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Stochastic resonance in a stochastic bistable system subject to additive white noise and dichotomous noise[☆]

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ABSTRACT

The stochastic resonance (SR) in a stochastic stable system driven by a static force and a periodic square-wave signal as well as by additive white noise and dichotomous noise is considered from the point of view of the signal-to-noise ratio (SNR). It is found that the SNR exhibits SR behavior when it is plotted as a function of the noise strength of the white noise and dichotomous noise, as well as when plotted as a function of the static force. Moreover, the influence of the strength of the stochastic potential force and the correlation rate of the dichotomous noise is investigated.

1. Introduction

The stochastic resonance (SR) phenomenon is a cooperative effect between a weak signal and noise in a nonlinear system, which leads to an enhanced response to the periodic force. It first appeared in 1981, when it was proposed by Benzi et al. [1] as a plausible mechanism for almost periodic occurrences of ice ages on Earth. SR was first demonstrated in a laboratory in Schmitt triggers, and it was experimentally observed in 1983 [2]. The SR phenomenon has been studied in a variety of nonlinear systems with additive and multiplicative noise [1–9]. It was concluded that nonlinearity, periodic forces and random forces are the essential ingredients for the onset of SR. On the other hand, behavior similar to SR has also been found in linear systems subject to multiplicative noise [10–13]. Gora [13] established a genuine SR in a linear system subject to mutually correlated additive and multiplicative white noises. It was suggested that noise multiplicativity and time correlation are the necessary conditions for SR to occur in a linear system.

In the last two decades, lots of work about SR has been referred to the consideration of fluctuations depending on the time, yet some research has studied the fluctuations depending on the state variable [3–9]. Dunlap and co-workers [3–5] studied the nonlinear mobility of a classical particle moving in an infinite one-dimensional space spanned by the coordinate x, and subject to a potential $U(x)$ with period L and external electric field, where the potential is a random stationary potential which depends on the state variable. Jia et al. [6] investigated the effects of random potential on the transport of two systems. Their results showed that the effects of the random potential on the transport process as the amplitude of random potential increased are much more remarkable than those as the correlation length of the random potential increased. Tongjun Zhao et al. [7] investigated the motion of an overdamped Brownian particle in a periodic potential subjected to

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a position-dependent perturbation and a sinusoidal external force. Bao-quan Ai and Liang-guang Liu [8] investigated the stochastic resonance in a bistable system in the presence of a stochastic potential. Li Jing-hui [9] investigated a single protein motor system with fluctuating potential barrier and subject to a sinusoidal electric field. They calculated the signal-to-noise ratio in the adiabatic limit and found the stochastic resonance phenomenon for the motor system.

The study of dichotomous noise (DN) is becoming increasingly popular due to its relatively simple calculation and the well-defined procedures leading from dichotomous noise to both Gaussian white noise and white shot noise, especially when the noise is introduced in a nonlinear equation. DN can be generated by a two-state Poisson process and it be formed by quasiparticles (defects, impurities, spins, etc.) jumping between two-level systems [14,15]. For example, a digital circuit may be perturbed by Gaussian white noise which has resulted from the background noise around the circuit; meanwhile, it can also be disturbed by random telegraph noise (one form of dichotomous noise) which is induced by other digital circuits close to it. Therefore, the study of a system with white noise and dichotomous noise is of practical significance. Casado-Pascual et al. [16] explored the SR in noisy bistable, symmetric systems driven by periodic rectangular signals. They depicted the nonmonotonic behavior versus the noise strength of several SR quantifiers.

In the present paper, with the introduction of a dichotomous noise into a stochastic bistable system driven by additive white noise as well as a constant and a square-wave periodic signal, we investigate the SR of the system's signal-to-noise ratio as a function of the parameter of the noise and of the input signal.

