Coherent x-ray source due to the quantum reflection of an electron beam from a laser-field phase lattice

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Within the scope of relativistic quantum theory for electron-laser interaction in a medium and using the resonant approximation for the two degenerated states of an electron in a monochromatic radiation field, a nonperturbative solution of the Dirac equation is obtained. The multiphoton cross sections of the electrons' coherent scattering on the plane monochromatic wave at the Cherenkov resonance are obtained, taking into account the specificity of an induced Cherenkov process and spin-laser interaction as well. As a result of coherent reflection from the "phase lattice" of a slowed plane wave in a medium, electron beam quantum modulation at high frequencies occurs. So, a coherent x-ray source in an induced Cherenkov process is expected, since such a beam is a potential source of coherent radiation itself.

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I. INTRODUCTION

The creation of coherent sources of electromagnetic radiation above optical region of frequencies up to the x ray or γ ray is mainly connected with the realization of free electron lasers (FELs) via free-free induced transitions. Because of smallness of the electron-photon interaction cross section and relatively large spreads of actual electron beams, the implementation of these problems is possible in the processes with the largest length of coherence, such as induced Cherenkov, Compton, and undulator processes (see, e.g., [1–3]). From this point of view, the schemes of undulator [4] and Cherenkov [5] lasers have first been investigated experimentally (at present the undulator scheme is actually developed and the first lasing has been carried out in this system). However, the amplifying frequencies are still far below x ray. On the other hand, the unavailability of the normal-incidence mirrors of high reflectivity at short wavelengths in order of x ray practically excludes a resonator scheme of radiation generation. In this case it is necessary to implement a single pass high gain FELs. The most attractive scheme that is presently considered is the self-amplified spontaneous emission [6], where the spontaneous undulator radiation from the first part of an undulator is used as an input signal to the downstream part.

The output intensity of FELs can be significantly increased in the super-radiant regime when the radiation intensity is scaled as ρ^2, where ρ is the electron beam density. The super-radiation in FELs can be initiated by the several mechanisms. The effects of electron and radiation pulses due to the relative slippage can lead to super-radiant behavior of FEL intensity [7]. This is a self-organizing phenomenon arising from the initially unbunched electron beam. Another process that initiates super-radiation is the coherent spontaneous emission when the radiation fields emitted by electrons are summed up coherently giving the ρ^2 scaling. The coherent spontaneous emission arising from the longitudinal spectral components of the electron pulse shape has been the subject of recent studies, both theoretical and experimental [8]. As has been shown, the latter can lead to significant enhancement of the start-up power of FELs [9] reducing the length of amplifiers, which is so crucial for single pass FELs.

Coherent spontaneous emission can arise from the initially prebunched electron beam in the klystron interaction scheme. This scheme is applicable in the classical interaction mode when the stimulated process of emission or absorption is determined by the phase synchronism condition. As a result, in the first interaction region the momenta of the particles in the beam become modulated and, simultaneously, the particles in the beam begin to be weakly bunched (see, e.g., [10]). In the free drift region particle bunching develops further and in the second interaction region bunched particle beam can be used to generate spontaneous super-radiation. However, employing this scheme for short wave radiation is problematic since the effective bunching length (the drift space length) $L\sim\lambda v/\delta v$ (λ is the wavelength of stimulated radiation-bunching wavelength, v is the particle velocity, and δv is the velocity modulation amplitude) is rather small, which poses obvious difficulties in maintaining the accuracy of the parameters of the scheme, in view of which the particle bunches are spread out. We also note that the klystron scheme is extremely sensitive to beam spreads (bunching is spread out even for monochromatic electron beam).

During the coherent interaction with the em wave, a quantum modulation of particle beam density occurs in difference to the classical one after the interaction remains unlimitedly long (for monochromatic beam). This is a result of the coherent superposition of particle states with various energy and momentum due to absorbed and emitted photons in the radiation field that is conserved after the interaction (due to wave property of a single particle). The quantum-modulated state of the particle leads to modulation of the beam density after the interaction at the frequency of the stimulating wave and its harmonics [11]. In stimulated processes the classical modulation or bunching of the beam occurs if the particle wave packet's size (Δx) is small enough: $\Delta x\ll\lambda$. In the opposite case, the quantum modulation of the beam takes place. The quantum modulated beam is the assembly of co-

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herently excited particles and can be used to produce super-radiation in various schemes. This radiation is analogous to free-induction decay and photon-echo in atomic systems [12]. The different radiation mechanisms of quantum-modulated beams have been investigated in the works [13].

