Quantum optics of lossy beam splitters

Stephen M. Barnett, John Jeffers, and Alessandra Gatti*
Department of Physics and Applied Physics, University of Strathclyde, Glasgow G4 0NG, Scotland

Rodney Loudon
Department of Physics, University of Essex, Colchester CO4 3SQ, England

(Received 8 August 1997)

The familiar input-output relations for an optical beam splitter are generalized to allow for linear absorption by the medium forming the mirror. Beam-splitter losses generally affect the noise levels detectable in experiments involving nonclassical light. When employed to investigate two-photon interference effects, a lossy beam splitter can lead to apparent nonlinear absorption, which, in the most extreme case, leads to either both or neither of the photons being absorbed. The degree of second-order coherence of antibunched light can be maintained on transmission through the beam splitter but any amplitude squeezing in the incident light is degraded. [S1050-2947(98)05303-7]

PACS number(s): 42.50.Dv

I. INTRODUCTION

Beam splitters play important roles in much of optical physics. They are key elements in interferometers, both the classical instruments whose fringes are controlled by first-order coherence and the Hanbury-Brown–Twiss variety used in measurements of second-order coherence [1]. They are frequently used in the detection of nonclassical effects, including antibunching [2], squeezing [3], and two-photon interference [4]. They are also of fundamental importance in investigations of the nature of light [5]. An extensive and up-to-date account of these effects and others is given in [6].

A beam splitter superposes two incident or input fields to produce two output fields. In its simplest form it may be thought of as a thin layer of dielectric with complex transmission and reflection coefficients determined by the usual boundary conditions on the electromagnetic field at dielectric–free-space interfaces. Such simple models have been discussed in some detail [7–10]. In this paper, however, we will be primarily concerned not with the forms of the fields in and around the dielectric but rather with the relationships between the incident and outgoing fields far from the beam splitter and with the effects of the superposition of fields at the beam splitter on their quantum properties.

Figure 1(a) depicts a beam splitter that superposes two independent incident modes, with the continuum annihilation operators [11,12] $\hat{a}_{\rm in}(\omega)$ and $\hat{b}_{\rm in}(\omega)$, to form two independent outgoing modes, with the operators $\hat{a}_{\rm out}(\omega)$ and $\hat{b}_{\rm out}(\omega)$. In addition to the square beam splitter, with propagation directions at right angles, the theory that follows applies to any four-port device with two input and two output ports, for example, the absorbing film shown in Fig. 1(b) with light incident normally on both sides. The incident and output modes propagate in free space and their annihilation and creation operators must satisfy the usual commutation relations

$$[\hat{a}_j(\omega),\hat{a}_j^\dagger(\omega')]=\delta(\omega-\omega')=[\hat{b}_j(\omega),\hat{b}_j^\dagger(\omega')]$$

$$[\hat{a}_j(\omega),\hat{b}_j^\dagger(\omega')]=0=[\hat{b}_j(\omega),\hat{a}_j^\dagger(\omega')], \qquad (1.1)$$

with j representing either in or out.

Almost all previous theoretical work is concerned with beam splitters in which none of the incident light is absorbed. For an ideal lossless beam splitter that is reciprocal (invariant under time reversal) and symmetric [13], the pairs of input and output operators are related by a unitary transformation of the form [14–16]

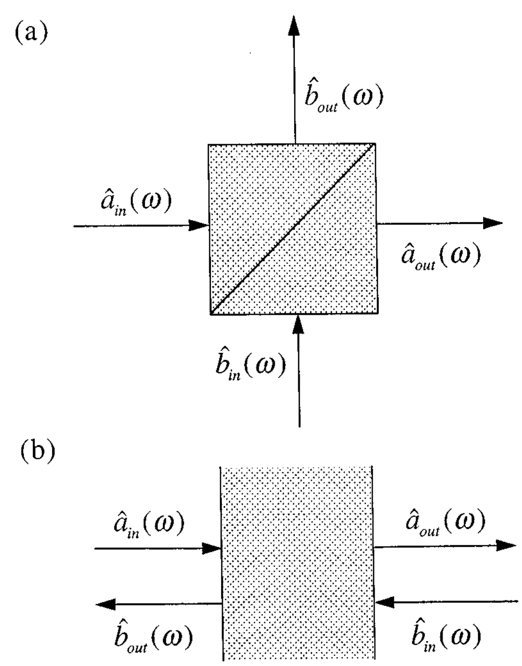

FIG. 1. Schematic diagrams of (a) a beam splitter and (b) a partially reflecting film.

*Present address: INFM Dipartimento di Fisica dell' Università di Milano, Via Celoria 16, Milano 20133, Italy.

$$\hat{a}_{\text{out}}(\omega)=t(\omega)\hat{a}_{\text{in}}(\omega)+r(\omega)\hat{b}_{\text{in}}(\omega),$$
$$\hat{b}_{\text{out}}(\omega)=t(\omega)\hat{b}_{\text{in}}(\omega)+r(\omega)\hat{a}_{\text{in}}(\omega), \tag{1.2}$$

where $r(\omega)$ and $t(\omega)$ are respectively the beam-splitter reflection and transmission coefficients. These have a frequency dependence that characterizes the construction of the beam splitter. The formal requirement for unitarity imposes two restrictions on the forms of the coefficients:

$$|t(\omega)|^2+|r(\omega)|^2=1,$$
$$t(\omega)r^*(\omega)+t^*(\omega)r(\omega)=0 \tag{1.3}$$

for all angular frequencies ω. These restrictions ensure that the commutation relations (1.1) are all satisfied; equivalently, they guarantee the conservation of energy and the orthogonality of the two outgoing modes, respectively. They can be reexpressed in a variety of useful forms, for example, the compact version

$$|t(\omega)\pm r(\omega)|^2=1. \tag{1.4}$$

Of course, apart from constant-intensity monochromatic incident light, most beams are time dependent and their potentially complicated properties on transmission through the beam splitter must be determined by Fourier transformation of the much simpler theory in frequency space.