2. A stochastic bistable system and its signal-to-noise ratio

We consider the Brownian motion of a particle in a bistable potential $U(x)$ driven by an external force $f(t)$ and a noise term $\eta(t)$, described by the following Langevin equation:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\mathrm{d}U(x)}{\mathrm{d}x} + \eta(t) + f(t), \tag{1}$$

where $\eta(t)$ is a Gaussian white noise with

$$\langle \eta(t)\rangle = 0, \qquad \langle \eta(t_1)\eta(t_2)\rangle = 2D\delta(t_1 - t_2) \tag{2}$$

and

$$f(t) = r + \varepsilon\Gamma(t) + As(t), \tag{3}$$

where r is a constant denoting a static force and $\Gamma(t)$ is a dichotomous noise with correlation rate λ, $\Gamma(t) = \pm 1$. $\Gamma(t)$ is uncorrelated with $\eta(t)$. $s(t)$ is a periodic square-wave signal with period T,

$$s(t) = \begin{cases} -1, & 0 < t \le T/2 \\ 1, & T/2 < t \le T. \end{cases} \tag{4}$$

The potential function of the system has the form

$$U(x) = U_0(x) + \xi(x), \tag{5}$$

where $U_0(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4$ is a deterministic bistable potential with unstable states $x_\pm = \pm\sqrt{a/b}$ and stable state $x_0 = 0$. The stochastic potential $\xi(x)$ is dichotomous [3,5–7]; it takes only two values separated by 2Δ, making discontinuous jumps at random points along the one-dimensional space, and

$$\xi(x) = \Delta(-1)^{n(x,0)}, \tag{6}$$

where the randomness of the function $\xi(x)$ is expressed in terms of the random function $n(x_2, x_1)$, which counts the number of jumps the potential makes between the values $+\Delta$ and $-\Delta$ in the interval between $x = x_1$ and $x = x_2$. The mean of the random function $n(x_2, x_1)$ is

$$\langle n(x_2, x_1)\rangle = \frac{|x_2 - x_1|}{l}, \tag{7}$$

where the correlation length l is the mean distance between jumps. The probability distribution of $n(x, 0)$ is Poissonian. Applying the properties of $n(x_2, x_1)$ and $\xi(x)$, one can obtain the following equations [17]:

$$\langle \xi(x)\rangle = 0, \tag{8}$$

$$\langle \xi(x_1)\xi(x_2)\rangle = \Delta^2 \exp\left(-\frac{2\,|x_1 - x_2|}{l}\right), \tag{9}$$

$$[\xi(x)]^{2n} = \Delta^{2n}, \qquad [\xi(x)]^{2n+1} = \Delta^{2n}\xi(x), \tag{10}$$

$$\left\langle \exp\left[\frac{\xi(x_1) - \xi(x_2)}{D}\right]\right\rangle_\xi = \exp\left\{\left(\frac{\Delta}{D}\right)^2\left[1 - \exp\left(-\frac{2\,|x_1 - x_2|}{l}\right)\right]\right\}. \tag{11}$$

From Eqs. (1), (3), (5) and (6), we can see that the system (1) is actually driven by two dichotomous noises: one noise is included in the force $f(t)$, and the other results from the differentiation of the potential $U(x)$, Eqs. (5) and (6).

We assume that the amplitude of the square wave $A \ll 1$ and that its frequency is so small that there is enough time for the system to reach the local equilibrium during the period of the square wave, i.e., we make the assumption that the system satisfies the adiabatic approximation condition [18]. The quasi-stationary distribution function can be written as

$$\rho_{st} = N \exp\left[-\frac{\Phi(x,t)}{D}\right], \tag{12}$$

where N is the normalization constant and $\Phi(x,t)$ is the rectified potential function

$$\Phi(x,t) = U_0(x) + \xi(x) - xf(t). \tag{13}$$

Under the adiabatic limit condition, the transition rates out of $x_\pm$ can be expressed as

$$\begin{aligned} W_\pm(f(t)) &= \frac{a}{\sqrt{2}\pi} \exp\left[-\frac{\Phi(x_0) - \Phi(x_\pm)}{D}\right] \\ &= \frac{a}{\sqrt{2}\pi} \exp\left\{\left(\frac{\Delta}{D}\right)^2 \left[1 - \exp\left(-\frac{2\sqrt{a}}{l\sqrt{b}}\right)\right]\right\} \exp\left[-\frac{a^2}{4Db} \mp \sqrt{\frac{a}{b}}\frac{f(t)}{D}\right] \\ &= W_0 \exp\left[\mp\sqrt{\frac{a}{b}}\frac{f(t)}{D}\right] \end{aligned} \tag{14}$$

where W_0 denotes the characteristic switching frequency of the bistable system when it is only driven by the additive noise $\eta(t)$,

$$W_0 = \frac{a}{\sqrt{2}\pi} \exp\left\{\left(\frac{\Delta}{D}\right)^2 \left[1 - \exp\left(-\frac{2\sqrt{a}}{l\sqrt{b}}\right)\right] - \frac{a^2}{4Db}\right\}. \tag{15}$$

