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Relaxation of a Thermal Perturbation in a Collisional Plasma

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Abstract—The relationship between the nonlocal and nonstationary effects in electron heat transport processes in a weakly collisional plasma is investigated by considering the problem of the relaxation of a thermal perturbation as an example. It is shown that, for small-scale perturbations, the electron thermal conductivity depends not only on the temperature scale length but also on time. The consequence is that there exist two qualitatively different characteristic relaxation regimes of thermal perturbations on small and large scale lengths. As a result, the evolution of hot spots in laser plasmas should be described with allowance for the nonstationary nature of electron heat transport. In the course of this evolution, relaxation on the collisional kinetic time scale is clearly seen to change into relaxation on the collisional hydrodynamic time scale.

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1. INTRODUCTION

The problem of electron heat transport is one of the key issues in laser fusion. The nonlocal nature of heat transport in a laser plasma is confirmed both by experimental data [1–3] and by theoretical results [4–9]. It is now well established that, on temperature scale lengths that are on the order of or shorter than 100 electron mean free paths, the electron heat flux differs from the local one in the classical theory [10, 11]. So far, theoretical models of electron heat transport have been developed based on the quasi-stationary approximation, in which the heat flux is considered to be nonlocal in space but local in time. This implies that electron thermal conductivity does not depend explicitly on time. On the other hand, in kinetic simulations, it was found that not only the spatially nonlocal nature of heat transport processes but also their nonstationary nature play an important role in a description of transport in a plasma with small-scale temperature variations [12]. It should be noted that the problem of nonstationary transport has a long history: it was formulated many years ago [13, 14] in a hydrodynamic model approach by supplementing the classical hydrodynamic equations with an equation for the temporal evolution of the heat flux. Even this simplest model showed that the character of transport processes can change substantially. The objective of the present paper is to investigate the nonstationary nature of electron heat transport by solving the kinetic equation with the exact collision integral. As an example, we consider the relaxation of a thermal perturbation that occurs initially on a spatial scale shorter than the electron mean free path.

The problem of the temperature relaxation under nonlocal heat transport conditions is of interest not only from the standpoint of fundamental studies but also because it is a challenging problem in the implementation of the idea of using speckled laser beams in laser fusion experiments in order to control unavoidable nonuniformities in laser radiation, to ensure a high efficiency of laser energy deposition in a fusion target, and to achieve a uniform compression of the targets. It should be noted, however, that the speckles (hot spots) arising in the plasma are fairly small—their transverse dimensions (about 1–3 μm) are less than the electron mean free path; consequently, the classical transport theory cannot be used to describe their relaxation, which occurs under intermediate collisional conditions, i.e., between the collisionless and collisional transport regimes. In addition, the characteristic relaxation time of such small-scale temperature inhomogeneities is comparable to the electron collision time. The problem of the relaxation of an individual laser hot spot was solved in [15] on the basis of the quasi-stationary transport theory. It is now clear that the initial size of such a spot is bounded from below. Moreover, the nonstationary effects have a great influence on the relaxation of a thermal perturbation that occurs initially on a spatial scale on the order of the electron mean free path.

The paper is organized as follows. In Section 2, we obtain an exact kinetic solution to the initial-value problem of the relaxation of a small thermal perturbation. In Section 3, we analyze the relationship between the nonlocal and nonstationary transport effects for the case of a periodic single-mode thermal perturbation. In Section 4, we study the relaxation of a spatially localized perturbation of the mean electron energy (effective temperature) of a hot spot. Finally, in Section 5, we draw conclusions about the conditions under which nonstationary effects can manifest themselves in electron heat transport and about their regular properties.

