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Improvement of signal-to-noise ratio by stochastic resonance in sigmoid function threshold systems, demonstrated using a CMOS inverter

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ABSTRACT

Stochastic resonance (SR) has become a well-known phenomenon that can enhance weak periodic signals with the help of noise. SR is an interesting phenomenon when applied to signal processing. Although it has been proven that SR does not always improve the signal-to-noise ratio (SNR), in a strongly nonlinear system such as simple threshold system, SR does in fact improve SNR for noisy pulsed signals at appropriate noise strength. However, even in such cases, when noise is weak, the SNR is degraded. Since the noise strength cannot be known in advance, it is difficult to apply SR to real signal processing. In this paper, we focused on the shape of the threshold at which SR did not degrade the SNR when noise was weak. To achieve output change when noise was weak, we numerically analyzed a sigmoid function threshold system. When the slope around the threshold was appropriate, SNR did not degrade when noise was weak and instead was improved at suitable noise strength. We also demonstrated SNR improvement for noisy pulsed voltages using a CMOS inverter, a very common threshold device. The input–output property of a CMOS inverter resembles the sigmoid function. By inputting the noisy signal voltage to a CMOS inverter, we measured the input and output voltages and analyzed the SNRs. The results showed that SNR was effectively improved over a wide range of noise strengths.

1. Introduction

In nonlinear systems, in which output cannot be obtained if there is only a very weak periodic signal input, the input is enhanced with the help of noise to allow it to be observed. This phenomenon, called stochastic resonance [1–9], is observed in a very wide range of fields, such as circuits [10], biology [11,12] and nano-systems [13,14]. Stochastic resonance is also an interesting phenomenon when applied to signal processing. There are numerous reports on how the properties of a signal-to-noise ratio (SNR) change as a result of stochastic resonance. However, it has been proved that within the regime of the validity of linear response theory, the ratio of the output SNR over the input SNR (SNR gain:$\mathrm{SNR}_{\mathrm{gain}}$) cannot exceed unity for a nonlinear system driven by a sinusoidal signal and Gaussian white noise [8]. This fact makes it difficult to apply stochastic resonance to signal processing. Meanwhile, in recent years, as Makra [15], Casado-Pascual [16], and Duan et al. [17,18] have reported, beyond regimes where linear response theory is applied, $\mathrm{SNR}_{\mathrm{gain}}$ can in fact exceed unity in non-dynamic and dynamic systems. These reports prompted us to investigate the possibility of building a novel signal and information processing system by applying stochastic resonance.

Simple threshold systems, the most basic system for analyzing stochastic resonance, have been widely examined using fundamental electrical elements such as comparators. We have also investigated the theoretical analysis of simple threshold systems [19,20]. Focusing on their stochastic characteristics, we applied stochastic comparators to circuits to estimate the distance between vectors [21,22]. We have also successfully expressed cellular migration using these stochastic computing systems [23].

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In this paper, we investigated how to improve $\mathrm{SNR}_{\mathrm{gain}}$ over a wide range of noise strengths, focusing on the shape of the threshold. Stochastic resonance shows peak SNR when the noise strength is appropriate, whereas the SNR of the output degrades at weak noise strengths. However, for actual signal processing, SNR should ideally be improved continuously from weak noise levels. From this starting point, we examined whether the SNR of an inverter output can surpass the SNR of an inverter input over a wide range from weak noise strength, focusing on the shape of threshold.

The article is organized as follows. In Section 2, we numerically analyze the fundamental responses of a simple threshold system for a rectangular signal input. We also investigate the sigmoid function threshold system to observe how $\mathrm{SNR}_{\mathrm{gain}}$ is improved. In Section 3, to examine SNR improvement of an actual noisy pulsed signal, analog signal processing is experimented using a CMOS inverter. Section 4 gives our concluding remarks.