In reality, beam splitters exhibit not only dispersive, or frequency-dependent, reflection and transmission coefficients but also losses, and these two features are inter-related by causal considerations embodied in the Kramers-Kronig relations [17]. Indeed, the presence of both dispersion and loss plays an important role in the canonical quantization of the field in the presence of dielectric media [9,18]. Despite this close relationship, and apart from some work on the analogous fibre couplers [19], there is very little published theory on beam splitters with losses [20]. The lack of attention to the quantum properties of lossy beam splitters is understandable, as losses tend to suppress nonclassical features such as squeezing [21] and the beam splitters in the relevant experiments are designed to minimize the loss. There are, nevertheless, interesting nonclassical effects for which the presence of losses is necessary [22,23].

In this paper we present a simple quantum theory of lossy beam splitters and apply it to model the effects of such devices on nonclassical states of light. We find that photon antibunching is unchanged by transmission through the beam splitter but that squeezing is, as expected, suppressed by the losses. More surprising is the modification of the famous two-photon interference effect [4,24] by an apparent nonlinear absorption in the linear medium forming the beam splitter, which restores the photon coincidences between the two output modes that do not occur for a lossless beam splitter.

II. LOSSY BEAM SPLITTERS

In practice, some of the light incident on a beam splitter is neither transmitted nor reflected but rather absorbed. This is an inevitable consequence of the frequency dependence of the transmission and reflection coefficients; the beam-splitter material must, for example, tend towards transparency, $r(\omega)\to 0$, as the frequency ω tends to infinity. The absorption can be very small in any chosen spectral range, but significant absorption then occurs at other frequencies. The presence of absorption means that some of the light incident on the beam splitter does not escape. The free-space commutation relations (1.1) remain in force but the input-output relations (1.2) need to be generalized and the transmission and reflection coefficients no longer satisfy the ideal beam-splitter relations (1.3) or (1.4).

These relations can, however, be replaced by a pair of inequalities. The first of these is

$$|t(\omega)|^2+|r(\omega)|^2\leqslant 1 \tag{2.1}$$

with the equality holding only for zero losses at the frequency ω. The left-hand side of this inequality can be interpreted as the probability of survival for a single photon incident on the beam splitter.

A second inequality is derived by considering the effect of the beam splitter on arbitrary input fields with classical or coherent amplitudes α and β. The requirement that the total output intensity (or mean photon flux) should be less than or equal to that at the input then gives

$$|t(\omega)\alpha+r(\omega)\beta|^2+|t(\omega)\beta+r(\omega)\alpha|^2\leqslant|\alpha|^2+|\beta|^2 \tag{2.2}$$

for any pair of complex numbers α and β. The equality again holds only if there are no absorption losses. For the special choices $\alpha=\pm\beta$, the inequality simplifies to

$$|t(\omega)\pm r(\omega)|^2\leqslant 1 \tag{2.3}$$

and combination with Eq. (2.1) provides a bound on the real part of $t(\omega)r^*(\omega)$ of the form

$$|t(\omega)r^*(\omega)+r(\omega)t^*(\omega)|\leqslant 1-|r(\omega)|^2-|t(\omega)|^2, \tag{2.4}$$

with both sides equal to zero for a lossless beam splitter.

The commutation relations (1.1) between the output creation and annihilation operators must remain valid in the presence of beam-splitter loss, as they represent basic properties of the free-space quantized fields. Their forms are maintained by the presence of Langevin noise operators associated with fluctuating currents within the medium forming the beam splitter [9]. The general relationships between the input and output operators with the inclusion of losses are [9,20]

$$\hat{a}_{\text{out}}(\omega)=t(\omega)\hat{a}_{\text{in}}(\omega)+r(\omega)\hat{b}_{\text{in}}(\omega)+\hat{F}_a(\omega),$$
$$\hat{b}_{\text{out}}(\omega)=t(\omega)\hat{b}_{\text{in}}(\omega)+r(\omega)\hat{a}_{\text{in}}(\omega)+\hat{F}_b(\omega). \tag{2.5}$$