2. SOLUTION OF THE INITIAL-VALUE PROBLEM OF THE RELAXATION OF A THERMAL PERTURBATION

As a background, we consider a plasma in which the electrons have the density n_e and temperature T_e and are characterized by a Maxwellian distribution function $F_0(v)$. Let us consider the Cauchy problem for a small deviation $\delta f_e(v, x, t)$ from the distribution function $F_0(v)$ due to perturbations of the equilibrium density and temperature [12, 15]: $\delta f_e(v, x, t = 0) = \delta F_0 = [\delta n(0)/n_e + (m_e v^2/2T_e - 3/2)\delta T(0)/T_e]F_0$. Since we are interested only in the relaxation of the thermal energy, we take into account exclusively the temperature perturbations $\delta T(0)$ and set $\delta n(0) = 0$. This corresponds, e.g., to the problem of the relaxation of a rapidly developing hot spot when the density perturbations due to the excitation of plasma waves are small, which is the case if the temperature is perturbed on spatial scales much larger than the Debye radius. Hence, we characterize the initial perturbation $\delta f_e(v, x, 0)$ by the perturbed mean electron energy (temperature) $\delta T(x, 0)$:

$$\delta f_e(v, x, t = 0) = \frac{\delta T(x, 0)}{T_e}\left(\frac{v^2}{2v_{Te}^2} - \frac{3}{2}\right)F_0(v), \quad (1)$$

where $v_{Te} = \sqrt{T_e/m_e}$ is the thermal velocity of an electron with mass m_e and charge e.

The linearized kinetic equation for the (ω, k) Fourier component of the perturbation δf has the form

$$-i(\omega - \mathbf{k}\cdot\mathbf{v})\delta f - \frac{e}{m_e}\mathbf{E}\frac{\partial F_0}{\partial \mathbf{v}} = C_{ei}[\delta f] + C_{ee}[\delta f, F_0] + \delta f_e\big|_{t=0}, \quad (2)$$

where $\mathbf{E}$ is the self-consistent longitudinal electric field and C_{ei} and C_{ee} are the electron–ion and electron–electron collision integrals. Expanding the perturbation δf in Legendre polynomials, $\delta f = \sum_{l=0}^{\infty} f_l(\omega, k, v)P_l(\theta)$, we obtain the following infinite set of equations for the angular harmonics f_l of the perturbed distribution function:

$$-i\omega f_0 + \frac{ikv}{3}f_1 = C_{ee}[f_0] + \delta f_e(v, t = 0) \quad (l = 0), \quad (3)$$

$$-i\omega f_1 + ikvf_0 + ikv\frac{2}{5}f_2 - \frac{eE}{m_e}\frac{\partial F_0}{\partial v} = -\nu_{ei}f_1 \quad (l = 1), \quad (4)$$

$$-i\omega f_l + ikv\frac{l}{2l-1}f_{l-1} + ikv\frac{l+1}{2l+3}f_{l+1} = -\frac{l(l+1)}{2}\nu_{ei}f_l \quad (l > 1), \quad (5)$$

where $\nu_{ei}(v) = 4\pi Z n_e e^4\Lambda/(m_e^2 v^3)$ is the velocity-dependent electron–ion collision frequency, Λ is the Coulomb logarithm, and Z is the ion charge number in the plasma. The set of equations is written for a highly ionized plasma, $Z \gg 1$; in this case, the electron–electron collisions can be ignored in comparison with the electron–ion collisions in the equations for all the angular harmonics of the distribution function, except in the equation for its symmetric ($l = 0$) part, because this equation does not incorporate electron–ion collisions.

The procedure of summation of an infinite series in order to obtain a solution to Eqs. (3)–(5) was described in [9, 16]. It implies introducing the modified electron collision frequency ν_l that satisfies the recurrence relation

$$\nu_l = -i\omega + \frac{1}{2}l(l+1)\nu_{ei} + \frac{(l+1)^2}{4(l+1)^2 - 1}\frac{k^2v^2}{\nu_{l+1}}. \quad (6)$$

Applying this procedure, we obtain the following expression for the first angular harmonic f_1 of the distribution function:

$$f_1 = -\frac{ikv}{\nu_1}f_0 + \frac{eE}{m_e\nu_1}\frac{\partial F_0}{\partial v}, \quad (7)$$

where the symmetric part f_0 of the function satisfies the equation

$$\left(-i\omega + \frac{k^2v^2}{3\nu_1}\right)f_0 + ikv\frac{eE}{3m_e\nu_1}\frac{\partial F_0}{\partial v} = C_{ee}[f_0] + \delta f_e(t = 0). \quad (8)$$