2. Numerical analysis

2.1. Simple threshold system

First, we investigate the performance of a simple threshold system using an equation model. The equation model of a simple threshold system is described as follows.

$$V_{in}(t) = s(t) + \xi(t) \tag{1}$$

$$V_{out}(t) = \begin{cases} 1 & (if\ V_{in} > \theta) \\ 0 & (otherwise) \end{cases} \tag{2}$$

where $s(t)$ is the original signal and θ is the threshold value. $\xi(t)$ is the zero-mean additive Gaussian white noise with autocorrelation $\langle \xi(t)\xi(0)\rangle = \sigma^2\delta(t)$, and σ is the standard deviation of the noise. In this paper, we call V_{in} the input signal and V_{out} the output signal. The original signal is a periodic rectangular signal as described below.

$$s(t) = \begin{cases} h & (nT \le t < (n+\gamma)T) \\ 0 & ((n+\gamma)T \le t < (n+1)T) \end{cases} \tag{3}$$

where h is signal height, T is the period, n is the period number, and γ is the duty cycle ratio.

We computed the averaged power spectral density (PSD), from which we obtained the SNR. We used the most common SNR definition, which is the ratio of signal power P_S at signal frequency f_s and the level of the background noise at identical frequency P_N. Here, we identified P_N as the mean power around f_s as follows.

$$P_N = \frac{1}{0.6f_s}\left[\int_{0.5f_s}^{0.8f_s} P(f)\mathrm{d}f + \int_{1.2f_s}^{1.5f_s} P(f)\mathrm{d}f\right]. \tag{4}$$

Here, $P(f)$ is the signal power at frequency f. From P_S and P_N, SNR can be calculated as follows.

$$\mathrm{SNR} = P_S/P_N. \tag{5}$$

Here, we term $\mathrm{SNR}_{\mathrm{in}}$ the SNR of the input signal and $\mathrm{SNR}_{\mathrm{out}}$ the SNR of the output signal. $\mathrm{SNR}_{\mathrm{gain}}$ is also defined as follows.

$$\mathrm{SNR}_{\mathrm{gain}} = \mathrm{SNR}_{\mathrm{out}}/\mathrm{SNR}_{\mathrm{in}}. \tag{6}$$

Hereinafter, the threshold value is normalized by signal height. We call it the effective threshold, θ_{ef}, which is defined as follows.

$$\theta_{ef} \equiv (\theta - s_{\mathrm{min}})/(s_{\mathrm{max}} - s_{\mathrm{min}}) \tag{7}$$

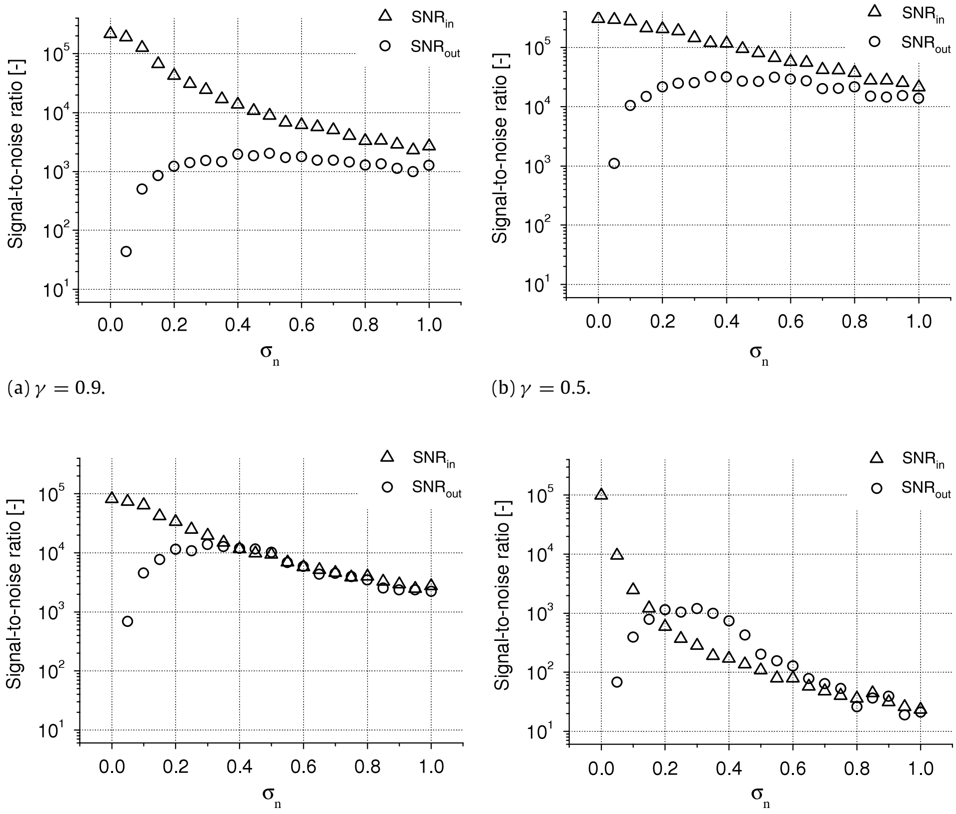
where s_{max} is the maximum signal value and s_{min} is the minimum signal value. Signal height $h = s_{\mathrm{max}} - s_{\mathrm{min}}$. Here, as $s_{\mathrm{min}} = 0$, Eq. (7) becomes