The input fields and the noise sources within the mirror are required to be independent so that the input operators must commute with the output Langevin operators:

$$[\hat{a}_{\text{in}}(\omega),\hat{F}_a^\dagger(\omega')]=[\hat{a}_{\text{in}}(\omega),\hat{F}_b^\dagger(\omega')]=[\hat{a}_{\text{in}}(\omega),\hat{F}_a(\omega')]$$
$$=[\hat{a}_{\text{in}}(\omega),\hat{F}_b(\omega')]=0 \tag{2.6}$$

and similarly for the b operators. Imposition of the commutation relations (1.1) on the output mode operators then leads to the requirements on the noise-operator commutation relations:

$$[\hat{F}_a(\omega),\hat{F}_a^\dagger(\omega')]=\delta(\omega-\omega')\{1-|t(\omega)|^2-|r(\omega)|^2\}$$
$$=[\hat{F}_b(\omega),\hat{F}_b^\dagger(\omega')],$$

$$[\hat{F}_a(\omega),\hat{F}_b^\dagger(\omega')]=-\delta(\omega-\omega')\{t(\omega)r^*(\omega)+r(\omega)t^*(\omega)\}$$
$$=[\hat{F}_b(\omega),\hat{F}_a^\dagger(\omega')]. \quad (2.7)$$

Note that these commutators are proportional to combinations of the transmission and reflection coefficients that are zero for the ideal beam splitter, according to Eq. (1.3).

At optical frequencies, the matter forming the beam splitter can be considered to be in its ground state. We use the ket $|0\rangle$ to represent the composite ground state of the material and the vacuum state of the incident electromagnetic modes, so that it is a zero right eigenstate of the corresponding destruction operators,

$$\hat{F}_a(\omega)|0\rangle=\hat{F}_b(\omega)|0\rangle=\hat{a}_{\rm in}(\omega)|0\rangle=\hat{b}_{\rm in}(\omega)|0\rangle=0. \quad (2.8)$$

It follows from the input-output relations (2.5) that $|0\rangle$ is also the vacuum state of the output electromagnetic modes, with

$$\hat{a}_{\rm out}(\omega)|0\rangle=\hat{b}_{\rm out}(\omega)|0\rangle=0. \quad (2.9)$$

The quantum averages of the Langevin operators vanish,

$$\langle\hat{F}_a(\omega)\rangle=\langle\hat{F}_b(\omega)\rangle=\langle\hat{F}_a^\dagger(\omega)\rangle=\langle\hat{F}_b^\dagger(\omega)\rangle=0, \quad (2.10)$$

and the only nonzero ground-state expectation values for products of pairs of noise operators are

$$\langle\hat{F}_a(\omega)\hat{F}_a^\dagger(\omega')\rangle=\delta(\omega-\omega')\{1-|t(\omega)|^2-|r(\omega)|^2\}$$
$$=\langle\hat{F}_b(\omega)\hat{F}_b^\dagger(\omega')\rangle,$$

$$\langle\hat{F}_a(\omega)\hat{F}_b^\dagger(\omega')\rangle=-\delta(\omega-\omega')\{t(\omega)r^*(\omega)+r(\omega)t^*(\omega)\}$$
$$=\langle\hat{F}_b(\omega)\hat{F}_a^\dagger(\omega')\rangle. \quad (2.11)$$

These relations may also be derived on the basis of a fully canonical one-dimensional theory applied to a dielectric slab [9] as summarized in Appendix A, where results for the limit of a very thin slab, the ''delta-function mirror,'' are also presented.

The relationships derived above are sufficient to model the effects of the lossy beam splitter on any given input states. It is useful, however, to describe its action on the pair of superposition modes associated with the annihilation operators

$$\hat{c}_j(\omega)=\frac{1}{\sqrt{2}}\{\hat{a}_j(\omega)+\hat{b}_j(\omega)\},$$

$$\hat{d}_j(\omega)=\frac{1}{\sqrt{2}}\{\hat{b}_j(\omega)-\hat{a}_j(\omega)\}, \quad (2.12)$$

with j representing either in or out. These superposition modes have the merit that they are not mixed by the action of the beam splitter. The outgoing symmetric and antisymmetric annihilation operators are related to their incident counterparts by

$$\hat{c}_{\rm out}(\omega)=[t(\omega)+r(\omega)]\hat{c}_{\rm in}(\omega)+\hat{F}_c(\omega),$$

$$\hat{d}_{\rm out}(\omega)=[t(\omega)-r(\omega)]\hat{d}_{\rm in}(\omega)+\hat{F}_d(\omega), \quad (2.13)$$

where we have introduced independent superposition Langevin noise operators

$$\hat{F}_c=\{\hat{F}_a+\hat{F}_b\}/\sqrt{2} \quad \text{and} \quad \hat{F}_d=\{\hat{F}_b-\hat{F}_a\}/\sqrt{2} \quad (2.14)$$

that satisfy the simple commutation relations

$$[\hat{F}_c(\omega),\hat{F}_c^\dagger(\omega')]=\delta(\omega-\omega')\{1-|t(\omega)+r(\omega)|^2\},$$

$$[\hat{F}_d(\omega),\hat{F}_d^\dagger(\omega')]=\delta(\omega-\omega')\{1-|t(\omega)-r(\omega)|^2\},$$

$$[\hat{F}_c(\omega),\hat{F}_d^\dagger(\omega')]=0=[\hat{F}_d(\omega),\hat{F}_c^\dagger(\omega')]. \quad (2.15)$$

For a lossless beam splitter, the relation (1.4) shows that the input-output relations (2.13) for the superposition modes amount to no more than a phase shift. For lossy beam splitters, the commutation relations (2.15) are of precisely the form required to restore the quantum fluctuations apparently reduced by the beam splitter and to retain the canonical commutation relations between the outgoing annihilation operators [12].