The general solution to Eq. (8) can be written as a linear combination of the basis functions ψ^A ($A = N, T$),

$$f_0 = i\frac{eE}{kT_e}F_0 - \omega\frac{eE}{kT_e}\psi^N F_0 + \frac{3}{2}\frac{\delta T_k(0)}{T_e}\psi^T F_0, \quad (9)$$

which satisfy two identical equations with different source terms S_A:

$$\left(-i\omega + \frac{k^2v^2}{3\nu_1}\right)\psi^A = F_0^{-1}C_{ee}[F_0\psi^A] + S_A, \quad (10)$$

with $S_N = 1$ and $S_T = v^2/(3v_{Te}^2) - 1$.

The thermal perturbations at an arbitrary time are determined by the moments J_B^A of the basis functions ψ^A because (cf. [9])

$$\delta T_k(\omega) = \frac{4\pi m_e}{3n_e}\int_0^\infty dv\, v^2(v^2 - 3v_{Te}^2)f_0 \equiv -\frac{\omega}{k}eEJ_T^N + \frac{3}{2}\delta T_k(0)J_T^T, \tag{11}$$

where

$$J_B^A = \frac{4\pi}{n_e}\int_0^\infty v^2 dv\,\psi^A F_0 S_B. \tag{12}$$

For potential perturbations, the current and electric field are related through Ampère's law:

$$\frac{\partial E}{\partial t} + 4\pi j = 0. \tag{13}$$

Substituting into relationship (13) the expression for the electric current, whose Fourier component is determined in accordance with expression (7) by

$$j_k(\omega) = -\frac{ie^2 n_e}{k^2 T_e}\omega(1 + i\omega J_N^N)E(\omega) + \frac{3en_e}{2k}\frac{\delta T_k(0)}{T_e}\omega J_N^T, \tag{14}$$

we can eliminate the electric field from relationship (11). As a result, we arrive at the final expression describing the relaxation of a given initial thermal perturbation:

$$\delta T(x, t) = \frac{3}{8\pi^2}\int_{-\infty}^{+\infty} dk\,\delta T_k(0)\int_{-\infty + i0}^{+\infty + i0} d\omega \exp(ikx - i\omega t) \times\left(J_T^T(k, \omega) - \frac{i\omega J_N^T(k, \omega)J_T^N(k, \omega)}{k^2\lambda_{\mathrm{De}}^2\epsilon(k, \omega)}\right). \tag{15}$$

Here, we have introduced the dielectric function for a collisional plasma [17],

$$\epsilon(k, \omega) = 1 + (1 + i\omega J_N^N)/(k^2\lambda_{\mathrm{De}}^2), \tag{16}$$

where $\lambda_{\mathrm{De}} = v_{Te}/\omega_{pe}$ is the electron Debye radius and $\omega_{pe} = \sqrt{4\pi e^2 n_e/m_e}$ is the electron plasma frequency.

Note that, in the thermal relaxation problem, the terms proportional to $\sim k\lambda_{\mathrm{De}} \ll 1$ are small corrections; they can be ignored for most applications because the characteristic spatial scales of the perturbations are much larger than the electron Debye radius λ_{De}. As a matter of fact, this corresponds to the quasineutral plasma approximation. In this case, expression (15) can be conveniently treated as a solution to the heat conduction equation in the k-representation,

$$\frac{\partial \delta T_k(t)}{\partial t} = \frac{2}{3n_e}i\mathbf{k}\cdot\mathbf{q}_k(t), \qquad \mathbf{q}_k(t) = \frac{i\mathbf{k}}{2\pi}\int_{-\infty + i0}^{+\infty + i0} d\omega \exp(-i\omega t)\kappa(k, \omega)\delta T_k(\omega), \tag{17}$$

with the thermal conductivity κ from nonlocal nonstationary transport theory [17]:

$$\kappa(k, \omega) = \frac{n_e}{k^2}\left(\frac{1 + i\omega J_N^N}{J_T^T + i\omega D_{NT}^{NT}} + i\frac{3}{2}\omega\right), \qquad D_{NT}^{NT} = J_N^N J_T^T - J_N^T J_T^N. \tag{18}$$

In what follows, we will analyze in detail the relaxation of different types of initial temperature perturbations $\delta T(x, 0)$.

