

Quantum Information Processing with Semiconductor Macroatoms

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(Received 24 July 2000; revised manuscript received 14 September 2000)

An *all optical* implementation of quantum information processing with semiconductor macroatoms is proposed. Our quantum hardware consists of an array of quantum dots and the computational degrees of freedom are energy-selected interband optical transitions. The quantum-computing strategy exploits exciton-exciton interactions driven by ultrafast multicolor laser pulses. Contrary to existing proposals based on charge excitations, our approach does not require time-dependent electric fields, thus allowing for a subpicosecond, decoherence-free, operation time scale in realistic semiconductor nanostructures.

PACS numbers: 03.67.Lx, 03.65.Fd, 71.35.Cc, 73.20.Dx

The introduction of quantum information/computation (QIC) [1] as an abstract concept has given birth to a great deal of new thinking about how to design and realize quantum information processing devices. This goal is extremely challenging: one should be able to perform, on a system with a well-defined quantum-state space (the *computational* space), precise quantum-state preparation, coherent quantum manipulations (*gating*) of arbitrary length, and state detection as well. It is well known that the major obstacle to implement this ideal scheme is *decoherence*: the spoiling of the unitary character of quantum evolution due to the uncontrollable coupling with environmental, i.e., noncomputational, degrees of freedom. Mostly due to the need of low decoherence rates, the first proposals for experimental realizations of quantum information processing devices originated from specialties in atomic physics [2], in quantum optics [3], and in nuclear and electron magnetic-resonance spectroscopy [4]. On the other hand, practically relevant quantum computations require a large number of quantum-hardware units (*qubits*) that are known to be hardly achievable in terms of such systems. In contrast, in spite of the serious difficulties related to the “fast” decoherence times, a solid-state implementation of QIC seems to be the only way to benefit synergistically from the recent progress in ultrafast optoelectronics [5] as well as in mesostructure and/or nanostructure fabrication and characterization [6]. Among the proposed solid-state implementations one should mention those in superconducting device physics [7] and in meso- and nanoscopic physics [8]. In particular, the first semiconductor-based proposal, by Loss and DiVincenzo, relies on spin dynamics in quantum dots; it exploits the low decoherence of spin degrees of freedom in comparison to the one of charge excitations.

As originally envisioned in [9], gating of charge excitations could be performed by exploiting *present* ultrafast laser technology [5] that allows one to generate and manipulate electron-hole quantum states on a subpicosecond time scale: *coherent carrier control* [10]. In this respect, decoherence times on nanosecond and/or microsecond scales can be regarded as “long” ones. Based on

this idea a few implementations have been recently put forward [11]. However, while in these proposals single-qubit operations are implemented by means of ultrafast optical spectroscopy, the control of two-qubit operations still involves the application of external fields and/or microcavity-mode couplings, whose switching times are much longer than decoherence times in semiconductor quantum dots (QDs). It clearly follows that such proposals are currently out of reach in terms of state-of-the-art optoelectronics technology.

As already pointed out in [9], in order to take full advantage from modern ultrafast laser spectroscopy one should be able to design fully optical gating schemes able to perform single- and two-qubit operations on a subpicosecond time scale. Following this spirit, in this Letter we propose the first *all optical* implementation with semiconductor macromolecules. Our analysis is based on a realistic, fully three-dimensional, description of multi-QD structures, whose many-body electron-hole Hamiltonian can be schematically written as [6]

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{H}' = (\mathbf{H}_c + \mathbf{H}_{cc}) + (\mathbf{H}_{cl} + \mathbf{H}_{env}), \quad (1)$$

where \mathbf{H}_c describes the noninteracting electron-hole system within the nanostructure confinement potential, \mathbf{H}_{cc} is the sum of the three (electron-electron, hole-hole, and electron-hole) Coulomb-interaction terms, \mathbf{H}_{cl} describes the coupling of the carrier system with a classical light field [12], while \mathbf{H}_{env} describes the interaction of the carrier system with environmental degrees of freedom, such as phonon and plasmon modes of the host material. The latter is responsible for decoherence processes and it will be accounted for within a density-matrix formalism (see below).