III. QUANTUM INTERFERENCE EFFECTS

A. Coherent input states

Classically, a beam splitter superposes the incident field amplitudes to produce outgoing fields. If the amplitudes incident from directions $a_{\rm in}$ and $b_{\rm in}$ for the frequency ω are $\alpha_{\rm in}(\omega)$ and $\beta_{\rm in}(\omega)$, respectively, then those leaving in the $a_{\rm out}$ and $b_{\rm out}$ modes are $\alpha_{\rm out}(\omega)$ and $\beta_{\rm out}(\omega)$ given by

$$\alpha_{\rm out}(\omega)=t(\omega)\alpha_{\rm in}(\omega)+r(\omega)\beta_{\rm in}(\omega),$$

$$\beta_{\rm out}(\omega)=t(\omega)\beta_{\rm in}(\omega)+r(\omega)\alpha_{\rm in}(\omega). \quad (3.1)$$

This simple behavior survives in the quantum description for the (continuum) coherent states [11,12] defined by a simple generalization of the familiar discrete mode coherent states [12,25]. The combined state of the input modes and the medium forming the beam splitter is then

$$|\{\alpha_{\mathrm{in}}(\omega)\},\{\beta_{\mathrm{in}}(\omega)\}\rangle = \exp\left\{\int d\omega[\alpha_{\mathrm{in}}(\omega)\hat{a}^{\dagger}_{\mathrm{in}}(\omega) - \alpha^{*}_{\mathrm{in}}(\omega)\hat{a}_{\mathrm{in}}(\omega)]\right\}\exp\left\{\int d\omega[\beta_{\mathrm{in}}(\omega)\hat{b}^{\dagger}_{\mathrm{in}}(\omega) - \beta^{*}_{\mathrm{in}}(\omega)\hat{b}_{\mathrm{in}}(\omega)]\right\}|0\rangle, \tag{3.2}$$

where $|0\rangle$ denotes the ground state defined in Eqs. (2.8) and (2.9). The continuum coherent state (3.2) is the right eigenstate of both $\hat{a}_{\mathrm{in}}(\omega)$ and $\hat{b}_{\mathrm{in}}(\omega)$, for all frequencies ω, with eigenvalues $\alpha_{\mathrm{in}}(\omega)$ and $\beta_{\mathrm{in}}(\omega)$, respectively. It is now straightforward to show that this state is also a right eigenstate of the output annihilation operators,

$$\hat{a}_{\mathrm{out}}(\omega')|\{\alpha_{\mathrm{in}}(\omega)\},\{\beta_{\mathrm{in}}(\omega)\}\rangle = \{t(\omega')\alpha_{\mathrm{in}}(\omega') + r(\omega')\beta_{\mathrm{in}}(\omega')\}|\{\alpha_{\mathrm{in}}(\omega)\},\{\beta_{\mathrm{in}}(\omega)\}\rangle,$$

$$\hat{b}_{\mathrm{out}}(\omega')|\{\alpha_{\mathrm{in}}(\omega)\},\{\beta_{\mathrm{in}}(\omega)\}\rangle = \{t(\omega')\beta_{\mathrm{in}}(\omega') + r(\omega')\alpha_{\mathrm{in}}(\omega')\}|\{\alpha_{\mathrm{in}}(\omega)\},\{\beta_{\mathrm{in}}(\omega)\}\rangle. \tag{3.3}$$

It follows that the two output fields are also in coherent states with amplitudes given by the classical expressions (3.1). These results look the same as for a lossless beam splitter, but of course the reflection and transmission coefficients that appear in the output amplitudes are reduced by the loss in accordance with the inequality (2.1).

The simplicity of these transmission characteristics is peculiar to the coherent states and mixtures of them [26]. Other input states produce entangled outputs that cannot be factorized into a product of separate states for the two outgoing fields [27]. Such states can exhibit explicitly quantum effects. In the remainder of this section we consider two examples of entangled outputs, we derive a general description of the state of the outgoing fields, and we discuss the effects of the beam splitter on nonclassical incident light.

B. Two-photon interference

A single photon incident on a lossless beam splitter, with frequency-independent transmission and reflection coefficients t and r, is transmitted with probability $|t|^2$ and reflected with probability $|r|^2 = 1-|t|^2$. A pair of photons exhibits a more interesting behavior. If the two photons are incident in the same beam, then they behave like independent *classical* particles in that the probabilities for both being transmitted or both reflected are $|t|^4$ and $|r|^4$, respectively, while the probability for one being reflected and the other transmitted is $2|t|^2|r|^2$ [14]. This behavior has been demonstrated using parametric fluorescence as a source of photon pairs [28,29].

A somewhat different result occurs if the two photons enter the beam splitter through different arms. Under suitable conditions the two photons can be made to leave in the same beam. This effect, which has been demonstrated experimentally [4,30], arises from destructive interference between the amplitudes for both photons to be reflected and for both photons to be transmitted and it is a consequence of the bosonic nature of the photons [31]. In this section we examine the expected modifications of these effects when a lossy beam splitter is used.