3. PERIODIC INITIAL THERMAL PERTURBATION

We consider a periodic initial single-mode perturbation $\delta T(x, 0) = T_0\cos(k_0 x)$. Studying such a perturbation will allow us to understand how the characteristic relaxation time depends on the temperature scale length, $L = 1/k_0$. Since the thermal perturbation that occurs on a given spatial scale can be qualitatively characterized by a certain Fourier component—the one that makes the main contribution to the temperature distribution—solving the relevant problem makes it possible to qualitatively predict the regular features of the time evolution of the perturbation.

In the classical local model [10, 11], as well as in the nonlocal quasi-stationary model [9], the thermal conductivity does not depend explicitly on time (in Eq. (18), this corresponds to $\omega = 0$) and its temporal evolution is determined only by the slow time dependence of the parameters of the main plasma state. Accordingly, the evolution of the temperature perturbation is in fact described by the expression

$$\delta T(x, t) = \delta T_{\mathrm{hydro}}(t)\cos(k_0 x), \qquad \delta T_{\mathrm{hydro}}(t) = T_0\exp(-t/\tau), \quad \tau = \frac{3n_e}{2\kappa(k_0)k_0^2}, \tag{19}$$

where τ is the characteristic relaxation time. In the classical model, the thermal conductivity in expression (19) is independent of the spatial scale k_0, $\kappa = \kappa_{SH} = 128n_e v_{Te}\lambda_{ei}/(3\pi)$, where $\lambda_{ei} = v_{Te}/\nu_{ei}^T$ is the electron mean free path (corresponding to $\nu_{ei}^T = \sqrt{2/(9\pi)}\,\nu_{ei}(v_{Te})$). The characteristic relaxation time decreases rapidly with k_0 (i.e., with inhomogeneity scale length), $\tau \propto k_0^{-2}$. However, the classical transport theory cannot be applied when $k_0\lambda_{ei} > 0.06/\sqrt{Z}$, because, in this case, it would greatly underestimate the characteristic relaxation time of the temperature perturbations, so the nonlocal theory should be used. In [15], it was shown that, with allowance for the nonlocal nature of heat transport, the characteristic thermal relaxation time should be redefined as follows:

$$\tau = 3n_e[1 + 10(\sqrt{Z}k_0\lambda_{ei})^{0.9}]/(2\kappa_{SH}k_0^2); \tag{20}$$

this corresponds to nonlocal thermal conductivity (18), which is well approximated in the quasi-stationary limit ($\omega = 0$) by the expression $\kappa = \kappa_{SH}/[1 + 10(\sqrt{Z}k_0\lambda_{ei})^{0.9}]$ [9]. Note that this quasi-stationary approach is applicable only to the case of sufficiently slow relaxation of the initial perturbation, $\tau > 1/\nu_{ee}^T$ ($\nu_{ee}^T = \nu_{ei}^T/Z$) [17], which corresponds to characteristic perturbation scale lengths of $k_0\lambda_{ei} < 1/\sqrt{Z}$ and refers to the strongly collisional limit.

Let us now consider the evolution of thermal perturbations in the collisionless limit, $k_0\lambda_{ei} \gg 6Z^{2/3}$ [17]. We begin with expression (15) and write the moments of the basis functions in the form

$$J_N^N = \frac{i}{\omega}J_+(p), \quad J_N^T = \frac{i}{3\omega}[(p^2-1)J_+(p) - p^2],$$
$$J_T^T = \frac{i}{9\omega}[(p^4 - 2p^2 + 5)J_+(p) - p^4 + p^2], \tag{21}$$

where $J_+(x) = x\exp(-x^2/2)\int_{i\infty}^x dt\exp(t^2/2)$ is the standard dispersion function used in collisionless plasma theory [18] and $p = \omega/(k_0 v_{Te})$. As a result, we obtain

$$\delta T = \frac{iT_0}{12\pi}\cos(k_0x)\int_{-\infty}^{\infty}\frac{dp}{p}e^{-iptk_0v_{Te}}$$
$$\times\left[J_+(p) + \frac{J_+ + k_0^2\lambda_{De}^2(p^2-1)^2J_+ - k_0^2\lambda_{De}^2p^4}{1 - J_+ + k_0^2\lambda_{De}^2}\right]. \tag{22}$$