Applying the method in Ref. [19], we can get the expression for the correlation function

$$K(\tau) = [B_1(r,\varepsilon,A) + B_3(r,\varepsilon,A)] \exp(-\lambda|t|) + B_2(r,\varepsilon,A)\varphi(t) + C(r,\varepsilon,A)\delta(t), \tag{16}$$

where

$$\varphi(t) = \frac{4}{\pi^2}\sum_{j=0}^{\infty}(2j+1)^{-2}\exp[-i(2j+1)\Omega t] \tag{17}$$

$$B_1(r,\varepsilon,A) = \frac{1}{16}\left[w(r+\varepsilon+A) - w(r-\varepsilon-A) + w(r+\varepsilon-A) - w(r-\varepsilon+A)\right]^2, \tag{18}$$

$$B_2(r,\varepsilon,A) = \frac{1}{16}\left[w(r+\varepsilon+A) - w(r-\varepsilon-A) - w(r+\varepsilon-A) + w(r-\varepsilon+A)\right]^2, \tag{19}$$

$$B_3(r,\varepsilon,A) = \frac{1}{16}\left[w(r+\varepsilon+A) + w(r-\varepsilon-A) - w(r+\varepsilon-A) - w(r-\varepsilon+A)\right]^2, \tag{20}$$

$$C(r,\varepsilon,A) = \frac{1}{4}\left\{C_0(r+\varepsilon+A) + C_0(r-\varepsilon-A) + C_0(r+\varepsilon-A) + C_0(r-\varepsilon+A)\right\}, \tag{21}$$

$$w(\mu) = \sqrt{\frac{a}{b}}\frac{W_- - W_+}{W_- - W_+} = \sqrt{\frac{a}{b}}\frac{\exp[2\sqrt{a}\mu/(D\sqrt{b})] - 1}{\exp[2\sqrt{a}\mu/(D\sqrt{b})] + 1}, \tag{22}$$

$$C_0(\mu) = \frac{8W_0^2}{[W_-(\mu) + W_+(\mu)]^3}. \tag{23}$$

The power spectrum, defined as the Fourier transform of the correlation function, is given by

$$S(\omega) = S_1(0) + S_2(\omega), \tag{24}$$

where

$$S_1(0) = \frac{2}{\lambda}[B_1(r,\varepsilon,A) + B_3(r,\varepsilon,A)] + C(r,\varepsilon,A), \tag{25}$$

$$S_2(\omega) = B_2(r,\varepsilon,A)\varphi(\omega), \tag{26}$$

$$\varphi(\omega) = \frac{8}{\pi}\sum_{j=0}^{\infty}(2j+1)^{-2}\delta[\omega - (2j+1)\Omega]. \tag{27}$$

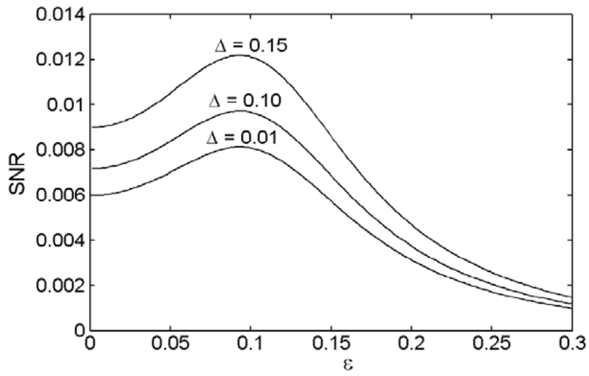

Fig. 1. The SNR versus the dichotomous noise strength ε for $a = 1$, $b = 1$, $D = 0.1$, $l = 10$, $\lambda = 100$, $A = 0.05$, $r = 0.1$ for varied intensity Δ of the stochastic potential.

