Effect of non-Fermi liquid behavior on the temperature dependence of the amplitude of de Haas-van Alphen oscillations

P. Schlottmann
Department of Physics, Florida State University, Tallahassee, Florida 32306, USA
(Received 13 January 2008; revised manuscript received 14 March 2008; published 14 May 2008)

We calculate the amplitudes of de Haas-van Alphen oscillations for a system of heavy fermions close to a quantum critical point (QCP). A nested Fermi surface, which consists of an electron and a hole pocket, together with the remaining interaction between the carriers after the heavy particles are formed, leads to itinerant antiferromagnetism. The order can be gradually suppressed by increasing the mismatch between the Fermi momenta, and a quantum critical point is obtained as $T_N \to 0$, giving rise to a heavy-fermion non-Fermi liquid behavior in the specific heat, quasiparticle linewidth, and resistivity. The Lifshitz–Kosevich expression for the de Haas-van Alphen oscillations is modified by the quasiparticle self-energy. The amplitudes are considerably reduced by the interactions. The magnetic field and temperature dependence of the amplitudes shows that the effects of the QCP extend over a large T interval.

DOI: 10.1103/PhysRevB.77.195111 PACS number(s): 71.27.+a, 71.28.+d, 72.15.Qm, 75.20.Hr

I. INTRODUCTION

Landau's Fermi liquid (FL) theory has been successful in describing the low energy properties of most normal metals. Numerous U, Ce, and Yb based heavy-fermion systems[1–3] display deviations from the FL behavior, which manifest themselves as, e.g., a $\log(T)$ dependence in the specific heat over T, a singular behavior at low T of the magnetic susceptibility, and a power-law dependence of the resistivity, with an exponent close to 1. These deviations from FL are known as non-Fermi liquid (NFL) behavior. The breakdown of the FL can be tuned by alloying (chemical pressure), hydrostatic pressure, or a magnetic field. In most cases, the systems are close to the onset of antiferromagnetism (AF) and the NFL behavior is attributed to a quantum critical point (QCP).[4–9]

NFL properties are usually measured in the disordered phase and inferred from the temperature and field dependence extrapolated to $T \to 0$. The QCP itself is elusive to experiment, and to explore its neighborhood requires the fine tuning of experimental parameters. From the theoretical point of view, the QCP is not a stable fixed point. There are differences between the behaviors of $CeCu_{5.9}Au_{0.1}$,[9,10] $Ce(Ru_{1-x}Fe_x)_2Ge_2$,[11] $CeAuSb_2$,[12] and $YbRh_2Si_2$ [13,14] close to the QCP, which suggests the nonuniversality of the NFL properties in heavy fermions.

In heavy-fermion systems, the low energy excitations in the disordered (paramagnetic) phase are well described by bands of quasiparticles with a large effective mass. In the sense of a Fermi liquid, the remaining repulsive interaction between the heavy quasiparticles (after the heavy particles have been formed) is assumed to be weak. Itinerant AF arises from the topology of the Fermi surface and the remaining interactions. For instance, the nesting of two parabolic pockets, one electronlike and the other holelike, leads to spin-density waves.[15] As the long-range order is suppressed, T_N is reduced and a QCP is obtained as $T_N \to 0$. The tuning parameter for the QCP, in this case, is the mismatch between the Fermi surfaces.

We previously studied the precritical region of the two-pocket model via renormalization group (RG) approaches.[15,16] Our main results are the following: For the tuned QCP, the effective mass m^* (specific heat over T) and the magnetic susceptibility logarithmically increase as T is lowered and diverge at the critical point, signaling the breakdown of the FL.[15,16] There is a crossover from the $-\ln(T)$ dependence of C/T to a constant γ as T is lowered if the QCP is not perfectly tuned. The quasiparticle linewidth, a quantity relevant to electrical resistivity, shows a crossover from NFL ($\sim T$) to FL ($\sim T^2$) behavior with increasing nesting mismatch and decreasing temperature.[17] The behaviors of the specific heat, susceptibility, effective mass, and resistivity (see Ref. 18) are in good qualitative agreement with experiments for numerous quantum critical heavy-fermion compounds. The predictions of our nested Fermi surface model are sufficiently robust to explain many of the consequences of the QCP. An exception is the dynamical susceptibility (relevant to inelastic neutron scattering),[19] which strongly depends on the local character of the f electrons, so that a single band model may not be sufficient. In Secs. VI and VII, we discuss the frequencies of oscillation as a function of the nesting mismatch, which vary little in a single band model, but may substantially vary if one would allow the f electrons to localize.

In this paper, we calculate the temperature and field dependence of the amplitudes of the de Haas-van Alphen (dHvA) oscillations in the neighborhood of the QCP. General expressions for the oscillatory part of the thermodynamic potential in the presence of interactions were previously derived.[20–22] They consist of summations over the fermionic Matsubara poles involving the quasiparticle self-energy. We evaluate the amplitudes by using the self-energy obtained in Ref. 17.