Unlike in the conventional problem of the relaxation of an initial field perturbation [18], the main contribution to the integral in expression (22) does not come from the zeros of the dispersion relation $\epsilon = 1 - J_+ + k_0^2\lambda_{De}^2 = 0$: these zeros lead merely to small oscillating corrections (corresponding to plasma oscillations) on the order of $k_0^2\lambda_{De}^2 \ll 1$, which are ignored in our analysis. In this case, the integral of the first term in expression (22) can be taken exactly to yield an exponential relaxation of the thermal perturbation, $\propto\exp(-k_0^2v_{Te}^2t^2/2)$, and the integral of the second term can be approximated by $\propto\exp(-3k_0^2v_{Te}^2t^2/2)$, so we have $\delta T(x, t) = \delta T_{kin}(t)\cos k_0x$, where

$$\delta T_{kin}(t) \approx \frac{T_0}{3}\left[2\exp\left(-\frac{k_0^2v_{Te}^2t^2}{2}\right) + \exp\left(-\frac{3k_0^2v_{Te}^2t^2}{2}\right)\right]. \tag{23}$$

Hence, in the kinetic limit, the relaxation of a thermal perturbation is an essentially nonstationary process occurring on a characteristic time scale of $\sim 1/(k_0v_{Te})$.

For characteristic initial perturbation scale lengths of $k_0\lambda_{ei} \gtrsim 1/\sqrt{Z}$, the relaxation time becomes $\tau \lesssim 1/\nu_{ee}^T$, which is at the limit of applicability of the quasi-stationary theory [17]. For such inhomogeneity scale lengths, the relaxation of thermal perturbations should be described with allowance for the time dependence of the thermal conductivity. In this case, there does not exist a simple analytic solution of form (19) and the relaxation of thermal perturbations cannot be described in terms of only one characteristic time scale, as was done for a strongly collisional plasma. In essence, the thermal perturbations behave as if they relax simultaneously in nonlocal hydrodynamic (19) and collisionless kinetic (23) regimes.

For arbitrary values of the parameter $k_0\lambda_{ei}$, the temperature relaxation is well described (with an accuracy of up to 10%; cf. the solid curves versus the circles in Fig. 1) by the following approximate expression, which combines the limiting formulas (19) and (23):

$$\delta T(t) = A\delta T_{hydro}(t) + (1 - A)\delta T_{kin}(t), \tag{24}$$

where the coefficient $0 < A < 1$ determines the relative contributions from collisionless and collisional transport. In Fig. 2, the coefficient A is given as a function of the collisionality parameter $k_0\lambda_{ei}$. For $k_0\lambda_{ei} < 0.3$, we have $A \approx 1$ and the temperature relaxes in accordance with the quasistatic theory. As the parameter $k_0\lambda_{ei}$ increases, the coefficient A decreases (Fig. 2); in this case, it can be approximated by the simple dependence $A = [1 + (k_0\lambda_{ei})^{0.8}]^{-1}$. For $k_0\lambda_{ei} > 1$, the fast kinetic and slow hydrodynamic (see representation (1)) stages show up clearly in the relaxation of the thermal perturbation, in accordance with relationship (24). The kinetic term in fact describes the relaxation of the thermal perturbation before the first collision, i.e., during the collisionless expansion of the electrons. On time scales $t \gtrsim 1/\nu_{ei}^T$, the temperature evolution is determined by the hydrodynamic time. For inhomogeneity scale length of $k_0\lambda_{ei} \gtrsim 1/\sqrt{Z}$, this time differs from that in the quasistatic theory [12] and can be approximated by the expression

$$\tau = 3n_e(1 + 12(\sqrt{Z}k_0\lambda_{ei})^{1.2})/(2\kappa_{SH}k_0^2). \tag{25}$$

