langle v\rangle = 2\mathrm{e}^{-2/\theta}\sinh(b\pi/\theta)\ , \qquad (4)$$

which, in the limit $b \ll \theta$, can be approximated by the expression given in Eq. (3). Clearly, Eqs. (2) and (4), both valid for $b \ll \theta$, are different. The ME dynamics cannot replace the Langevin dynamics without further restrictions on the parameters. Namely, both descriptions coincide when $b\to 0$, $\theta\to 0$ and $b \ll \theta$. But, then, the peak in the velocity predicted by Eq. (3) takes place for a value of θ for which this equation is not valid.

The authors acknowledge the Dirección General de Investigación Científica y Técnica (DGICYT) of Spain (Project No. PS92-0682) for support.

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