Quantum modulation of the particle beams at the interaction with a laser radiation at the above optical frequencies can be obtained through multiphoton transitions in the presence of a "third body" (to realize the energy-momentum exchange with the coherent field providing coherent superpositions of $N \gg 1$ particles' quantum states). The good candidates for this goal may serve induced Cherenkov, Compton, and undulator processes due to these above-mentioned properties. For the stimulated Cherenkov process at the $\vartheta = 0$ interaction angle the possibility of quantum modulation at hard x-ray frequencies due to the particle "reflection" from the laser pulse [14] has been studied in the work [15]. At the arbitrary interaction angle the exact solution of the problem is very complicated as the quantum equation of motion is reduced to the Matheu-type equation. An analysis using the perturbation theory has been done in the work [16].

In recent studies [17,18] it has been shown that coherent interaction of electrons with a plane monochromatic wave in a dielectric medium can be described as a resonant scattering of a particle on the "phase lattice" of a traveling wave similar to the Bragg scattering of the particle on the crystal lattice. The latter is obvious in the frame of reference (FR) of rest of the wave. Since the index of refraction of a medium $n > 1$ ($n(\omega) \equiv n$ as the wave is monochromatic) in this FR there is only a static periodic magnetic field and an elastic scattering of a particle takes place. The law of conservation for the Cherenkov process taking into account the quantum recoil translates into the Bragg resonance condition between the de Broglie wave of the particle and this static periodic structure. Hence, in the induced Cherenkov process the interaction resonantly connects two states of the particle, which are degenerated over the longitudinal momentum with respect to the direction of the wave propagation. These are the states with the longitudinal momentum p_x (incident particle) and $p_x + \ell \hbar k$ (scattered particle), as far as the conservation law of this process is $|p_x| = |p_x + \ell \hbar k|$ (ℓ- number of absorbed or radiated photons with a wave vector $k = k_x$). The latter is the same as the Bragg condition of the electron coherent elastic scattering on crystal lattice. Therefore, in the stimulated Cherenkov process, no matter how weak the wave field is, the usual perturbation theory is not applicable because of such degeneration of the states. So, the interaction near the resonance is necessary to describe by the secular equation [17]. The latter, in particular, reveals zone structure of the particle states in the field of the transverse electromagnetic (em) wave in a dielectric medium [17,18]. Note that the application of the perturbation theory ignoring the above-mentioned degeneration in this process has reduced to essentially incorrect results that have been elucidated in the paper [19].

Hence, to reveal the above-mentioned quantum properties of an electron state in the induced Cherenkov process, particularly those leading to quantum modulation effect for an electron beam, it is necessary to solve the Dirac equation in a plane em wave in a medium at an arbitrary interaction angle beyond the scope of the perturbation theory. In the present paper the case of strong radiation field is considered. Using the resonance approximation for the above-mentioned two degenerated states in a monochromatic radiation field [17], a nonperturbative solution of the Dirac equation (nonlinear over field solution of the Hill-type equation) is obtained. The multiphoton probabilities of free electrons' coherent scattering on a strong monochromatic wave at the Cherenkov resonance are calculated, taking into account the above-mentioned specificity of the induced Cherenkov process [17,18] and spin-laser interaction as well. In the result of this resonant scattering the electron beam quantum modulation at high frequencies occurs that corresponds to the electron energy exchange at the coherent reflection from the "phase lattice" of the slowed wave in a medium. By the proper choice of electron-laser parameters one can achieve the energy exchange corresponding to x-ray frequencies and we can expect to have, in principle, a coherent x-ray source in the induced Cherenkov process, since such a quantum modulated beam is a potential source of coherent super-radiation.

This paper is organized as follows. In Sec. II we consider the Dirac equation for an electron in the strong em radiation field in the medium and obtain a set of ordinary differential equations for transition amplitudes. In Sec. III by the resonant approximation we obtain transition amplitudes and probabilities of multiphoton emission or absorption. A summary of results is given in Sec. IV, and implications for short wave super-radiation are discussed.