For any given number of electron-hole pairs N , a direct diagonalization of \mathbf{H}_0 will provide the many-body states of the interacting electron-hole system; they, in turn, allow one to evaluate many-exciton optical spectra, i.e., the absorption probability corresponding to the generic $N \rightarrow N'$ transition.

The above theoretical scheme has been applied to realistic state-of-the-art QD arrays. In particular, as quantum hardware, we consider a GaAs-based structure with in-plane parabolic confinement potential [13]; moreover, as discussed below (see Fig. 2), in order to induce a significant exciton-exciton dipole coupling, an in-plane static electric field F is applied.

Let us discuss first the optical response of the *semiconductor macromolecule* ($a + b$) in Fig. 1. The excitonic ($0 \rightarrow 1$) optical spectrum in the presence of an in-plane electric field $F = 15$ kV/cm is shown in Fig. 2(A), where the two lowest optical transitions correspond to the formation of direct ground-state excitons in dots a and b , respectively (see Fig. 1). In contrast, the high-energy peaks correspond to optical transitions involving excited states of the in-plane parabolic potential. Because of the strong in-plane carrier confinement, the low-energy excitonic states are expected to closely resemble the corresponding single-particle ones, thus involving the parabolic-potential ground state only. This is confirmed by the excitonic spectrum (solid curve) in Fig. 2(B), which has been obtained limiting our single-particle basis set to the parabolic-potential ground state. As we can see, apart from a small rigid shift, the relative position of the lowest transitions is the same. This suggests to use as a basis of our computational space the set formed by the lowest excitonic transition in each QD.

Let us now come to the biexcitonic spectrum [dashed line in Fig. 2(B)]; it describes the generation of a second electron-hole pair in the presence of a previously created exciton ($1 \rightarrow 2$ optical transitions). The crucial feature in Fig. 2(B) is the magnitude of the “biexcitonic shift” [6], i.e., the energy difference between the excitonic and the biexcitonic transition (see solid and dashed curves). For the QD structure under investigation we get energy splittings up to 5 meV [see inset of Fig. 2(B)], which is by far

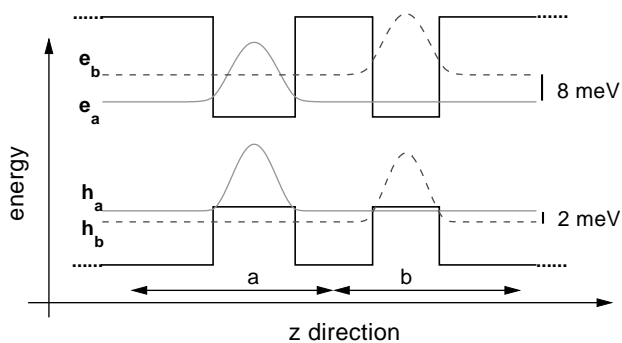


FIG. 1. Schematic representation of the squarelike potential profile for electrons (e) and holes (h) along the growth (z) direction of our QD array. This is tailored in such a way to allow for an energy selective creation/destruction of bound electron-hole pairs (i.e., excitons) in dots a and b . Moreover, the interdot barrier width ($w \sim 50$ Å) is such to prevent single-particle tunneling and at the same time to allow for significant interdot Coulomb coupling.