Here $S_1(0)$ denotes the power density at the zero frequency associated with the noise background, and $S_2(\omega)$ is the power spectrum of the output signal. The output signal-to-noise ratio (SNR) is the ratio between the power density of the signal and the noise at the signal frequency, which has the form

$$\mathrm{SNR} = \frac{8}{\pi} \frac{B_2(r, \varepsilon, A)}{C(r, \varepsilon, A) + 2[B_1(r, \varepsilon, A) + B_3(r, \varepsilon, A)]/\lambda}. \tag{28}$$

3. Discussion and conclusions

Dybiec et al. [20,21] investigated the escape of a Brownian particle over a dichotomously fluctuating barrier for various shapes of barrier. They analyzed the characteristic features of resonant behavior for situations when the barrier switches either between different heights, representing erection of a barrier and formation of a well, respectively, or when it proceeds through "on" and "off" positions. They presented numerically results for the mean first passage time (MFPT) over the fluctuating barrier. We point out that our system is different from those studied in Refs. [20,21]. (i) In Refs. [20,21], the derivative of the stochastic potential function at state variable x is a product of a dichotomous noise with respect to the time variable t and a function $g(x)$ of x, while in our stochastic system the stochastic potential function is a dichotomous potential with respect to the state variable x. (ii) The driving force in Refs. [20,21] is only a Gaussian white noise, while in our system it also includes an additive dichotomous noise and a statistic force. (iii) The authors in Refs. [20,21] found the nonmonotonic behavior of the MFPT as a function of the barrier fluctuation rate, i.e., as a function of the correlation rate of the dichotomous noise, while in the present paper, our aim is to obtain the expression of the output signal-to-noise ratio (SNR) and analyze its dependence on the correlation rate of the dichotomous noise, on the statistic force as well as on the noise intensity.

The influence of the dichotomous noise strength on the system output signal-to-noise ratio (SNR) is analyzed in Fig. 1. As seen from Fig. 1, the SNR varies nonmonotonically with the increase of the strength ε of the dichotomous noise; for fixed other parameters, the SNR reaches its maximum at some intermediate value of ε, and so the SR phenomenon takes place in this case. Meanwhile, the SNR increases monotonically with the variety of the increase of the intensity Δ of the dichotomous potential $\xi(x)$, which means that the dichotomous potential can improve the output signal-to-noise ratio.

In practice, physical systems are usually asymmetric. Therefore, some new phenomena may occur: for example, the nonzero currents induced in stochastic ratchets [22] and the occurrence of even and odd spectral harmonics in the power spectral density of a bistable system [23–26]. Gora [13] studied the SR in a linear system with constant driving. He believed that constant driving is needed to product the SR in the linear system he investigated. In this study, we consider the case of the stochastic bistable potential $U(x)$ subject to a symmetry-breaking direct-current signal, i.e., the constant r, which can be also regarded as a static force. Since r can break the asymmetry of the system potential function, it will consequently influence the potential barrier, and thus affect the system output. From Fig. 2, we see that the SNR also varies nonmonotonically with the variety of the constant r. The SNR curve is almost symmetric for positive and negative values of r. When the absolute value $|r|$ increases, the SNR curve reaches a peak and then decreases for larger value $|r|$. In addition, the SNR increases with the increase of the correlation rate λ of the dichotomous noise.

For a bistable system subject to additive noise, when a small periodic modulation is applied to the potential at a modulation frequency much smaller than the intrawell relaxation rate, the thermal activation rates are modulated periodically in time. At some optimal noise level, the transitions between the two wells also occur almost periodically in time; the timescale associated with the Kramers rate equals approximately half the signal period. In this case SR takes place. In this work, we analyze the SR phenomenon in Figs. 3–5. From these figures, one immediately concludes that the SR phenomenon occurs in the SNR curves versus the additive noise intensity D. From Figs. 3 and 4, one can see two peaks on the SNR curve. With the increase of the correlation rate λ of the dichotomous noise and the intensity Δ of the dichotomous potential $\xi(x)$, the height of first peak increases. Therefore, for small strength of the additive noise, the increase of the

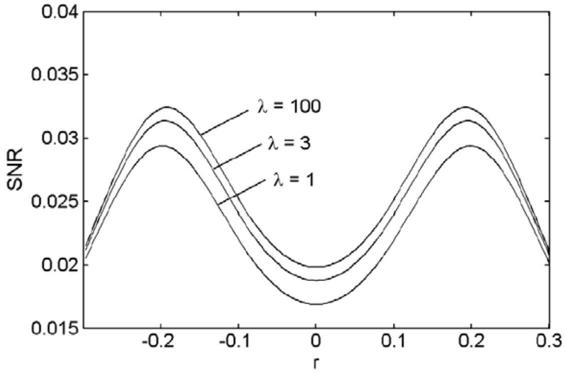

Fig. 2. The SNR versus the system bias r for $\varepsilon = 0.2$, $a = 1$, $b = 1$, $D = 0.18$, $l = 10$, $\Delta = 0.1$, $A = 0.1$ for varied correlation rate λ of the dichotomous noise.