II. NONLINEAR SOLUTION OF THE DIRAC EQUATION FOR THE ELECTRON IN STRONG EM RADIATION FIELD IN A MEDIUM

In this section we shall solve the Dirac equation for a spinor particle in the given radiation field in a medium

$$i\frac{\partial \Psi}{\partial t} = [\hat{\boldsymbol{\alpha}}\{\hat{\mathbf{p}} - e\mathbf{A}(\tau)\} + \hat{\beta}m]\Psi, \tag{1}$$

where

$$\hat{\boldsymbol{\alpha}} = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \quad \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2}$$

are the Dirac matrices, with the σ Pauli matrices, m and e are the mass and charge of a particle, respectively (here we set $\hbar = c = 1$), $\hat{\mathbf{p}} = -i\boldsymbol{\nabla}$ is the operator of the generalized momentum, $\mathbf{A} = \mathbf{A}(t - nx)$ is the vector potential of a linearly polarized plane wave propagating in the OX direction in a medium,

$$\mathbf{A} = \{0, A_0(\tau)\cos(\omega\tau), 0\}, \quad \tau = t - nx. \tag{3}$$

We shall assume that the em wave is adiabatically switched on at $\tau = -\infty$ and switched off at $\tau = +\infty$ [$\mathbf{A}(\tau = \mp\infty) = 0$].

To solve the problem, it is more convenient to pass to the FR of the rest of the wave (R frame moving with the velocity $V = 1/n$). As is noticed, in this FR there is only the static magnetic field that will be described according to Eq. (3) by the following vector potential:

$$\mathbf{A}_R = \{0, A_0(x')\cos k'x', 0\}, \tag{4}$$

where

$$k' = \omega\sqrt{n^2-1}. \tag{5}$$

The wave function of a particle in the R frame is connected with the wave function in the laboratory frame Λ by the Lorentz transformation of the bispinors

$$\Psi = \hat{S}(\vartheta)\Psi_R, \tag{6}$$

where

$$\hat{S}(\vartheta) = ch\frac{\vartheta}{2} + \alpha_x sh\frac{\vartheta}{2}, \quad th\vartheta = V = \frac{1}{n} \tag{7}$$

is the transformation operator. For Ψ_R we have the following equation:

$$i\frac{\partial\Psi_R}{\partial t'} = [\hat{\alpha}\{\hat{\mathbf{p}}' - e\mathbf{A}_R(\mathbf{x}')\} + \hat{\beta}m]\Psi_R. \tag{8}$$

Since the interaction Hamiltonian does not depend on the time and transverse (to the direction of the wave propagation) coordinates the eigenvalues of the operators $\hat{H}'$, $\hat{p}_y'$, $\hat{p}_z'$ are conserved: $E' = \text{const}$, $p_y' = \text{const}$, $p_z' = \text{const}$ and the solution of Eq. (8) can be represented in the form of a linear combination of free solutions of the Dirac equation with amplitudes $a_i(x')$ depending only on x':

$$\Psi_R(\mathbf{r}', t') = \sum_{i=1}^{4} a_i(x')\Psi_i^{(0)}. \tag{9}$$

Here

$$\Psi_{1,2}^{(0)} = \left(\frac{E'+m}{2E'}\right)^{1/2}\begin{bmatrix}\varphi_{1,2}\\ \dfrac{\sigma_x p_x' + \sigma_y p_y'}{E'+m}\varphi_{1,2}\end{bmatrix} \times \exp[i(p_x'x' + p_y'y' - E't)],$$

$$\Psi_{3,4}^{(0)} = \left(\frac{E'+m}{2E'}\right)^{1/2}\begin{bmatrix}\varphi_{1,2}\\ \dfrac{-\sigma_x p_x' + \sigma_y p_y'}{E'+m}\varphi_{1,2}\end{bmatrix} \times \exp[i(-p_x'x' + p_y'y' - E't)], \tag{10}$$

where

$$p_x' = (E'^2 - p_y'^2 - m^2)^{1/2}, \quad \varphi_1 = \begin{pmatrix}1\\0\end{pmatrix}, \quad \varphi_2 = \begin{pmatrix}0\\1\end{pmatrix}. \tag{11}$$

The solution of Eq. (8) in the form (9) corresponds to an expansion of the wave function in a complete set of the wave functions of an electron with certain energy and transverse momentum p_y' [with longitudinal momenta $\pm(E'^2 - p_y'^2 - m^2)^{1/2}$ and spin projections $S_x = \pm\frac{1}{2}$]. The latter are normalized to one particle per unit volume. Since there is symmetry with respect to the direction $\mathbf{A}_R$ (the OY axis) we have taken, without loss of generality, the vector $\mathbf{p}'$ in the XY plane ($p_z' = 0$).