Consider a pair of photons incident in the same input arm a_{in} while arm b_{in} is left in its vacuum state. The general (pure) state can then be written in the form

$$|\phi\rangle = \int_0^{\infty} d\omega_a \int_0^{\infty} d\omega'_a \phi(\omega_a,\omega'_a)\hat{a}^{\dagger}_{\mathrm{in}}(\omega_a)\hat{a}^{\dagger}_{\mathrm{in}}(\omega'_a)|0\rangle, \tag{3.4}$$

where $|0\rangle$ again denotes the electromagnetic vacuum state and ground state of the medium forming the beam splitter. Without loss of generality we can choose the superposition amplitude $\phi(\omega_a,\omega'_a)$ to be symmetric under interchange of its arguments. Normalization of the state vector then imposes the condition

$$2\int_0^{\infty} d\omega_a \int_0^{\infty} d\omega'_a |\phi(\omega_a,\omega'_a)|^2 = 1. \tag{3.5}$$

It is convenient to introduce continuum number operators [12] for the two outputs. These have states of well-defined photon number as their eigenstates and they may be expressed in the forms

$$\hat{N}_a = \int_0^{\infty} d\omega \hat{a}^{\dagger}_{\mathrm{out}}(\omega)\hat{a}_{\mathrm{out}}(\omega),$$

$$\hat{N}_b = \int_0^{\infty} d\omega \hat{b}^{\dagger}_{\mathrm{out}}(\omega)\hat{b}_{\mathrm{out}}(\omega) \tag{3.6}$$

for the a_{out} and b_{out} modes, respectively. The probabilities for finding given numbers of photons in the outputs are then obtained from the Kelley-Kleiner counting formula, specialized to unit quantum efficiency and infinite counting time [25]. In particular, the probability for finding two photons in mode a_{out} and none in mode b_{out} is

$$P(2_a,0_b) = \tfrac{1}{2}\langle \hat{N}_a(\hat{N}_a-1)\rangle. \tag{3.7}$$

Similarly, the remaining nonzero probabilities are

$$P(0_a,2_b) = \frac{1}{2}\langle \hat{N}_b(\hat{N}_b-1)\rangle,$$

$$P(1_a,1_b) = \langle \hat{N}_a\hat{N}_b\rangle,$$

$$P(1_a,0_b) = \langle \hat{N}_a\rangle - \langle \hat{N}_a(\hat{N}_a-1)\rangle - \langle \hat{N}_a\hat{N}_b\rangle,$$

$$P(0_a,1_b) = \langle \hat{N}_b\rangle - \langle \hat{N}_b(\hat{N}_b-1)\rangle - \langle \hat{N}_a\hat{N}_b\rangle,$$

$$P(0_a,0_b) = 1 - \langle \hat{N}_a\rangle - \langle \hat{N}_b\rangle + \langle \hat{N}_a\hat{N}_b\rangle + \frac{1}{2}\langle \hat{N}_a(\hat{N}_a-1)\rangle + \frac{1}{2}\langle \hat{N}_b(\hat{N}_b-1)\rangle. \tag{3.8}$$

For a lossless beam splitter, the number of photons leaving the beam splitter is strictly equal to the number incident upon it and the last three of the probabilities listed in Eq. (3.8) are all zero. Inclusion of losses introduces a probability for absorption so that these probabilities need not vanish. The probabilities can be calculated by using the relationships (2.5) to express the output continuum number operators (3.6) in terms of the input continuum annihilation and creation operators and then applying the commutation relations (1.1). Evaluations of some of the required photon-number factorial moments are presented in Appendix B.

It is quite straightforward to calculate the moments for any given forms of frequency dependence of the transmission and reflection coefficients. For simplicity, however, as well as for each of comparison with earlier work, we restrict our discussion to coefficients that do not vary appreciably over the bandwidth of the light, and can thus be approximated as independent of frequency. With these restrictions, we find the following forms for the required probabilities:

$$P(2_a,0_b)=|t|^4, \quad P(0_a,2_b)=|r|^4,$$

$$P(1_a,1_b)=2|t|^2|r|^2,$$

$$P(1_a,0_b)=2|t|^2(1-|t|^2-|r|^2),$$

$$P(0_a,1_b)=2|r|^2(1-|t|^2-|r|^2),$$

$$P(0_a,0_b)=(1-|t|^2-|r|^2)^2. \tag{3.9}$$

These results are fully consistent with the property noted for a lossless beam splitter that the photons behave like independent classical particles. Thus each photon is transmitted with probability $|t|^2$, reflected with probability $|r|^2$ and absorbed with probability $1-|t|^2-|r|^2$. In particular, the probability that precisely one photon survives is the sum of $P(1_a,0_b)$ and $P(0_a,1_b)$, with the value of twice the product of the probability $1-|t|^2-|r|^2$ that a single photon is absorbed with the probability $|t|^2+|r|^2$ that a single photon is not absorbed. Note that the probabilities in Eq. (3.9) are all independent of the form of the superposition amplitude $\phi(\omega_a,\omega_a')$ used in the state (3.4). We illustrate Eq. (3.9) in Fig. 2, which is a plot of the probabilities of obtaining four possible outcomes for a two-photon input against the single-photon survival probability $2|t|^2$ for a symmetric lossy beam splitter. The probability of obtaining no photons at either output, curve (0,0) in the figure, decays with increasing survival probability, while the probability of obtaining a photon in one of the output arms and none in the other, curve $(1,0)=(0,1)$, vanishes at both ends of the range. The probabilities of obtaining one photon in each arm and of obtaining both photons in one of the output arms, curves (1,1) and $(2,0)=(0,2)$, respectively, increase with the single photon survival probability. These results show that the naive assumption that the photons behave as independent classical particles is adequate to explain their behavior in this case.