The rest of the paper is organized as follows: In Sec. II, we restate the expression for the oscillatory part of the thermodynamic potential. In Sec. III, the dHvA amplitudes are evaluated for a marginal Fermi liquid. In Sec. IV, we introduce the two-pocket model[15,16] and provide a brief summary of the temperature and frequency dependence of the quasiparticle linewidth.[17] In Sec. V, we present our results for the dHvA amplitudes. In Sec. VI, we discuss the continuous

change in the oscillation frequencies across the AF transition for the two-pocket model. Concluding remarks follow in Sec. VII.

II. DE HAAS-VAN ALPHEN EFFECT IN INTERACTING SYSTEMS

The Lifshitz–Kosevich expression of the thermodynamic potential for a free fermion gas in a homogeneous magnetic field B describes the dHvA oscillations of most normal metals. The oscillatory part of the thermodynamic potential is given by[22]

$$\Omega_{\rm osc} = 2k_B T\left(\frac{eB}{2\pi\hbar}\right)^{3/2}\left|\frac{\partial^2 S_i}{\partial k_z^2}\right|^{-1/2} \times\sum_{r=1}^{\infty}\frac{(-1)^r}{r^{3/2}}\frac{\cos\left[2\pi r\left(\frac{F_i}{B}-\gamma\right)\pm\frac{\pi}{4}\right]}{\sinh(2\pi^2 k_B T r/\hbar\omega_c)}. \quad (1)$$

Here, $\omega_c = eB/m^*$ is the cyclotron frequency, where m^* is the effective mass that contains the effects of the periodic potential, γ is a phase factor, and r is the index labeling the harmonics of the oscillations. The thermodynamic potential, and hence the magnetization, exhibits a periodicity in $1/B$ with a frequency $F_i(\theta,\phi)$ when the magnetic field is directed along the direction (θ,ϕ). The dHvA frequency is related through the Onsager relation to an extremal cross-sectional area $S_i(\theta,\phi)$ of the Fermi surface as follows:

$$F_i(\theta,\phi) = \frac{\hbar S_i(\theta,\phi)}{2\pi e}, \quad (2)$$

where the subindex i labels the different possible orbits. The sign $\pm$ in the phase $\pi/4$ depends on whether the extremal orbit is a maximum or a minimum. For simplicity, we have neglected the Zeeman splitting of the electron states and the Dingle factor arising from damping that results from the scattering off of the imperfections in the crystal.

The influence of interactions between the carriers on the de Haas-van Alphen effect has been studied in numerous papers.[20–22] The interaction introduces a quasiparticle self-energy, which reduces the amplitude of the oscillations. In other words, the quasiparticle linewidth changes the thermal damping factor. We limit ourselves to the situation in which the frequency dependence of the imaginary part of the self-energy is symmetric about the Fermi surface (electron-hole symmetry), and we neglect the effect of the Zeeman splitting on the self-energy. Under these circumstances, the frequency of the oscillations is not modified and it is sufficient to study the amplitude of the oscillations, which is now given by[22]

$$A_r = \sum_{\xi_n>0}\exp\left\{-\frac{2\pi r}{\hbar\omega_c}[\xi_n - X(\xi_n)]\right\}. \quad (3)$$

Here, $i\xi_n = i\pi T(2n+1)$ are the Matsubara frequencies for fermions and $X(\xi_n)$ is the value of the imaginary part of the self-energy at this frequency. For $X=0$, i.e., no interactions, A_r reduces to $A_r^0 = 1/[2\sinh(2\pi^2 k_B T r/\hbar\omega_c)]$, which is in agreement with Eq. (1). For interacting electrons, $X<0$, so that A_r is always reduced with respect to the noninteracting system.

A proper definition of quasiparticles requires that their linewidth at low energies is small compared to their resonance energy. This is satisfied for a Fermi liquid, wherein the width grows proportionally to ω^2/D or T^2/D, where D is the bandwidth. Due to the approximately linear increase in the linewidth with $|\omega|$ and T for the tuned QCP, the width of the quasiparticles grows as fast as their energy and the quasiparticles are not well defined according to the above-mentioned criterion. By using the Kubo formula, we have shown that this has no effect on the resistivity, which turns out to be approximately proportional to the quasiparticle linewidth.[18]

We can more formally argue that there is a one-to-one correspondence between the Fermi liquid excitations and those of the system with the tuned QCP. We will use the Fermi surface mismatch as the parameter that can be continuously varied by interpolating between the FL and the tuned QCP. Assume first that the mismatch parameter (denoted below as δ) is large; then, at low energies the excitation spectrum is FL-like (see Ref. 17), i.e., the quasiparticle linewidth is proportional to ω^2/D or T^2/D and the quasiparticles are well defined. We now gradually reduce the Fermi surface mismatch. This reduces the energy range of the FL behavior, which tends to zero at the tuned QCP. As a function of the mismatch parameter, we can follow each excitation into the region of the tuned QCP. Hence, there is still a one-to-one correspondence between the states and the free Fermi gas statistics still applies (with different a dispersion) despite of the broad linewidth of the excitations. Hence, the extended Lifshitz–Kosevich equation is applicable to the present situation.