$$\theta_{ef} = \theta/h. \tag{8}$$

In this paper, we analyze the subthreshold case ($\theta_{ef} > 1$), under conditions where the input cannot reach the threshold value when noise strength is zero. Here, we define the normalized standard deviation of noise as follows.

$$\sigma_n = \sigma/h. \tag{9}$$

We examined the relation between the duty cycle and the $\mathrm{SNR}_{\mathrm{gain}}$ for $n = 300$. In Fig. 1, the analyzed $\mathrm{SNR}_{\mathrm{in}}$ and $\mathrm{SNR}_{\mathrm{out}}$ at each noise strength are shown at several γ values. For all γ values, $\mathrm{SNR}_{\mathrm{in}}$ decreases monotonically as σ_n increases, but $\mathrm{SNR}_{\mathrm{out}}$ peaks at the appropriate noise strength, which is the stochastic resonance. The degradation of $\mathrm{SNR}_{\mathrm{in}}$ is the smallest when $\gamma = 0.5$, while $\mathrm{SNR}_{\mathrm{out}}$ shows a rather complicated response according to the γ value. Note that when γ is as small as 0.01, $\mathrm{SNR}_{\mathrm{gain}}$ can exceed unity as shown in Fig. 1(d). SR with periodic pulse trains have been also reported for simple threshold system [15,24] and for double well potential [25,26]. They showed that the SNR gain also exceeded unity. However, when noise strength was weak, since the $\mathrm{SNR}_{\mathrm{out}}$ became zero, the SNR gain also became zero.



(a) $\gamma = 0.9$. (b) $\gamma = 0.5$. (c) $\gamma = 0.1$. (d) $\gamma = 0.01$.

Fig. 1. SNR$_{in}$ and SNR$_{out}$ as functions of σ_n for different duty cycles when $\theta_{ef} = 1.1$.

The similar results were also seen in our results. Even if $\gamma = 0.01$, SNR$_{out}$ is much smaller than SNR$_{in}$ when noise strength is weak. Fig. 2(c) shows the waveforms of input and output signal for various σ_n values when $\gamma = 0.01$, which corresponds to Fig. 1(d). Fig. 2(c) shows the case of $\sigma_n = 0.3$, the SNR$_{out}$ of which was approximately maximal. In this case, a very regular output can be observed according to the input of the noisy pulsed signal. Meanwhile, when σ_n is smaller (Fig. 2(a) and (b)), although the pulsed input signal becomes clear, the periodicity of the output degrades. When noise is weak, and the input signal does not reach the simple threshold, there is no change in output signal. This is why SNR$_{gain}$ degrades in weak noise regions.