Substituting Eq. (9) into Eq. (8) then multiplying by the Hermitian conjugate functions and taking into account (10) and (2) we obtain a set of differential equations for the unknown functions $a_i(x')$. The equations for a_1, a_3 and a_2, a_4 are separated and for these amplitudes we have the following set of equations:

$$p_x'\frac{da_1(x')}{dx'} = iep_yA_y(x')a_1(x') - eA_y(x') \times (p_x' - ip_y')a_3(x')\exp(-2ip_x'x'),$$

$$p_x'\frac{da_3(x')}{dx'} = -iep_yA_y(x')a_3(x') - eA_y(x') \times (p_x' + ip_y')a_1(x')\exp(2ip_x'x'). \tag{12}$$

A similar set of equations is also obtained for the amplitudes $a_2(x')$ and $a_4(x')$. For simplicity, we shall assume that before the interaction there are only electrons with specified longitudinal momentum and spin state, i.e.,

$$|a_1(-\infty)|^2 = 1, \quad |a_3(+\infty)|^2 = 0, \quad |a_2(-\infty)|^2 = 0, \quad |a_4(+\infty)|^2 = 0. \tag{13}$$

From the condition of conservation of the norm we have

$$|a_1(x')|^2 - |a_3(x')|^2 = \text{const} \tag{14}$$

and the probability of reflection is $|a_{3,4}(-\infty)|^2$.

Application of the unitarian transformation

$$a_1(x') = b_1(x')\exp\left(i\frac{ep_y'}{p_x'}\int_{-\infty}^{x'}A_y(\eta)d\eta - i\frac{\vartheta'}{2}\right),$$

$$a_3(x') = b_3(x')\exp\left(-i\frac{ep_y'}{p_x'}\int_{-\infty}^{x'}A_y(\eta)d\eta + i\frac{\vartheta'}{2}\right) \tag{15}$$

simplifies Eq. (12). Here ϑ' is the angle between the momentum of electron and the direction of the wave propagation in the R frame. The new amplitudes $b_1(x')$ and $b_3(x')$ satisfy the same initial conditions: $|b_1(-\infty)|^2 = 1$, $|b_3(+\infty)|^2 = 0$, according to Eq. (13).

From Eqs. (12) and (15) for the $b_1(x')$ and $b_3(x')$ we obtain the following set of equations:

$$\frac{db_1(x')}{dx'} = -f(x')b_3(x'),$$

$$\frac{db_3(x')}{dx'} = -f^*(x')b_3(x'), \tag{16}$$

where

$$f(x')=\frac{eA_y(t)p'}{p'_x}\exp\left(-2ip'_xx'-i\frac{2ep'_y}{p'_x}\int_{-\infty}^{x'}A_y(\eta)d\eta\right),$$
$$p'=\sqrt{p'^2_y+p'^2_x}. \tag{17}$$

Using the following expansion by the Bessel functions

$$\exp(-i\alpha\sin k'x')=\sum_{N=-\infty}^{\infty}J_N(\alpha)\exp(-iNk'x'),$$

we can reduce Eq. (16) to the form

$$\frac{db_1(x')}{dx'}=-\sum_{N=-\infty}^{\infty}f_N\exp[-i(2p'_x-Nk')x']b_3(x'),$$
$$\frac{db_3(x')}{dx'}=-\sum_{N=-\infty}^{\infty}f_N\exp[i(2p'_x-Nk')x']b_1(x'), \tag{18}$$

where

$$f_N=\frac{p'}{2p'_y}Nk'J_N\left(2\xi\frac{m}{p'_x}\frac{p'_y}{k'}\right),\quad \xi=eA/m. \tag{19}$$

III. RESONANT APPROXIMATION FOR TRANSITION AMPLITUDES

Because of conservation of particle energy and transverse momentum (in R frame) the real transitions in the field will occur from a p'_x state to the $-p'_x$ one and, consequently, the probabilities of multiphoton scattering will have maximal values for the resonant transitions

$$2p'_x=sk' \quad (s=\pm1,\pm2\ldots). \tag{20}$$

The latter expresses the condition of exact resonance between the electron de Broglie wave and the incident "wave lattice." In the Λ frame, inelastic scattering takes place and Eq. (20) corresponds to the well-known Cherenkov conservation law

$$\frac{2E(1-nv\cos\vartheta)}{(n^2-1)}=s\omega, \tag{21}$$

where ϑ is the angle between the electron momentum and the wave propagation direction in the Λ frame (the Cherenkov angle), v and E are the electron velocity and energy.