III. MARGINAL FERMI LIQUID

In this section, we apply the above formalism to the simple case of a marginal Fermi liquid.[23] In a marginal FL, the interactions are sufficiently strong so that the discontinuity in the Fermi distribution has shrunk to zero. Consequently, the FL behavior, i.e., the ω^2 and T^2 dependences of the self-energy, is replaced by a linear dependence on $|\omega|$. This situation is quite similar to the behavior close to a QCP.

At $T=0$, the quasiparticle self-energy for a marginal FL can be parametrized as $\Gamma(\omega) = V^2\rho_F^2\pi|\omega|$, where V is the interaction strength and ρ_F is the density of states at the Fermi level. This functional dependence can be used to calculate $X(\xi_n)$ as follows:

$$iX(\xi_n) = -\int_{-D}^{D}\frac{d\omega}{\pi}\frac{\Gamma(\omega)}{\omega - i\xi_n} = -2i\xi_n V^2\rho_F^2\ln\frac{\sqrt{D^2+\xi_n^2}}{|\xi_n|}, \quad (4)$$

where D is an electronic cutoff. Note that X is real and negative if $\xi>0$. The same expression of X can be obtained by analytical continuation of $\Gamma(\omega)$ into the complex plane and then replacing the frequency by $i\xi_n$.

By inserting $X(\xi_n)$ into expression (3) and numerically evaluating the sum, we obtain the A_r for the marginal FL case. The result is to be compared to that for the noninteracting system, i.e., A_r^0, for the same effective mass and at the same temperature and magnetic field. The ratio A_r/A_r^0 is shown in Fig. 1 for the first five amplitudes as a function of

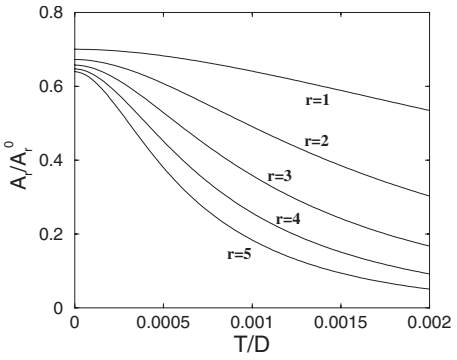

FIG. 1. de Haas-van Alphen amplitudes for the first five harmonics r as a function of T for a fixed magnetic field for a marginal Fermi liquid normalized to the amplitude for the noninteracting system. Here, $V\rho_F=0.2$ and $D=10$ and if D is equated to 1000 K, the magnetic field is 2 T with m^* being the free electron mass.

T for a fixed magnetic field. This quantity is always smaller than 1. The effect of the self-energy grows with the index r of the harmonic, as well as with the temperature. As seen in the resistivities of many compounds, the effect of a QCP extends out to quite high temperatures. This is also the case for the amplitudes of the dHvA oscillations. Increasing the field, on the other hand, reduces the effect of the interactions. If D corresponds to 1000 K, the magnetic field for the curves in Fig. 1 is 2 T if m^* is the free electron mass m. For other values of m^*, B has to be renormalized by m^*/m; i.e., for $m^*/m=10$, the field corresponds to 20 T.

IV. TWO-POCKET MODEL AND QUASIPARTICLE LINEWIDTH

The model consists of two pockets, one electronlike and the other holelike, that are separated by a wave vector $\mathbf{Q}$. The kinetic energy of the carriers is given by[15,16]

$$H_0=\sum_{\mathbf{k}\sigma}[\epsilon_1(\mathbf{k})c^\dagger_{1\mathbf{k}\sigma}c_{1\mathbf{k}\sigma}+\epsilon_2(\mathbf{k})c^\dagger_{2\mathbf{k}\sigma}c_{2\mathbf{k}\sigma}], \tag{5}$$

where $\mathbf{k}$ is measured from the center of each pocket and assumed to be small compared to the nesting vector $\mathbf{Q}$. Here, $\epsilon_1(\mathbf{k})=v_F(k-k_{F1})$ and $\epsilon_2(\mathbf{k})=v_F(k_{F2}-k)$, where k_{F1} and k_{F2} are the respective Fermi momenta. For simplicity, we assume that the Fermi velocity is the same for both pockets.