We now turn to the case of two photons incident in different input arms, one in arm a_{in} and the other in arm b_{in}. The general (pure) state can be written in the form

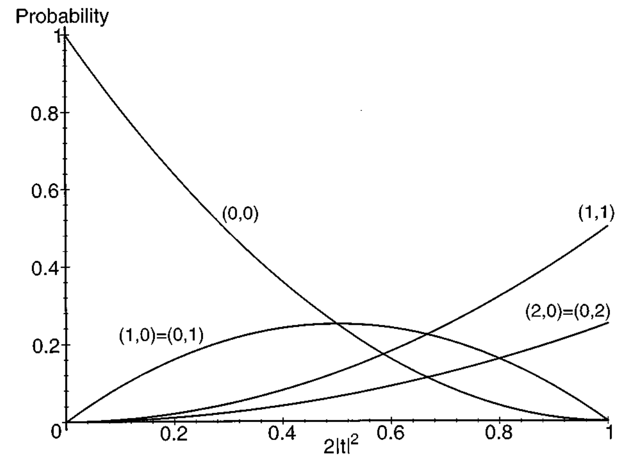

FIG. 2. Plots of the probabilities of the various possible outcomes for two photons incident from the same arm against the single photon survival probability. We assume that the transmission and reflection coefficients have the same modulus. The bracketed numbers show the elements of the output probability distribution in arms a and b to which the curves refer.

$$|\psi\rangle=\int_0^\infty d\omega_a\int_0^\infty d\omega_b\psi(\omega_a,\omega_b)\hat{a}_{\text{in}}^\dagger(\omega_a)\hat{b}_{\text{in}}^\dagger(\omega_b)|0\rangle, \tag{3.10}$$

where there is no required symmetry for the function $\psi(\omega_a,\omega_b)$. Normalization of the state vector imposes a normalization on $\psi(\omega_a,\omega_b)$ so that

$$\int_0^\infty d\omega_a\int_0^\infty d\omega_b|\psi(\omega_a,\omega_b)|^2=1. \tag{3.11}$$

The probabilities for finding given numbers of photons in the two outputs are obtained from Eqs. (3.7) and (3.8) and they may be calculated by the method outlined in Appendix B. If we again specialize to reflection and transmission coefficients that may be approximated as frequency independent, then we find

$$P(2_a,0_b)=|t|^2|r|^2[1+I]=P(0_a,2_b),$$

$$P(1_a,1_b)=|t|^4+|r|^4+[t^2r^{*2}+r^2t^{*2}]I,$$

$$P(1_a,0_b)=(|t|^2+|r|^2)(1-|t|^2-|r|^2)-(tr^*+rt^*)^2I$$
$$=P(0_a,1_b),$$

$$P(0_a,0_b)=(1-|t|^2-|r|^2)^2+(rt^*+tr^*)^2I, \tag{3.12}$$

where we have introduced the (real) overlap integral

$$I=\int_0^\infty d\omega_a\int_0^\infty d\omega_b\psi(\omega_a,\omega_b)\psi^*(\omega_b,\omega_a). \tag{3.13}$$

The probabilities in Eq. (3.12) are now crucially dependent on the form of the superposition amplitude $\psi(\omega_a,\omega_b)$ used in the state (3.10). If $I=0$, the photon amplitudes for the two input modes do not overlap in time at the beam splitter, and the probabilities (3.12) reduce to the forms expected for two independent particles, each with probability $|t|^2$ for transmission, $|r|^2$ for reflection and $1-|t|^2-|r|^2$ for absorption. At the opposite extreme, for $I=1$, the photon amplitudes overlap completely in time and the probability for finding precisely one photon in each of the two outputs is

$$P(1_a,1_b)=|t^2+r^2|^2. \tag{3.14}$$

This quantity vanishes for $t=\pm ir$, when the second equality in Eq. (1.3) is satisfied even in the presence of loss. The famous Hong-Ou-Mandel interference effect [4] thus survives for a lossy beam splitter provided that the complex t and r remain orthogonal.

It is interesting to note that the quantum interference effects embodied in the overlap integral I generally affect the probabilities for one or two of the photons to be absorbed. In particular, the probabilities for two, one, or no photons to survive are

$$\begin{aligned}
P(2\ \text{survive})&=P(2_a,0_b)+P(0_a,2_b)+P(1_a,1_b)=(|t|^2+|r|^2)^2+(tr^*+rt^*)^2I,\\
P(1\ \text{survives})&=P(1_a,0_b)+P(0_a,1_b)=2(|t|^2+|r|^2)(1-|t|^2-|r|^2)-2(tr^*+rt^*)^2I,\\
P(0\ \text{survive})&=P(0_a,0_b)=(1-|t|^2-|r|^2)^2+(tr^*+rt^*)^2I.
\end{aligned} \tag{3.15}$$