So, we can utilize the resonant approximation keeping only resonant terms in the Eq. (18). Generally, in this approximation, at detuning of resonance $|\delta_s|=|2p'_x-sk'|\ll k'$, we have the following set of equations for the certain s-photon transition amplitudes $b_1^{(s)}(x')$ and $b_3^{(s)}(x')$:

$$\frac{db_1^{(s)}(x')}{dx'}=-f_s\exp[-i\delta_sx']b_3^{(s)}(x'),$$

$$\frac{db_3^{(s)}(x')}{dx'}=-f_s\exp[i\delta_sx']b_1^{(s)}(x'). \tag{22}$$

This resonant approximation is valid for the slowly varying functions $b_1^{(s)}(x')$ and $b_3^{(s)}(x')$, i.e., by the condition

$$\left|\frac{db_{1,3}^{(s)}(x')}{dx'}\right|\ll|b_{1,3}^{(s)}(x')|k'. \tag{23}$$

At first we shall solve the case of exact resonance ($\delta_s=0$). According to the boundary conditions (14) we have the following solutions for the amplitudes:

$$b_1^{(s)}(x')=\frac{ch\left[\int_{x'}^{\infty}f_sd\eta\right]}{ch\left[\int_{-\infty}^{\infty}f_sd\eta\right]},\quad b_3^{(s)}(x')=\frac{sh\left[\int_{x'}^{\infty}f_sd\eta\right]}{ch\left[\int_{-\infty}^{\infty}f_sd\eta\right]} \tag{24}$$

and for the reflection coefficient

$$R^{(s)}=|b_3^{(s)}(-\infty)|^2=th^2[f_s\triangle x'], \tag{25}$$

where $\triangle x'$ is the coherent interaction length. The reflection coefficient in the laboratory frame of reference is the probability of absorption at $v<1/n$ or emission at $v>1/n$. The latter can be obtained by expressing the quantities f_s and $\triangle x'$ by the quantities in this frame since the reflection coefficient is Lorentz invariant. So

$$R^{(s)}=th^2[F_s\triangle\tau], \tag{26}$$

where

$$F_s=\left[\frac{(1-nv\cos\vartheta)^2}{n^2-1}+v^2\sin^2\vartheta\right]^{1/2}\times\frac{s\omega}{2v\sin\vartheta}J_s\left(\xi\frac{2mv\sin\vartheta}{\omega(1-nv\cos\vartheta)}\right) \tag{27}$$

and $\triangle\tau$ for actual cases is the laser pulse duration in the Λ frame. The condition of applicability of this resonant approximation (23) is equivalent to the condition

$$|F_s|\ll\omega, \tag{28}$$

which restricts as the intensity of the wave as well as the Cherenkov angle. Besides, to satisfy the condition (28) we must take into account the very sensitivity of the parameter F_s towards the argument of Bessel function, according to Eq. (27). For the wave intensities when $F_s\triangle\tau\gtrsim1$, the reflection coefficient is of the order of unit that can occur for a large number of photons $s\gg1$ for the argument of Bessel function $Z\sim s\gg1$ in Eq. (27) [according to the asymptotic behavior of Bessel function $J_s(Z)$ at $Z\simeq s\gg1$].

Let us estimate the reflection coefficient of an electron from the laser pulse or the most probable number of absorbed/emitted photons due to resonance interaction in the induced Cherenkov process. For the typical values of experimental parameters of this process in the gaseous medium with the index of refraction $n-1\sim 10^{-4}$, at the initial electron energy $E\sim 50$ MeV and Cherenkov angle $\vartheta \sim 1$ mrad, during the "Bragg reflection" from the neodymium laser pulse ($\omega\triangle\tau\sim 10^2, \hbar\omega=1.17$ eV) with an intensity 10^{10} W/cm^2 ($\xi\sim 10^{-4}$) the electron absorbs or emits about 10^5 photons.