A strong interaction between electrons gives rise to heavy-fermion bands. In the spirit of the FL theory, there are weak remaining interactions between the heavy quasiparticles after the heavy particles are formed. The heavy electron bands are described by Eq. (5) and the weak interactions between quasiparticles are given by[15,16]

$$H_{12}=V\sum_{\mathbf{k}\mathbf{k}'\mathbf{q}\sigma\sigma'}c^\dagger_{1\mathbf{k}+\mathbf{q}\sigma}c_{1\mathbf{k}\sigma}c^\dagger_{2\mathbf{k}'-\mathbf{q}\sigma'}c_{2\mathbf{k}'\sigma'}+U\sum_{\mathbf{k}\mathbf{k}'\mathbf{q}\sigma\sigma'}c^\dagger_{1\mathbf{k}+\mathbf{q}\sigma}c^\dagger_{2\mathbf{k}'-\mathbf{q}\sigma'}c_{1\mathbf{k}\sigma'}c_{2\mathbf{k}'\sigma}. \tag{6}$$

Here, V and U represent the interaction strength for a small ($|\mathbf{q}|\ll|\mathbf{Q}|$) and a large (on the order of $\mathbf{Q}$) momentum transfer between the pockets, respectively. The limit of the Hubbard model is obtained by choosing $V=U$.

The interaction between the electrons and the nesting of the Fermi surface induces itinerant AF or charge-density waves (CDWs). For perfect nesting (electron-hole symmetry), an arbitrarily small interaction is sufficient for a ground state with long-range order. The degree of nesting is controlled by the mismatch parameter, $\delta=v_F|k_{F1}-k_{F2}|/2$. By increasing δ, the ordering temperature can be tuned to zero, leading to a QCP. The QCP is an unstable fixed point and can be reached only by perfectly tuning the system. Otherwise, the RG flow will deviate to a phase with long-range order or the compound remains a paramagnet.[15]

The leading order corrections to the vertex are the bubble diagrams of the zero-sound type (antiparallel propagator lines), which are logarithmic in the external energy ω. Assuming that ω is small compared to the cutoff energy D and that the density of states for electrons and holes, ρ_F, is constant, we have

$$\tilde{V}=\frac{V}{1-\rho_F V\xi},\quad 2\tilde{U}-\tilde{V}=\frac{(2U-V)}{1+\rho_F(2U-V)\xi}, \tag{7}$$

where $\xi=\ln[D/(|\omega|+2T+|\delta|)]$.[16] A divergent vertex indicates a strong coupling and signals an instability.[15,16] For a repulsive interaction, the vertex $\tilde{V}$ increases under renormalization and can give rise to AF long-range order. The vertex $(2\tilde{U}-\tilde{V})$, on the other hand, leads to a strong coupling if $2U<V$ and could lead to a CDW. Due to the mismatched Fermi surfaces, the renormalization ends either at a divergence in Eq. (7) or at $\xi=\ln(D/\delta)$, whichever is reached first. Thus, for a sufficiently large Fermi surface mismatch, the renormalization does not lead to an instability. The temperature smears the Fermi surface and introduces an effect similar to the mismatch.[16]

The key quantity to understand the low T specific heat, the enhancement of the effective mass, and the resistivity is the quasiparticle self-energy. In a FL, the damping of a quasiparticle is proportional to T^2, while the nesting condition changes this behavior to a linear dependence on T (with logarithmic corrections).[17] The contributions to the linewidth can be separated into additive terms of the NFL and FL types.

The NFL contribution Γ_{NFL} can be written as[17]

$$\Gamma_{\mathrm{NFL}}(\omega,T)=\frac{T}{2}\int dx\left[\coth(x)-\tanh\left(x-\frac{\omega}{2T}\right)\right]\chi''_S(x)\times[3|\tilde{V}|^2+|2\tilde{U}-\tilde{V}|^2]\rho_F, \tag{8}$$

where $\chi''_S(\omega/2T)$ is a staggered susceptibility (containing the nesting property), ω is the external frequency, and the frequency in the vertices is $2T|x|+|\omega|/2$. Here, we use the analytic continuation of the vertex functions; i.e., $i\pi/2$ is added to ξ. $\chi''_S(\omega/2T)$ gives rise to the NFL behavior and is self-consistently calculated by using the quasiparticle linewidth as follows:

$$\chi_S''(\omega/2T) \approx \frac{\rho_F}{2}\mathrm{Im}\ \psi\left[\frac{1}{2}+\frac{\Gamma_{\mathrm{NFL}}}{2\pi T}+i\frac{\omega-2(\delta-\delta_0)}{4\pi T}\right] + \frac{\rho_F}{2}\mathrm{Im}\ \psi\left[\frac{1}{2}+\frac{\Gamma_{\mathrm{NFL}}}{2\pi T}+i\frac{\omega+2(\delta-\delta_0)}{4\pi T}\right], \quad (9)$$

where Im ψ is the imaginary part of the digamma function, δ is the nesting mismatch defined above, and δ_0 is the mismatch corresponding to the QCP. The frequency of Γ_{NFL} in Im ψ is $2T|x|$. The self-consistent solution of Eqs. (8) and (9) yields the quasiparticle NFL linewidth as a function of ω and T.[17]

The FL contribution to the linewidth is[17]

$$\Gamma_{\mathrm{FL}}(\omega,T) = \frac{\pi}{8}[\omega^2+(\pi T)^2][3V^2+(2U-V^2)]\rho_F^3. \quad (10)$$

The vertices in Γ_{FL} are not dressed since this contribution does not arise from the nesting condition.