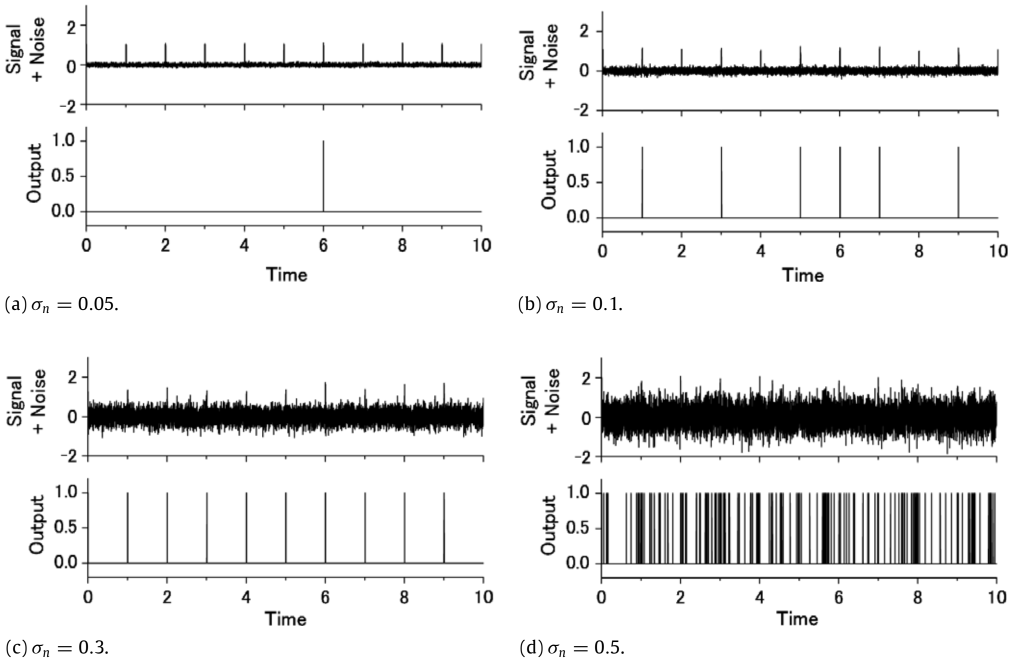
Here, for the SR effect, a theory exists as considered in Refs. [24,27]. This theory applies to any nonlinearity. For simple threshold system with the input of pulse trains, we compared the numerical results to the theoretical value and confirmed that almost the same value was achieved though the P_N was approximately calculated by Eq. (4).

2.2. *Sigmoid function threshold system*

How can we make SNR$_{out}$ larger than SNR$_{in}$ when σ_n is small? One answer is to apply a new threshold system so that V_{out} changes even if $\sigma_n = 0$. The sigmoid function is one candidate. If a sigmoid function has an appropriate gradient, V_{out} will change when V_{in} is lower than the threshold. Here, we define the threshold θ_s of the sigmoid function as the input value when the output value is half of the output swing. The sigmoid threshold is expressed as follows.

$$V_{out} = 1/\left[1 + \exp\{-\alpha(V_{in} - \theta_s)\}\right]. \tag{10}$$

For simplicity, in the following, we set $\theta_s = 0$. Here, α determines the slope around the threshold. The input–output properties for different α values are shown in Fig. 3. Using this sigmoid threshold, SNR$_{in}$ and SNR$_{out}$ are computed for $\alpha = 1$, 10 and 100. The results are shown in Fig. 4. Here, since the values of SNR$_{in}$ for $\alpha = 10$ and 100 were very similar to that of SNR$_{in}$ for $\alpha = 1$, these plots are omitted from Fig. 4. When α is as small as 1, SNR$_{out}$ has almost the same value as the SNR$_{in}$ for the whole range of noise strength. This resembles a linear response. When α is as large as 100, the SNR$_{out}$ is much larger than SNR$_{in}$ around $\sigma_n = 0.3$; however, the SNR$_{out}$ degrades with weak noise ($\sigma_n < 0.2$). This result resembles that of a simple threshold system. When $\alpha = 10$, the SNR$_{out}$ does not degrade for weak noise and SNR$_{out}$ is larger than SNR$_{in}$ over the whole region of noise strength.



(a) $\sigma_n = 0.05$.

(b) $\sigma_n = 0.1$.

(c) $\sigma_n = 0.3$.

(d) $\sigma_n = 0.5$.

Fig. 2. Input and output waveform for different noise strengths ($\gamma = 0.01$), which corresponds to Fig. 1(d).