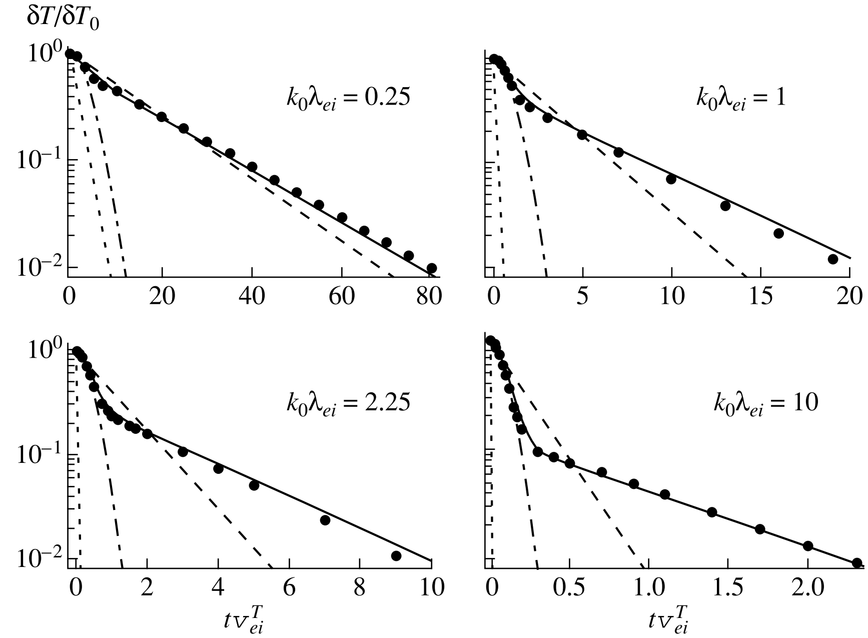

Fig. 1. Time evolution of periodic temperature perturbations at different initial spatial scales k_0 (circles) for $Z = 10$ and the corresponding results from quasistatic theory (dashed curves), classical collisional theory (dotted curves), and collisionless approximation (dashed-and-dotted curves). The solid curves were calculated from approximate formula (24).

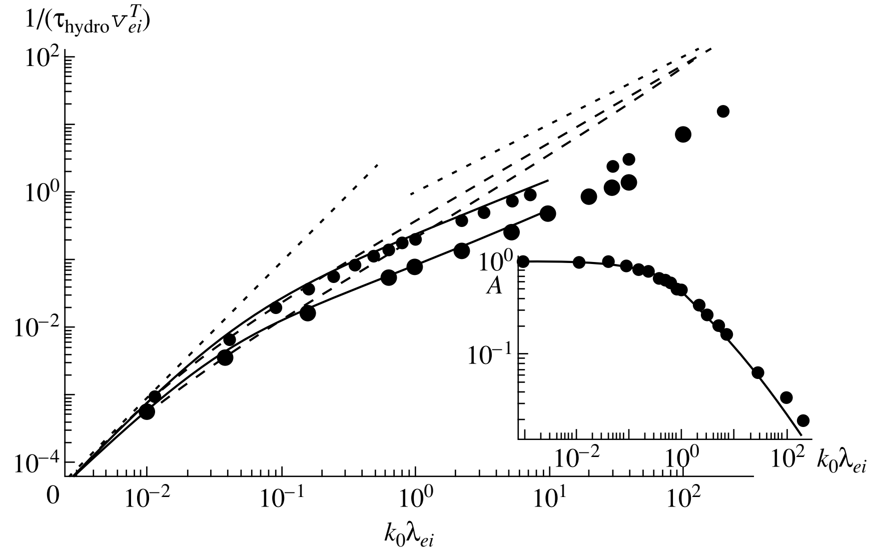

Fig. 2. Inverse hydrodynamic relaxation time $1/(\nu_{ei}^T\tau_{\text{hydro}})$ vs. thermal perturbation scale $k_0\lambda_{ei}$ for $Z = 10$ (small circles) and $Z = 50$ (large circles) and the corresponding results from quasistatic theory (dashed curves) and classical theories of a strongly collisional and a collisionless plasma (dotted curves). The solid curves were calculated from approximate formula (24). In the inset, the dependence of the coefficient A vs. thermal perturbation scale $k_0\lambda_{ei}$ (circles) is shown, the solid curve being the approximation proposed here.