The ω and T dependence of the self-consistent Γ_{NFL} can be understood from the analysis of limiting cases.[17] First, for the perfectly tuned QCP, i.e., $\delta=\delta_0$, we set $\omega=0$ and neglect Γ_{NFL} in the digamma function and the vertex renormalizations. The integral on the right-hand side of Eq. (8) is then independent of T; hence, $\Gamma_{\mathrm{NFL}} \propto T$, and not proportional to T^2, as for a FL. Similarly, as $T \to 0$, neglecting Γ_{NFL} in the digamma function and neglecting the vertex renormalizations, we obtain for $\delta=\delta_0$ that the right-hand side of Eq. (8) is proportional to $|\omega|$, which again differs from the FL behavior
($\propto\omega^2$). By including the vertex functions, we obtain additional logarithmic corrections, so that approximately

$$\Gamma_{\mathrm{NFL}} \propto [3|\tilde{V}|^2+|2\tilde{U}-\tilde{V}|^2]\rho_F^2 \max(|\omega|,T). \quad (11)$$

This result is quite similar to the marginal FL discussed in Sec. III. Second, for $\delta \neq \delta_0$, neglecting again the self-consistency and the vertex corrections, Γ_{NFL} is exponentially activated at low T and gradually crosses over to a linear T dependence with increasing T. Hence, at low T the FL contribution (proportional to T^2) dominates and at higher T there is NFL behavior. Third, at $T=0$, Γ_{NFL} identically vanishes for $|\omega|<2(\delta-\delta_0)$ and is proportional to $|\omega|-2(\delta-\delta_0)$ at larger frequencies. Hence, again, the FL contribution (proportional to ω^2) dominates at low energies.[17]

The ω and T dependence of the linewidth has the following physical consequences: In the disordered phase for the tuned QCP, the effective mass (specific heat over T) and the magnetic susceptibility logarithmically increase as T is lowered and signal the breakdown of the FL.[15,16] There is a crossover from the $-\ln(T)$ dependence of C/T to a constant γ as T is lowered when the QCP is not perfectly tuned, which is in agreement with experiments on numerous systems. The electrical resistivity is roughly proportional to $\Gamma_{\mathrm{NFL}}+\Gamma_{\mathrm{FL}}$ for zero frequency; i.e., $\rho(T)$ is slightly sublinear in T for the tuned QCP. For a Fermi surface mismatch that is larger than the critical one, the resistivity displays a crossover from a NFL ($\sim T$) to a FL ($\sim T^2$) behavior with increasing nesting mismatch and decreasing temperature, which is in agreement with experiments.[17] The quasiparticle damping also affects the linewidth of the quasielastic peak in neutron scattering experiments. A linear T dependence of the linewidth has been observed in $CeRu_xFe_{2-x}Ge_2$ for $x=0.48$,[11] and a crossover from NFL to FL behavior occurs as T is lowered when x substantially differs from the critical value.

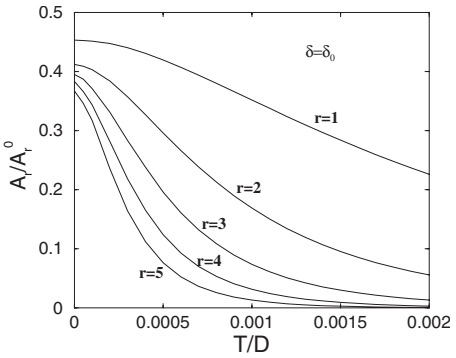

FIG. 2. de Haas-van Alphen amplitudes for the first five harmonics r as a function of T for a fixed magnetic field for the two-pocket model tuned to the QCP, $\delta_0=0.07$. The amplitudes are normalized to the amplitude for the noninteracting system. Here, $V\rho_F=U\rho_F=0.2$ and $D=10$ and if D is equated to 1000 K, the magnetic field is 2 T with m^* being the free electron mass. For other values of m^*, B has to be renormalized by m^*/m, i.e., for $m^*/m=10$ the field corresponds to 20 T.

V. AMPLITUDE OF DE HAAS-VAN ALPHEN OSCILLATIONS

In this section, we study the effects of the quasiparticle linewidth on the dHvA oscillation amplitude close to the QCP. Our model consists of two Fermi surfaces, namely, the electron and hole pockets. There are then two orbits corresponding to the extremal cross-sectional areas of the Fermi surface, which are both circular in our model, with radii k_{F1} and k_{F2}, respectively. There are then two fundamental frequencies of oscillation. In the absence of interactions, the oscillatory part of the thermodynamic potential in a uniform magnetic field B is given by the Lifshitz–Kosevich expression [Eq. (1)]. The quasiparticle linewidth induced by the interactions substantially modifies this expression and the amplitude of the oscillations is now given by Eq. (3). To simplify, we assume that both pockets have the same self-energy and neglect the effect of the Zeeman splitting.