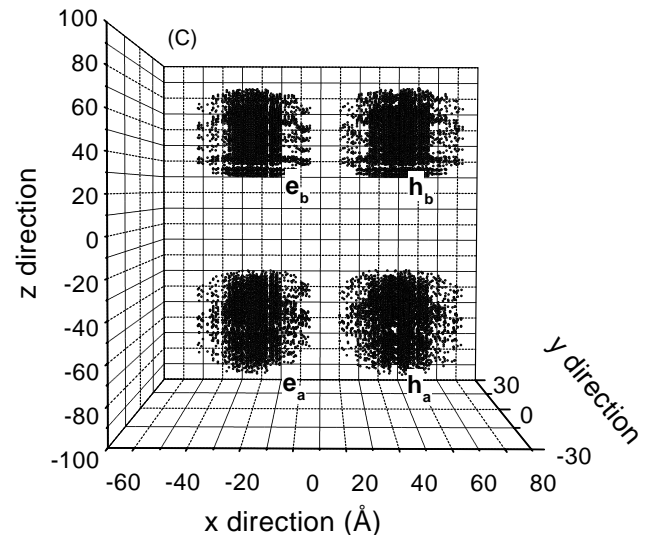
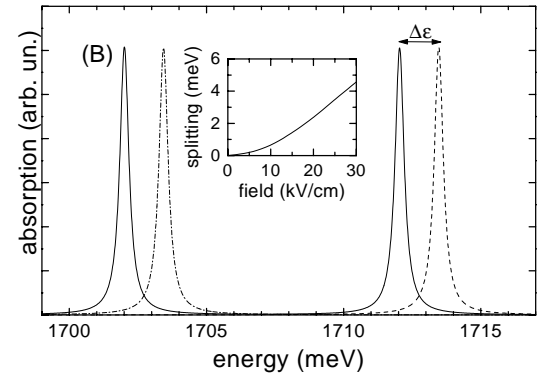
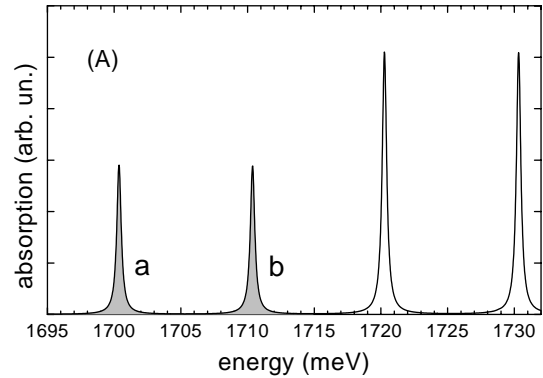


FIG. 2. Optical response of the array unit cell ($a + b$) in Fig. 1. (a) Excitonic spectrum obtained including the realistic multilevel structure of the in-plane parabolic potential. (b) Excitonic (solid curve) and biexcitonic spectrum (dashed curve) obtained including the in-plane ground state only. Because of the well-defined polarization of our laser source, the two structures in the biexcitonic spectrum correspond to the formation of an exciton in dot a given an exciton in dot b and *vice versa*. (c) Three-dimensional view of the spatial charge distributions of the two electrons (e_a and e_b) and holes (h_a and h_b) corresponding to the biexcitonic ground state in (b). As we can see, the charge separation induced by the static field significantly increases the average distance between electrons and holes, thus decreasing their attractive interaction. On the other hand, the repulsive terms are basically field independent. This is the origin of the positive energy difference $\Delta \mathcal{E}$ in (b).

larger than typical biexcitonic splittings in single QDs [6]. This is due to the in-plane static field F , which induces a charge separation between electrons and holes. This, in turn, gives rise to significant dipole-dipole coupling between adjacent excitonic states. The microscopic origin of such exciton-exciton coupling is the same as the Forster process exploited by Quiroga and Johnson [14].

The physical origin of the biexcitonic shift $\Delta\mathcal{E}$ is qualitatively described in Fig. 2(C), where we show the electron and hole charge distribution corresponding to the biexcitonic ground state.

The central idea in our QIC proposal is to exploit the few-exciton effect to design *conditional operations*. To this end let us introduce the excitonic occupation number operators \hat{n}_l , where l denotes the generic QD in our array. The two states with eigenvalues $n_l = 0$ and $n_l = 1$ correspond, respectively, to the absence (no conduction-band electrons) and to the presence of a ground-state exciton (a Coulomb-correlated electron-hole pair) in dot l ; they constitute our single-qubit basis: $|0\rangle_l$ and $|1\rangle_l$. The whole computational state space \mathcal{H} is then spanned by the basis $|\mathbf{n}\rangle = \otimes_l |n_l\rangle$, ($n_l = 0, 1$).