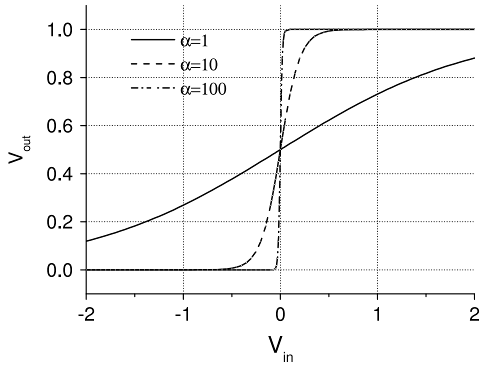


Fig. 3. Input–output properties for sigmoid function threshold systems.

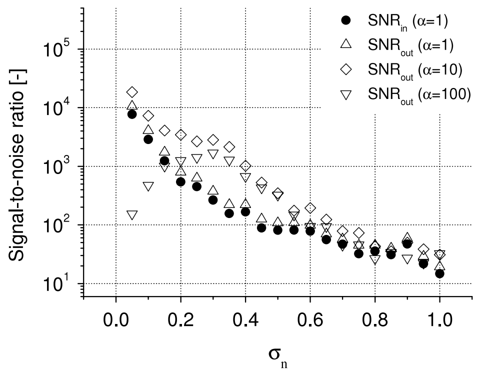


Fig. 4. SNR_{gain} for the sigmoid function threshold system for different α values.

Such nonlinearity as sigmoid function is called the smooth nonlinearity. Smooth nonlinearities have also been reported by Chapeau-Blondeau et al. [6,28]. In the Ref. [6], sigmoidal nonlinear system is discussed against the sine wave input. When

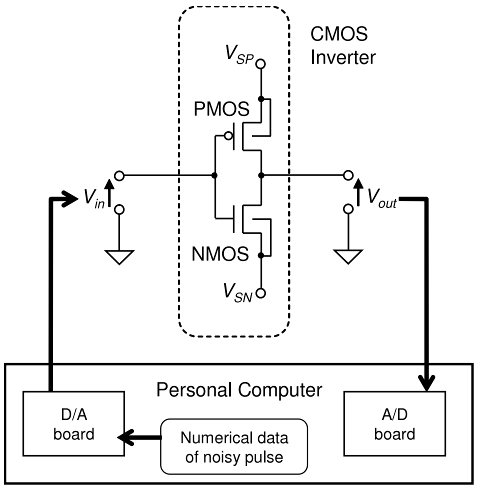


Fig. 5. Schematic view of experimental setup.

Table 1
Parameters for MOS transistors used in the experiment.

Name	Value
Gate length	10 µm
Gate width (PMOS)	20 µm
Gate width (NMOS)	10 µm
Gate oxide thickness	10 nm
$V_{th(PMOS)}$	−0.8 V
$V_{th(NMOS)}$	0.8 V
V_{SP}	1.53 V
V_{SN}	−1.77 V

the nonlinearity is almost simple threshold, SNR_{out} exhibits maximal value at appropriate noise strength and stochastic resonance occurs. However, when the nonlinearity becomes smoother, the input is visible at the output in the absence of the noise. When the system is smoother, SNR_{out} becomes monotonically larger and become close to linear response, which is the same as our result, though the input was pulse in our analysis.

As above, there is an appropriate range of α values. This means that the appropriate gradient of the slope around the threshold improves the SNR_{out} properties. This makes a difference in the small noise region and maintains SNR_{out} at the same level as SNR_{in}. When $\alpha = 10$, the sloping region around the threshold voltage functions so well that SNR_{out} has nearly the same value as SNR_{in} when the noise is weak; and SNR_{out} exceeds SNR_{in} when σ_n is about 0.2 to 0.6. When $\alpha = 10$, it may seem that stochastic resonance is not occurring, since SNR_{out} has no maximal peak at the appropriate noise strengths. However, this behavior of SNR_{out} can be understood as the superposition of two factors. When noise is weak, an almost linear response helps keep SNR_{out} at the same level as SNR_{in}. This prevents degradation of SNR_{gain}. When the noise is stronger, stochastic resonance occurs due to threshold nonlinearity and a marked improvement of properties of $SNR_{gain} \approx 10$ is realized. Thus, by using the shape of the slope around the threshold, it is possible to realize improvement of SNR_{gain} over a large range of noise strengths.