The total linewidth, $\Gamma(\omega)=\Gamma_{\mathrm{NFL}}(\omega)+\Gamma_{\mathrm{FL}}(\omega)$, is an even function of ω. Hence, all of the conditions mentioned in Sec. II are satisfied. The quantity $X(\xi_n)$ is then given by the Cauchy transform of $\Gamma(\omega)$,

$$X(\xi_n) = -\xi_n \int_{-D}^{D} \frac{d\omega}{\pi} \frac{\Gamma(\omega)}{\omega^2+[\pi T(2n+1)]^2}. \quad (12)$$

It is straightforward to numerically carry out this integration. By inserting $X(\xi_n)$ into Eq. (3), we evaluate the amplitude A_r relative to the amplitude for the noninteracting system, $A_r^0 = 1/[2\ \sinh(2\pi^2 k_B T r/\hbar\omega_c)]$.

Figure 2 shows the oscillation amplitude for the first five harmonics as a function of temperature for $\delta_0=0.07$, which

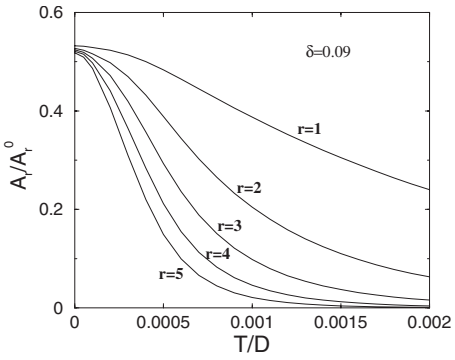

FIG. 3. de Haas-van Alphen amplitudes for the first five harmonics r as a function of T for a fixed magnetic field for the two-pocket model tuned away from the QCP, with $\delta=0.09$. The amplitudes are normalized to the amplitude for the noninteracting system. The remaining parameters are the same as those in Fig. 2.

corresponds to the tuned QCP. The T dependence is qualitatively similar to that of the marginal FL (see Fig. 1). The overall reduction in the amplitudes is, however, considerably larger close to the QCP. Also, as a function of T, the amplitudes decrease much faster as in Fig. 1 for the marginal FL. If the bandwidth D of the heavy carriers in Eq. (5) is 1000 K and the unrenormalized effective mass is 20 times that of free electrons, then Fig. 2 corresponds to an external field of 40 T and $T/D=0.001$ to 1 K. It is then necessary to have a very low Dingle temperature to observe more than the fundamental frequency. The situation is more favorable at lower T.

In Figs. 3 and 4, we present the amplitudes for larger mismatches of the Fermi surfaces, namely, $\delta=0.09$ and $\delta=0.11$, respectively. As we move away from the QCP, the amplitudes slightly increase but the general trends remain the same. Since expression (3) involves an integration over the frequency (actually a sum over the fermion Matsubara poles), the crossover between the FL and NFL regions is smeared and cannot be seen in the amplitudes. The overall trend, which is independent of the value of δ, is that A_r/A_r^0 is an increasing function as T is lowered.

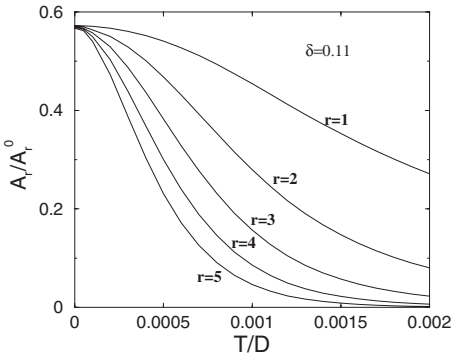

FIG. 4. de Haas-van Alphen amplitudes for the first five harmonics r as a function of T for a fixed magnetic field for the two-pocket model tuned away from the QCP, with $\delta=0.11$. The amplitudes are normalized to the amplitude for the noninteracting system. The remaining parameters are the same as those in Fig. 2.

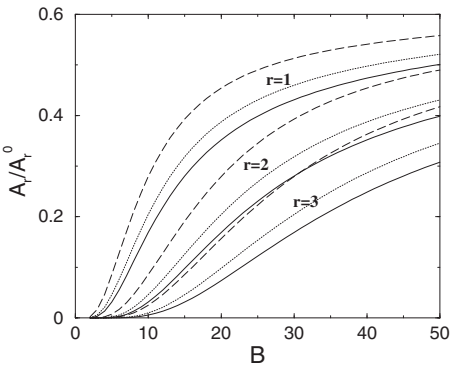

FIG. 5. Magnetic field dependence of the oscillation amplitudes (normalized to the amplitude for the noninteracting system) for constant $T=0.001$ and $r=1$, 2, and 3. The solid lines correspond to the tuned QCP with $\delta_0=0.07$, the dotted curves to that with $\delta=0.09$ and the long dashed curves to that with $\delta=0.11$. The remaining parameters are the same as those in Fig. 2.