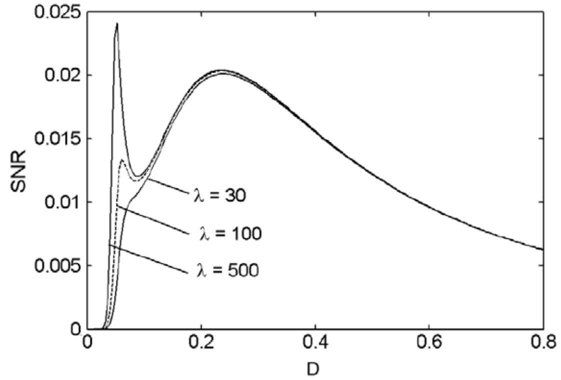

Fig. 3. The SNR versus the additive noise strength D for $a = 1$, $b = 1$, $\varepsilon = 0.3$, $l = 10$, $\Delta = 0.3$, $r = 0.1$, $A = 0.1$ for varied correlation rate λ of the dichotomous noise.

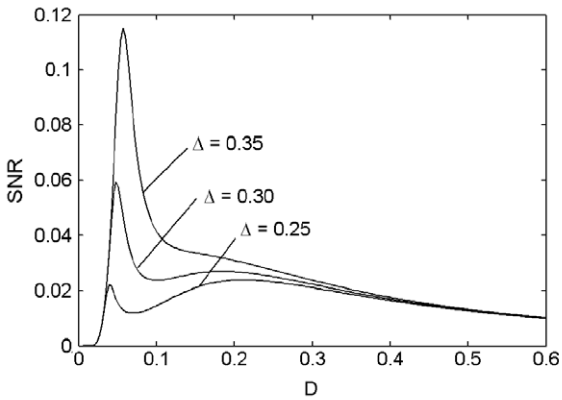

Fig. 4. The SNR versus the additive noise strength D for $a = 1$, $b = 1$, $\varepsilon = 0.25$, $l = 10$, $\lambda = 20$, $r = 0.1$, $A = 0.1$ for varied intensity Δ of the stochastic potential.

correlation rate λ, i.e., the flipping frequency of the barrier fluctuations and the increase of the intensity of the dichotomous potential, can dramatically improve the output SNR, while for large intensity of the additive noise, the effect of the correlation rate λ of the dichotomous noise and the intensity Δ of the dichotomous potential on the SNR is very weak.

In the above we have studied the stochastic resonance in a stochastic bistable system driven simultaneously by a static force, an additive white noise, a square-wave signal and by a dichotomous noise. Under the adiabatic approximation condition, the analytical expression of the signal-to-noise ratio (SNR) is obtained. The results show that the SNR presents stochastic resonance behavior when it is plotted versus the intensity of the additive noise and dichotomous noise as well as versus the valued of the static force. In addition, the intensity of the dichotomous potential and the correlation rate of the dichotomous noise can improve the output signal-to-noise ratio.

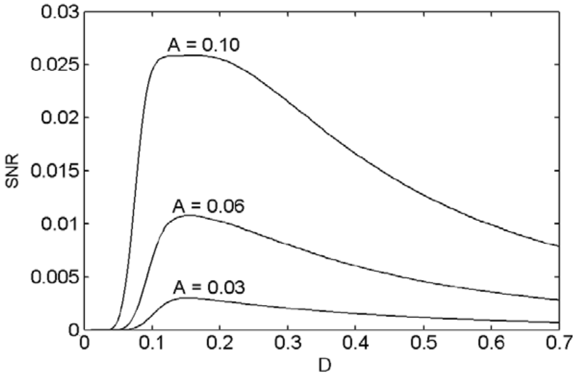

Fig. 5. The SNR versus the additive noise strength D $a = 1$, $b = 1$, $\varepsilon = 0.3$, $l = 5$, $\lambda = 10$, $\Delta = 0.3$, $r = 0.1$ for varied amplitude of the square-wave signal.

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