3. Experiment using a CMOS inverter

To apply stochastic resonance to noisy signal processing, we investigated the improvement of SNR_{gain} using actual devices. From the results for the previous section, the slope around the threshold was found to effectively improve the SNR_{out}. Accordingly, in this section, we investigate whether SNR_{gain} exceeds unity with an actual device which has a slope around the threshold. One of the most commonly used threshold devices is a complementary metal-oxide-semiconductor (CMOS) inverter. In general, CMOS inverters are used for binary logic circuits and output two voltages: the "High" and "Low" states. However, since a CMOS inverter has a slope around its threshold, if an analog signal is input to a CMOS inverter, the output gradually changes around the threshold. By making use of this property, we investigated whether a similar effect to that of the sigmoid function could be achieved.

The analysis was carried out by inputting a noisy pulsed signal to a CMOS inverter (Fig. 5). The parameters for the MOSFETs used in the experiments are listed in Table 1. $V_{th(PMOS)}$ is the threshold voltage of PMOS, and $V_{th(NMOS)}$ is that of NMOS. For convenience, the source voltages of p-channel MOSFET (PMOS) and n-channel MOSFET (NMOS), which comprise the

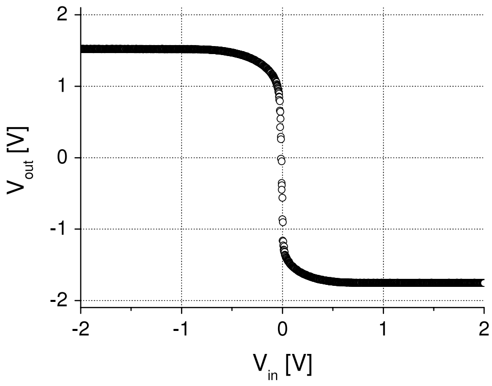


Fig. 6. $V_{in} - V_{out}$ properties of the CMOS inverter used for experiments. Its threshold is tuned to about 0.0 V.

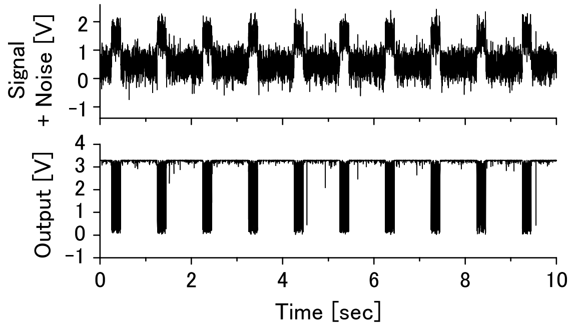


Fig. 7. Measured waveform of the noisy signal and the inverter output ($\gamma = 0.2$, $\sigma_n = 0.3$, $\theta_{ef} = 1.1$).

CMOS inverter, are tuned such that the threshold voltage of the inverter ($V_{th(inv)}$) becomes zero volts: specifically, when the voltage at the source electrode of PMOS (V_{SP}) is 1.53 V and that of NMOS (V_{SN}) is −1.77 V, $V_{th(inv)}$ of 0.0 V is realized, as shown in Fig. 6. From Fig. 6, it can be seen that V_{out} changes gradually around the threshold of the inverter.

We analyzed this system under conditions where the signal height = 1.0 V. From Eq. (7), since the threshold voltage of the inverter is 0.0 V, $s_{\min}$ is calculated to be −1.1 V to achieve $\theta_{ef} = 1.1$. To match the equation model of the previous section, the analysis is executed under conditions where the signal period is 1 s and the noise is applied at 1 ms intervals. The input signal is prepared as numerical data by adding Gaussian noise to the pulsed signals (Fig. 5). According to this numerical data, the input voltage signal is generated by a D/A board on a desktop computer. Simultaneously, input and output signals are measured by an A/D board on a desktop computer and sampled at 1 ms intervals. Here, the D/A board resolution is 2.4 mV/digit and that of the A/D board is 0.5 mV/digit. Experimental analysis is executed for 300 periods, the same as for the numerical analysis.