The magnetic field dependence of the oscillation amplitude for a constant T is shown in Fig. 5 for the first three harmonics and $\delta=0.07$, 0.09, and 0.11. The amplitudes relative to the noninteracting system increase with increasing field. In other words, the effects of the self-energy are much more dramatic at small fields. The amplitude is most reduced for the tuned QCP. As we increase the nesting mismatch, the NFL behavior is suppressed at low T and the smaller linewidth due to the FL behavior plays a dominant role.

VI. OSCILLATIONS IN THE ORDERED PHASE

Due to the spin waves that are thermally excited close to T_N, it is difficult to calculate the damping of the dHvA amplitudes near the transition. We will limit ourselves to following the periods of oscillations through the transition. A mean-field approach should be sufficient to obtain the cross sections of the Fermi surface for the ordered phase. The mean-field approximation for the present model[15] yields a BCS-type equation for the AF order parameter Δ,

$$1 = -\frac{V\rho_F}{2}\int_{-D}^{D} d\epsilon \frac{f(\omega_+) - f(\omega_-)}{\sqrt{[\epsilon - (k_{F1}+k_{F2})v_F/2]^2 + \Delta^2}}, \quad (13)$$

where

$$\omega_\pm = \delta \pm \sqrt{[\epsilon - (k_{F1}+k_{F2})v_F/2]^2 + \Delta^2} \quad (14)$$

are the dispersions in the presence of long-range order. Since the total number of electrons is conserved, the Fermi volume has to be invariant. Hence, if $k_{F1}=k_{F2}$, the system becomes an insulator due to the AF gap opening at the Fermi level. If $\delta\neq 0$, on the other hand, the system remains a metal. If $k_{F2}>k_{F1}$, the ω_+ band is empty and ω_- is partially filled; while if $k_{F2}<k_{F1}$, the ω_- band is full and ω_+ is partially populated. The Fermi surface still consists of two spheres, one electronlike and the other holelike. The new Fermi momenta are determined by the solutions of the quadratic equation $\omega=\mu$ for $\epsilon=v_F k$, where ω is the partially filled band and μ is the chemical potential so that the Fermi volume is conserved.

With increasing AF order, the Fermi momenta gradually evolve from the Fermi momenta in the paramagnetic phase, k_{F1} and k_{F2}. There is no abrupt change in the dHvA frequencies of oscillation at the QCP. This is in contrast to experimental evidence for $YbRh_2Si_2$, for which the Hall constant indicates that the heavy electrons are abruptly localized by the magnetism.[24] The sudden jump in R_H is obtained as an extrapolation to zero temperature. Changes in the Fermi volume at the QCP can only be achieved within a two-band model, i.e., the Anderson lattice with conduction and localized f electrons. In the paramagnetic heavy-fermion phase, the two type of states merge into a single heavy-fermion band. The present model is sufficient to describe the heavy fermions but not their breaking up. If at the QCP the heavy electron states are broken up, giving rise to the AF order of local magnetic moments rather than itinerant AF as for a spin-density wave, then the more complex Anderson lattice model needs to be considered.

VII. CONCLUSIONS

We calculated the amplitudes of the dHvA oscillations in the FL and NFL phases for a model consisting of spherical Fermi surfaces of one parabolic electron pocket and one parabolic hole pocket, which are assumed to be part of the heavy electron bands of a rare earth (actinide) compound. The electrons of both pockets interact with each other via a weak repulsive force, which (in the sense of a Fermi liquid) is the remainder of the strong correlations after the heavy particles are formed. The interaction and the nesting of the two Fermi surfaces give rise to instabilities of the spin-density and charge-density wave type. For perfect nesting (electron-hole symmetry), an arbitrarily small interaction is sufficient for a ground state with long-range order. The degree of nesting is controlled by the mismatch parameter δ and by varying δ the ordering temperature can be tuned to zero, leading to a QCP.

The QCP itself is elusive to experiment, and the NFL behavior is concluded by extrapolating the data to zero temperature. Landau's Fermi liquid theory predicts that for normal metals, the quasiparticle linewidth is proportional to ω^2 and T^2. The nesting condition of the Fermi surface modifies this behavior to a $|\omega|$ and T dependence for the tuned QCP.[17] For $\delta > \delta_0$, at low T there is a crossover from FL to NFL behavior with increasing T.