The full many-body Hamiltonian \mathbf{H}_0 in (1) restricted to the above computational space \mathcal{H} reduces to

$$\tilde{\mathbf{H}}_0 = \sum_l \mathcal{E}_l \hat{n}_l + \frac{1}{2} \sum_{l \neq l'} \Delta\mathcal{E}_{ll'} \hat{n}_l \hat{n}_{l'}. \quad (2)$$

Here, \mathcal{E}_l denotes the energy of the ground-state exciton in dot l while $\Delta\mathcal{E}_{ll'}$ is the biexcitonic shift due to the Coulomb interaction between dots l and l' , previously introduced (see Fig. 2). The effective Hamiltonian in (2) has exactly the same structure as the one proposed by Lloyd in his pioneering paper on quantum cellular automata [15], and it is the model Hamiltonian currently used in most of the NMR quantum-computing schemes [16]. This fact is extremely important since it tells us that (i) the present semiconductor-based implementation contains all relevant ingredients for the realization of basic QIC processing; (ii) it allows one to establish a one-to-one correspondence between our semiconductor-based scheme and much more mature implementations, like NMR [16].

According to (2), the single-exciton energy \mathcal{E}_l is renormalized by the biexcitonic shift $\Delta\mathcal{E}_{ll'}$, induced by the presence of a second exciton in dot l' ($\tilde{\mathcal{E}}_l = \mathcal{E}_l + \sum_{l' \neq l} \Delta\mathcal{E}_{ll'} n_{l'}$). In order to better illustrate this idea, let us focus again on the two-QD structure, i.e., two-qubit system, of Fig. 1 and fix our attention on one of the two dots, say dot b . The effective energy gap between $|0\rangle_b$ and $|1\rangle_b$ depends now on the occupation of dot a . This elementary remark suggests to design properly tailored laser-pulse sequences to implement controlled-not logic gates among the two QDs as well as single-qubit rotations. Indeed, by sending an ultrafast laser π pulse with central energy $\hbar\omega_b[n_a] = \mathcal{E}_b + \Delta\mathcal{E}_{ba}n_a$, the transition $|n_b\rangle_b \rightarrow |1 - n_b\rangle_b$ (π rotation) of the *target* qubit (dot b) is obtained if and only if the *control* qubit (dot a) is in the

state $|n\rangle_a$. Notice that the above scheme corresponds to the so-called selective population transfer in NMR [16]; alternative procedures used in that field can be adapted to the present proposal as well.

Moreover, by denoting with $U_b^{n_a}$ the generic unitary transformation induced by the laser π pulse of central frequency $\omega_b[n_a]$, it is easy to check that the two-color pulse sequence $U_b^0 U_b^1$ achieves the unconditional π rotation of qubit b .

In order to test the viability of the proposed quantum-computation strategy, we have performed a few simulated experiments of basic quantum information processing. Our time-dependent simulations are based on the realistic state-of-the-art QD structure of Fig. 1: $\mathcal{E}_a = 1.70$ eV, $\mathcal{E}_b = 1.71$ eV, $\Delta\mathcal{E} = 4,5$ meV, which correspond to $F = 30$ kV/cm [see inset of Fig. 2(B)]. They are based on a numerical solution of the Liouville–von Neumann equation describing the exact quantum-mechanical evolution of the many-exciton system (2) within our computational subspace \mathcal{H} in the presence of environment-induced decoherence processes [see term \mathbf{H}_{env} in Eq. (1)] [17]. Figure 3 shows a simulated sequence of two-qubit operations driven by two-color laser pulses.

Initially the system is in the state $|0, 0\rangle \equiv |0\rangle_a \otimes |0\rangle_b$. The first laser pulse (at $t = 0.2$ ps) is tailored in such a way to induce a $\frac{\pi}{2}$ rotation of the qubit a : $|0, 0\rangle \rightarrow (|0, 0\rangle + |1, 0\rangle)/\sqrt{2}$. At time $t = 0.8$ ps a second pulse induces a conditional π rotation of the qubit b : $|0, 0\rangle + |1, 0\rangle \rightarrow |0, 0\rangle + |1, 1\rangle$. This last operation plays a central role in any quantum information processing, since it transforms a *factorized* state $(|0\rangle + |1\rangle) \otimes |0\rangle$ into an *entangled* state. The scenario described so far is confirmed by the time