Fig. 7 shows the measured waveform of the noisy signal and the inverter output when $\gamma = 0.2$ and $\sigma_n = 0.3$. Although the output waveform is inverted because of the function of the inverter circuit, very similar results to the equation model were obtained (Fig. 2). Here, the SNR shows no change when the output is reversed, since the PSD is the same.

Fig. 8 shows the relationship between noise strength and SNRs in the CMOS inverter experiments. In this analysis, to set $\gamma = 0.01$, the "high" state is set at 10 ms. Noise data are updated at 1 ms intervals. This result corresponds to that of the sigmoid-threshold system shown in Fig. 4. SNR_{in} shows very similar results to that of Fig. 4, which indicates that the noisy signal has almost the same properties. SNR_{out} agrees closely with the sigmoid-threshold system for $\alpha = 10$. When the noise is large ($\sigma_n > 0.4$), SNR_{out} shows much higher values than those of SNR_{in} due to the effect of stochastic resonance. Moreover, when the noise is small ($\sigma_n < 0.2$), SNR_{out} does not degrade, but holds its value at almost the same level as SNR_{in}. As a result, SNR_{in} is larger than SNR_{out} for almost the whole region of noise strength.

Using a CMOS inverter, we were able to obtain high SNR_{gain} for a wider range of noise strengths. Numerous studies have been made of MOS transistors, which support the high performance and stability of today's LSIs. We clarified that when a noisy pulse signal is input to a CMOS inverter, the SNR_{gain} exceeds unity when noise is weak and is improved around $\sigma_n = 0.3$. These are novel characteristics of noisy signal processing by a CMOS inverter.

One of the examples for practical analog signal processing is the electrocardiograph. Strong impulse is measured for each heartbeat, from which we can know heart rate etc. In general, since the human body is high impedance, the measured signal is mixed with noises. To analyze the frequency of heartbeat from such a noisy signal, this CMOS inverter system will be useful.

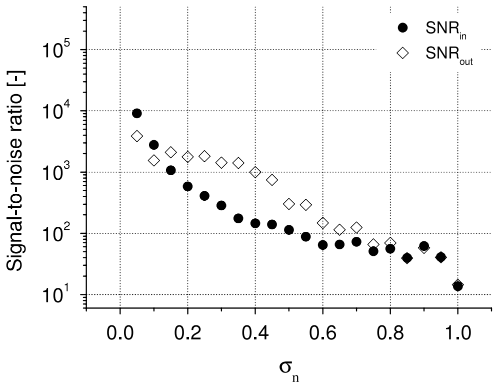


Fig. 8. SNR_{in} and SNR_{out} for experiments using a CMOS inverter ($\gamma = 0.01$, $\theta_{ef} = 1.1$).

4. Conclusion

We examined the effect of processing a noisy pulse signal by a threshold system from the point of view of improving SNR. Although SNR_{gain} could not be improved for a noisy rectangular signal with a duty cycle of 0.5 using a simple threshold system, when the duty cycle of the signal is small, SNR_{gain} exceeded unity in appropriate noise strength. However, SNR_{gain} was lower than unity when noise was weak.

To improve this limitation, we investigated the sigmoid function threshold system. Due to the effect of the linear response caused by the slope around the threshold, the SNR_{gain} for weak noise was found to be improved. Moreover, at appropriate noise strengths, the SNR_{gain} was significantly improved due to the effect of stochastic resonance.

To investigate potential applications for analog signal processing, we examined SNR improvement using an actual threshold device. Since the threshold properties resembled the sigmoid function, a CMOS inverter was applied to the threshold device. Signal processing experiments were carried out by applying voltages to the CMOS inverter using a D/A board. The results showed that SNR was effectively improved at appropriate noise strengths by stochastic resonance. Meanwhile, when noise was weak, SNR_{out} maintained the same level as SNR_{in}. A CMOS inverter showed improved SNR over a wide range of noise strengths in actual noisy signal processing. Since MOS transistors are ideal for large-scale integration, this signal processing method is likely to have numerous benefits.

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