The Lifshitz–Kosevich expression for the oscillatory part of the thermodynamic potential is modified due to the presence of interactions between the carriers.[22] The oscillation amplitudes can be expressed as a sum over the quasiparticle self-energy evaluated at the fermion Matsubara poles. The interactions always reduce the oscillation amplitudes. In the present case, the sum over the Matsubara poles smears the details of the crossover from FL to NFL behavior. Hence, qualitatively, the temperature dependence of the amplitudes is quite similar to that of a marginal FL. The suppression of the amplitudes is strongest for the tuned QCP. The NFL effects can be seen even far away from the QCP. For the present model, the frequencies of oscillation smoothly change through the QCP. This is in contrast to the experimental observations for the Hall constant of $YbRh_2Si_2$, wherein a change of carrier density at the QCP was concluded.[24]

The dHvA oscillations in the magnetization or the de Haas–Shubnikov oscillations in the resistivity are periodic as a function of B^{-1}. Hence, they are measured over a magnetic field interval. The amplitude of oscillation cannot be then associated with a given field, but it corresponds to that interval. On the other hand, the magnetic field also frequently acts as a tuning parameter for the QCP. Hence, a discussion of the oscillation amplitudes is only meaningful if the magnetic field, within the regime of measurement, does not affect the tuning of the QCP.

ACKNOWLEDGMENTS

The support by the U.S. Department of Energy under Grant No. DE-FG02-98ER45707 is acknowledged.

[1] G. R. Stewart, Rev. Mod. Phys. **73**, 797 (2001).
[2] H. v. Löhneysen, Physica B **206-207**, 101 (1994).
[3] M. B. Maple, C. L. Seaman, D. A. Gajewski, Y. Dalichaouch, V. B. Barbetta, M. C. de Andrade, H. A. Mook, H. G. Lukefahr, O. O. Bernal, and D. E. MacLaughlin, J. Low Temp. Phys. **95**, 225 (1994).
[4] B. Andraka and A. M. Tsvelik, Phys. Rev. Lett. **67**, 2886 (1991).
[5] A. M. Tsvelik and M. Reizer, Phys. Rev. B **48**, 9887 (1993).
[6] A. J. Millis, Phys. Rev. B **48**, 7183 (1993).
[7] M. A. Continentino, Phys. Rev. B **47**, 11587 (1993).
[8] T. Moriya and T. Takimoto, J. Phys. Soc. Jpn. **64**, 960 (1995); T. Takimoto and T. Moriya, Solid State Commun. **99**, 457 (1996).
[9] A. Rosch, A. Schröder, O. Stockert, and H. v. Löhneysen, Phys. Rev. Lett. **79**, 159 (1997).
[10] O. Stockert, H. v. Löhneysen, A. Rosch, N. Pyka, and M. Loewenhaupt, Phys. Rev. Lett. **80**, 5627 (1998).
[11] W. Montfrooij, M. C. Aronson, B. D. Rainford, J. A. Mydosh, A. P. Murani, P. Haen, and T. Fukuhara, Phys. Rev. Lett. **91**, 087202 (2003); W. Montfrooij, M. C. Aronson, B. D. Rainford, J. A. Mydosh, R. Hendrikx, T. Gortenmulder, A. P. Murani, P. Haen, I. Swainson, and A. de Visser, Phys. Rev. B **73**, 140401(R) (2006).
[12] L. Balicas, S. Nakatsuji, H. Lee, P. Schlottmann, T. P. Murphy, and Z. Fisk, Phys. Rev. B **72**, 064422 (2005).
[13] O. Trovarelli, C. Geibel, S. Mederle, C. Langhammer, F. M. Grosche, P. Gegenwart, M. Lang, G. Sparn, and F. Steglich, Phys. Rev. Lett. **85**, 626 (2000).
[14] P. Gegenwart, J. Custers, C. Geibel, K. Neumaier, T. Tayama, K. Tenya, O. Trovarelli, and F. Steglich, Phys. Rev. Lett. **89**, 056402 (2002).
[15] P. Schlottmann, Phys. Rev. B **59**, 12379 (1999).
[16] P. Schlottmann, Phys. Rev. B **68**, 125105 (2003).

[17] P. Schlottmann, Phys. Rev. B **73**, 085110 (2006).
[18] P. Schlottmann, Phys. Rev. B **74**, 235103 (2006).
[19] P. Schlottmann, Phys. Rev. B **75**, 205108 (2007).
[20] S. Engelsberg and G. Simpson, Phys. Rev. B **2**, 1657 (1970).
[21] S. Engelsberg, Phys. Rev. B **18**, 966 (1978).
[22] A. Wasserman and M. Springford, Adv. Phys. **45**, 471 (1996).
[23] A. Wasserman, M. Springford, and F. Han, J. Phys.: Condens. Matter **3**, 5335 (1991).
[24] S. Paschen, T. Lühmann, S. Wirth, P. Gegenwart, O. Trovarelli, C. Geibel, F. Steglich, P. Coleman, and Q. Si, Nature (London) **432**, 881 (2004).