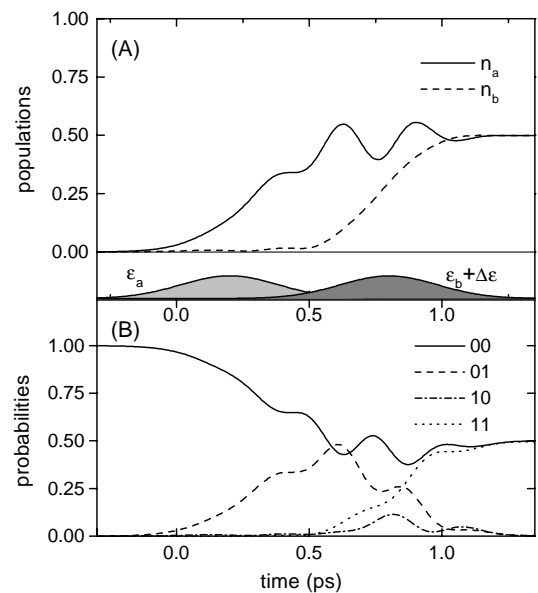


FIG. 3. Time-dependent simulation of two-qubit operations (see text). (a) Exciton populations (n_a and n_b), and (b) diagonal elements of the density matrix as a function of time. The two-color pulse sequence is also sketched schematically.

evolution of the exciton occupation numbers n_a and n_b reported in 3(A) as well as of the diagonal elements of the density matrix (in our four-dimensional computational basis) reported in 3(B).

The experiment simulated above clearly shows that the energy scale of the biexcitonic splitting in our quantum-dot molecule (see Fig. 2) is compatible with the subpicosecond operation time scale of Fig. 3.

At this point a few comments are in order. First we stress a very important feature of the proposed semiconductor-based implementation: as for NMR quantum computing, two-body interactions are always switched on (this should be compared to the schemes in which two-qubit gates are realized by turning on and off the coupling between subsystems, e.g., by means of slowly varying fields and cavity-mode couplings); conditional as well as unconditional dynamics is realized by means of sequences of ultrafast single-qubit operations whose length does not scale as a function of the total number of QDs in the array [18].

Let us now come to the *state measurement*. In view of the few-exciton character of the proposed quantum hardware, the conventional measurement of the carrier subsystem by spectrally resolved luminescence needs to be replaced by more sensitive detection schemes. To this end, a viable strategy could be to apply to our semiconductor-based structure the well-known recycling techniques commonly used in quantum-optics experiments [19].

Our nanometric scale does not allow for space selective optical addressing of individual qubits. For this reason, at least for our basic QD molecule ($a + b$), we resorted to an energy selective addressing scheme. However, extending such strategy to the whole QD array would imply different values of the excitonic transition in each QD, i.e., $\mathcal{E}_l \neq \mathcal{E}_l'$. This, besides obvious technological difficulties, would constitute a conceptual limitation of scalability towards massive quantum computations. The problem can be avoided following a completely different strategy originally proposed by Lloyd [15] and recently improved in [20]: by properly designed sequences of multicolor global pulses within a cellular-automaton scheme, local addressing is replaced by information-encoding transfer along our QD array.

Finally, a present limitation of the proposed quantum hardware is the nonuniform structural and geometrical properties of the QDs in the array, which may give rise to energy broadenings larger than the biexcitonic shift. However, recent progress in QD fabrication—including the realization of QD structures in microcavities—will allow one, we believe, to overcome this purely technological (nonconceptual) limitation.

In summary, the first *all optical implementation of QIC with a semiconductor-based quantum hardware* has been proposed. Our analysis has shown that energy-selected

optical transitions in realistic state-of-the-art QD structures are good candidates for quantum information encoding and manipulation. The subpicosecond time scale of ultrafast laser spectroscopy allows for a relatively large number of elementary operations within the exciton decoherence time.

We are grateful to David DiVincenzo, Neil Johnson, Seth Lloyd, and Mario Rasetti for stimulating and fruitful discussions. This work has been supported in part by the European Commission through the Research Project *SQID* within the *Future and Emerging Technologies (FET)* programme.

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