Figure 2 shows the asymptotic behavior of the inverse hydrodynamic time of relaxation of a thermal perturbation, $d\ln\delta T_{\text{hydro}}(t)/dt$. For $k_0\lambda_{ei} \lesssim 10$, this time is well described by approximate expression (25). Strictly speaking, for $k_0\lambda_{ei} < 1/\sqrt{Z}$, the relaxation of a thermal perturbation should be described in terms of characteristic relaxation time (20) by the quasi-stationary theory. On the other hand, for $k_0\lambda_{ei} < 1/\sqrt{Z}$, approximate expression (25) yields relaxation times differing from those given by more exact expression (20) by no more

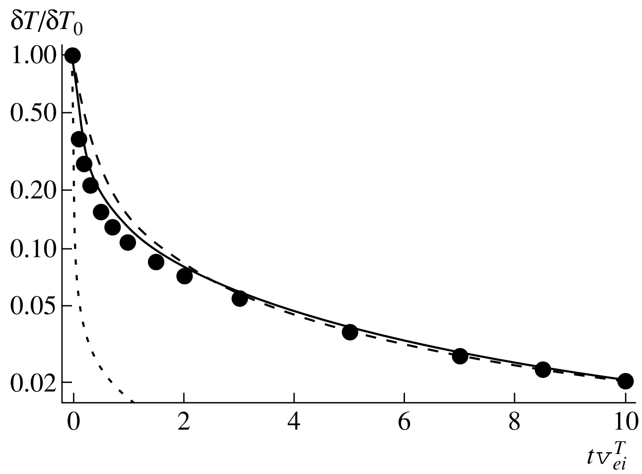

Fig. 3. Time evolution of the temperature at the center of a hot spot for $Z = 10$ (circles) in the case of a thermal perturbation with an initial scale length of $L/\lambda_{ei} = 0.1$ and the corresponding results from quasistatic theory (dashed curve) and classical theory of a strongly collisional plasma (dotted curve). The solid curve represents the temperature calculated from approximate formula (24).

than 30%. This justifies the use of approximate expression (25) for determining the characteristic hydrodynamic time of relaxation of temperature perturbation (24) for almost all spatial scales within the range $0 < k_0\lambda_{ei} \lesssim 10$.

4. LOCALIZED INITIAL THERMAL PERTURBATION

Note that the one-dimensional temperature relaxation model proposed in Section 3 in fact describes the evolution of one Fourier component of an arbitrary initial spatial thermal perturbation δT. Consequently, the above results can also be applied to the general case if we perform an inverse spatial Fourier transform. In this section, this is exemplified by the case of relaxation of a localized initial perturbation. To be specific, we consider a one-dimensional laser hot spot characterized by the Gaussian temperature profile

$$\delta T(x, 0) = T_0 \exp\left(-\frac{x^2}{L^2}\right), \tag{26}$$

where L is the characteristic size of the spot.

Figure 3 illustrates the time evolution of the amplitude of a thermal perturbation in a small laser spot with the initial size $L = 0.1\lambda_{ei}$. As in the case of an initially periodic perturbation, the electron energy relaxes in two regimes—kinetic and hydrodynamic. However, a transition from the first to the second regime is governed not only by nonstationary effects but also by the change in the characteristic size of the hot spot (the decrease in the effective spatial scale k_0), i.e., by the expansion of the spot. It is because of this latter effect that the characteristic relaxation time of the thermal perturbation changes during its evolution.