For the off resonant solution, when $\delta_s\neq 0$, but $f_s^2>\delta_s^2/4$ from Eq. (22), we obtain the following expression for $R^{(s)}$:

$$R^{(s)}=\frac{f_s^2}{\Omega_s^2}\frac{sh^2[\Omega_s\triangle x']}{1+\frac{f_s^2}{\Omega_s^2}sh^2[\Omega_s\triangle x']}, \tag{29}$$

where $\Omega_s=\sqrt{f_s^2-\delta_s^2/4}$, which has the same behavior as in the case of exact resonance . In opposite case when $f_s^2 \leqslant \delta_s^2/4$ the reflection coefficient is an oscillating function on interaction length.

As has been mentioned in the Introduction, during the coherent interaction with em wave the quantum modulation of particle beam density occurs. From Eqs. (9) and (26) for the electron wave function after the reflection from the wave pulse we have the superposition of incident and reflected electron waves (in the R frame)

$$\Psi_R=a_1(-\infty)\Psi_1^{(0)}+a_3(-\infty)\Psi_3^{(0)} \tag{30}$$

and in the result the probability density $\rho_R=\Psi_R^+\Psi_R$ is modulated at the x-ray frequencies

$$\rho_R^{(s)}=1+th^2[f_s\triangle x']+2\left[1-\frac{p_x'^2}{E'^2}\right]th[f_s\triangle x'] \times\cos(sk'x'-\varphi_0), \tag{31}$$

where

$$\sin\varphi_0=\frac{\sin\vartheta'}{\left[1-\frac{p_x'^2}{E'^2}\right]}.$$

In the laboratory frame of reference from Eqs. (7) and (30) we have

$$\rho^{(s)}\simeq\frac{1}{\sqrt{n^2-1}}(1+th^2[F_s\triangle\tau]+2th[F_s\triangle\tau]\cos(s\omega\tau-\vartheta'), \tag{32}$$

where it is taken into account that actually $|s\omega|\ll E$. As is seen from Eq. (30) the modulation depth is of the order of unit for the intensities when $F_s\triangle\tau\sim 1$, which can be satisfied for the moderate intensities of the laser radiation of the order of 10^{10} W/cm^2.

IV. DISCUSSION

So far we have dealt with a one particle problem. It is clear that for the monochromatic beam we will have density modulation on a spatial harmonic defined by Eq. (21). For the actual electron beam the fate of this modulation depends on the collimation and monochromaticity of the electron beam. So, in actual cases the modulation produced will not be pure in the sense that it will contain all integer multiples of the laser frequency. However, the laser field strengths and duration can be chosen in such a manner as to maximize the contribution from the higher harmonics. Due to angular ($\Delta\vartheta$) and energetic ($\Delta\varepsilon$) spreads of the electron beam, the modulation will persist for a distance of order $L_d=\lambda/(s\Delta_{\theta,\varepsilon})$ following the electron beam–laser interaction, where

$$\Delta_{\theta,\varepsilon}=\max\left\{\frac{\Delta\varepsilon}{\varepsilon\gamma^2},tg\,\vartheta\Delta\vartheta\right\}$$

is the Cherenkov resonance width. For distances larger than L_d the modulation washes out quickly as a result of Doppler dephasing. The grating structure is not lost, however (as the quantum modulation is a single particle effect). If the electrons interact with a second laser field, the various spatial harmonics will rephase at different distances following the interaction with the second field (this scheme is analogous to echo techniques for atomic systems).

Once the modulated beam is created the question remains as to how to use the modulation at some distance $L<L_d$ from the electron beam–laser interaction zone to produce super-radiation. Typically, $\Delta_{\theta,\varepsilon}$ is of order of 10^{-8}–10^{-6} and we have $L_d\sim 1$–10^2 cm (for different harmonics). The most direct scheme is to employ the Compton or undulator interaction scheme immediately following the electron beam–laser interaction. Due to the modulation of the electron beam, we will have a macroscopic transition current and without any initial seeding power such a beam will produce radiation that will scale as ρ^2, that is, super-radiation. This scheme is analogous to free-induction decay in coherent transient spectroscopy for atomic systems [12].

In summary, we have outlined a method for generating and detecting the density modulated electron beam having space period λ/s ($s\gg 1$) using optical fields with wavelength λ. Higher harmonics with shorter periods can be produced by a proper choice of electron-laser parameters.

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