These are the same as the probabilities found for two photons incident in the same input only if $I=0$ or if t and r are orthogonal (tr^* pure imaginary). In all other cases, the two-photon interference affects the survival probabilities. This means that an apparent *nonlinear absorption* occurs in the linear medium forming the beam splitter. It takes its most extreme form when the overlap is ideal so that $I=1$ and the transmission and reflection coefficients are equal or opposite ($t=\pm r$). It then follows from the inequality (2.3) that the maximum values of the moduli of the transmission and reflection coefficients are $|t|=|r|=1/2$. Under these conditions, the probability that precisely one of the photons survives is zero. The complete set of output photon-number probabilities is

$$\begin{aligned}
P(2_a,0_b)&=\frac{1}{8}=P(0_a,2_b),\\
P(1_a,1_b)&=\frac{1}{4},\\
P(1_a,0_b)&=0=P(0_a,1_b),\\
P(0_a,0_b)&=\frac{1}{2}.
\end{aligned} \tag{3.16}$$

Clearly either both photons are absorbed or neither is, and this occurs even though the absorption is a linear process. Figure 3 is a plot of the survival probabilities for various numbers of photons (3.15), after propagation through the beam splitter, against the survival probability for single independent photons. It assumes complete overlap of the two photons at the beam splitter, and equal or opposite reflection and transmission coefficients. The probability that no photons survive, curve (0), decreases from a maximum of unity at $2|t|^2=0$ to 0.5 at $2|t|^2=0.5$, and the probability that both photons survive, curve (2), increases from zero to 0.5. The probability that only one of the photons survives, shown in curve (1), increases to a maximum at $2|t|^2=0.25$ and then decreases to zero again at $2|t|^2=0.5$. This right-hand end of the plot corresponds to the results in Eq. (3.16).

The apparent two-photon absorption is, in fact, a manifestation of quantum interference as may be demonstrated by considering the superposition modes Eq. (2.12). When written in terms of these modes, the input state becomes

$$|\psi\rangle=\int_0^\infty d\omega_a\int_0^\infty d\omega_b\psi(\omega_a,\omega_b)\,\frac{1}{2}\,\{\hat{c}^\dagger_{\text{in}}(\omega_a)\hat{c}^\dagger_{\text{in}}(\omega_b) - \hat{d}^\dagger_{\text{in}}(\omega_a)\hat{d}^\dagger_{\text{in}}(\omega_b)\}|0\rangle, \tag{3.17}$$

where we have used the symmetry of $\psi(\omega_a,\omega_b)$ with respect to interchange of its arguments when $I=1$. The input state in this case has both photons in one or other of the two superpositions of the input fields. The output annihilation operators for the superposition modes are related to the input operators by Eq. (2.13). For the conditions under consideration, with $t=\pm r$ and $|t|=1/2$, light in one of the two superposition modes is completely absorbed while that in the other superposition merely undergoes a phase shift. It again follows, therefore, that either both or neither of the photons is absorbed by the beam splitter.

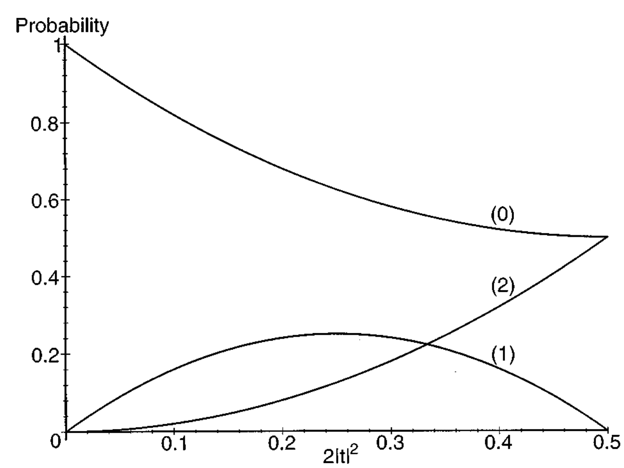

FIG. 3. Plots of the survival probabilities for two photons incident in different input arms, with full overlap between them and reflection and transmission coefficients related by $t=\pm r$. The numbers of surviving photons are shown in brackets adjacent to the appropriate curves.

C. General quantum statistical description

All the measurable statistical properties of the fields can be expressed in terms of normally ordered moments of the continuum annihilation and creation operators. These moments are conveniently expressed in terms of the normally ordered characteristic functional, with the form

$$\chi_k[\xi(\omega_a),\eta(\omega_b)]=\left\langle \exp\left[\int_0^\infty d\omega_a \xi(\omega_a)\hat{a}_k^\dagger(\omega_a)\right]\exp\left[\int_0^\infty d\omega_b \eta(\omega_b)\hat{b}_k^\dagger(\omega_b)\right]\exp\left[-\int_0^\infty d\omega_a \xi^*(\omega_a)\hat{a}_k(\omega_a)\right]\right.$$
$$\left.\times\exp\left[-\int_0^\infty d\omega_b \eta^*(\omega_b)\hat{b}_k(\omega_b)\right]\right\rangle, \tag{3.18}$$

where k denotes either in or out. Any normally ordered moment of the creation and annihilation operators is found by functional differentiation [12,32–34], for example,

$$\langle \hat{a}_k^\dagger(\omega)\hat{b}_k^\dagger(\omega')\hat{b}_k(\omega'')\hat{b}_k(\omega''')\rangle=\frac{\delta}{\delta\xi(\omega)}\,\frac{\delta}{\delta\eta(\omega')}\,\frac{-\delta}{\delta\eta^*(\omega'')}\,\frac{-\delta}{\delta\eta^*(\omega''')}\,\chi_k[\xi(\omega_a),\eta(\omega_b)]\Big|_{\xi=\eta=0}. \tag{3.19}$$