Figure 4 shows the spatial profiles of the effective temperature calculated numerically for different times (circles) from exact formula (15). The spatiotemporal distribution of the perturbed temperature $\delta T(x, t)$ is well described by approximate formula $\delta T(x, t) = \int dk \delta T(t) \exp(ikx)/(2\pi)$, where by T_0 in the definition of $\delta T(t)$ (see Eqs. (19), (23), (24)) is meant the spatial Fourier component of initial perturbation (26). The results from the approximate model coincide with those from the exact theory to within several percent, except for a narrow central region, where they differ by no more than 30%; moreover, this difference takes place only over a short time interval. On the whole, the evolution of a Gaussian thermal perturbation differs from that described by the quasi-stationary theory [15] to a lesser extent than from the evolution of a periodic perturbation. The reason is that the characteristic size of the spot increases rapidly and forces the relaxation of the thermal perturbation to proceed in the quasistatic hydrodynamic regime. This result, as well as the above investigation, shows that the quasi-stationary theory describes

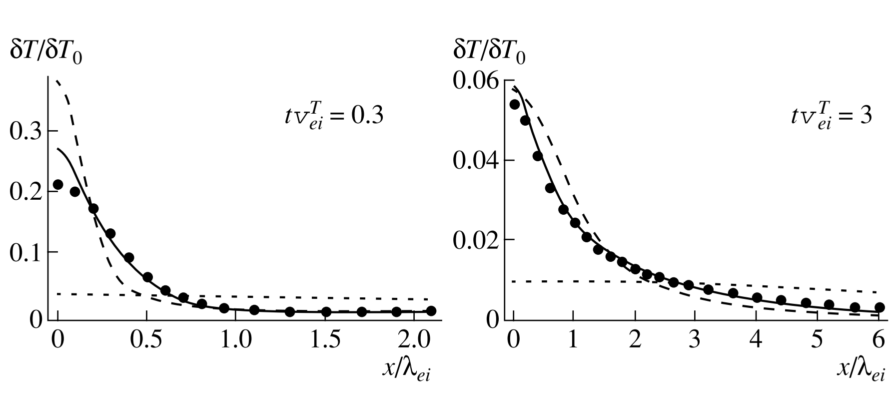

Fig. 4. Spatial temperature profiles for $Z = 10$ (closed circles) in the case of a thermal perturbation with an initial scale length of $L/\lambda_{ei} = 0.1$ and the corresponding results from quasistatic theory (dashed curves) and classical theory of a strongly collisional plasma (dotted curves). The solid curves represent the temperature calculated from approximate formula (24).

fairly accurately (with an accuracy of 30%) a localized thermal perturbation at initial spatial scales of $L \gtrsim \lambda_{ei}$.

5. CONCLUSIONS

In the present paper, we have obtained an analytic solution to the linear nonlocal problem of the relaxation of an initial thermal perturbation having an arbitrary shape and occurring on an arbitrary spatial scale. We have investigated the relationship between the nonlocal and nonstationary effects in electron heat transport. It has been found that, when the temperature varies on spatial scales $L \lesssim \sqrt{Z}\lambda_{ei}$, the nonstationary nature of the heat flux begins to play an important role in transport processes and should be incorporated in their description. The result of including nonstationary effects in the problem of evolution of a small-scale thermal perturbation is that the plasma evolves in two regimes—kinetic and hydrodynamic.

The general solution of the initial-value electron temperature relaxation problem has been exemplified by considering the relaxation of a periodic single-mode thermal perturbation and a spatially localized thermal perturbation. It is shown that the rapid expansion of a Gaussian initial thermal perturbation causes the relaxation of a hot spot to proceed in the quasistatic hydrodynamic regime. The solution obtained in the approximate model based on the exact solution to the kinetic equation is shown to agree well (with an accuracy of 30%) with the exact solution.

The above study has made use of the linear theory and, strictly speaking, cannot pretend to provide an exact quantitative description of the relaxation of large-amplitude thermal perturbations. The model, however, revealed a qualitatively new feature of the behavior of thermal perturbations: the existence of two qualitatively different relaxation regimes, resulting from the nonstationary nature of electron heat transport. This makes it possible to refine the limits of applicability of transport models based on the quasi-stationary theory [19].

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