We can write the characteristic functional for the outputs in terms of that for the inputs by using the relations (2.5), together with the fact that the normally ordered moments of the Langevin noise operators are zero in the ground state of the absorbing medium. The required relationship is then

$$\chi_{\text{out}}[\xi(\omega_a),\eta(\omega_b)]=\chi_{\text{in}}[\{t^*(\omega_a)\xi(\omega_a)+r^*(\omega_a)\eta(\omega_a)\},\{t^*(\omega_b)\eta(\omega_b)+r^*(\omega_b)\xi(\omega_b)\}]. \tag{3.20}$$

The characteristic functional for the outputs can thus be constructed from that of the inputs when the form of the input state is specified.

As a special case of these relations, consider an experiment in which the input mode b_{in} is in its vacuum state, when the second and fourth moments of the a_{out} output mode are

$$\langle \hat{a}_{\text{out}}^\dagger(\omega)\hat{a}_{\text{out}}(\omega')\rangle=t^*(\omega)t(\omega')\langle \hat{a}_{\text{in}}^\dagger(\omega)\hat{a}_{\text{in}}(\omega')\rangle \tag{3.21}$$

and

$$\langle \hat{a}_{\text{out}}^\dagger(\omega)\hat{a}_{\text{out}}^\dagger(\omega')\hat{a}_{\text{out}}(\omega'')\hat{a}_{\text{out}}(\omega''')\rangle=t^*(\omega)t^*(\omega')t(\omega'')t(\omega''')\langle \hat{a}_{\text{in}}^\dagger(\omega)\hat{a}_{\text{in}}^\dagger(\omega')\hat{a}_{\text{in}}(\omega'')\hat{a}_{\text{in}}(\omega''')\rangle. \tag{3.22}$$

The degree of second-order coherence of the a_{out} output is obtained by division of the fourth moment by the square of the second moment, with both moments Fourier transformed to the time domain. These time-domain moments can in principle be calculated for given input states and known frequency variations of the beam-splitter transmission coefficient $t(\omega)$. However, it is seen from the forms of Eqs. (3.21) and (3.22) that the output degree of second-order coherence equals that of the input in cases where $t(\omega)$ varies by a negligible amount over the bandwidth of the input state. In particular, any photon antibunching in the input state, measured by a deviation of the degree of second-order coherence below unity, survives with the same magnitude in the output state in such cases, even with loss in the beam splitter.

D. Homodyne detection of squeezed light

Besides its role as an integral component in quantum interference experiments, the beam splitter is also essential to the homodyne detection scheme. This is the preferred method for the detection of squeezed light, as the moments of the difference photocount distribution obtained at the detectors are proportional to the moments of the electric field distribution. The basic scheme uses the beam splitter shown in Fig. 1(a). The signal light to be investigated falls upon the beam splitter from input arm a_{in}, where it is mixed with an intense coherent local oscillator from arm b_{in}. The output light falls upon two detectors, and the difference photocount between arms a_{out} and b_{out} is obtained. The measurement is represented by the operator

$$\hat{O}=\int_0^{T_0} d\tau[\hat{a}_{\text{out}}^\dagger(\tau)\hat{a}_{\text{out}}(\tau)-\hat{b}_{\text{out}}^\dagger(\tau)\hat{b}_{\text{out}}(\tau)], \tag{3.23}$$

where the time-dependent operator $\hat{a}_{\text{out}}(\tau)$ is the Fourier transform of $\hat{a}_{\text{out}}(\omega)$, and so on, and T_0 is the detector integration time. The measurement operator is thus expressed in terms of the frequency-dependent input operators with the use of Eq. (2.5) as

$$\hat{O}=\int_0^{T_0}d\tau\int d\omega\int d\omega'\ \exp[i(\omega-\omega')\tau]\{[t^*(\omega)\hat{a}_{\rm in}^\dagger(\omega)+r^*(\omega)\hat{b}_{\rm in}^\dagger(\omega)+\hat{F}_a^\dagger(\omega)][t(\omega')\hat{a}_{\rm in}(\omega')+r(\omega')\hat{b}_{\rm in}(\omega')+\hat{F}_a(\omega')]$$
$$-[t^*(\omega)\hat{b}_{\rm in}^\dagger(\omega)+r^*(\omega)\hat{a}_{\rm in}^\dagger(\omega)+\hat{F}_b^\dagger(\omega)][t(\omega')\hat{b}_{\rm in}(\omega')+r(\omega')\hat{a}_{\rm in}(\omega')+\hat{F}_b(\omega')]. \tag{3.24}$$

Expectation values of this operator determine the moments of the difference photocount distribution.

In order to take things further, we assume that the local oscillator is a large-amplitude coherent beam at the central squeezing frequency ω_0 and that the reflection and transmission coefficients of the beam splitter are equal in magnitude and constant at the frequencies of interest. The noise fields of the beam-splitter medium are taken to be in the ground state, as before, and expectation values that involve only the two noise operator products then vanish. Also, products of noise operators with the local oscillator dominate those with the signal, so the latter may be ignored. The homodyne detection operator reduces to

$$\hat{O}=(t^*r-r^*t)f_L^{1/2}\int_0^{T_0}d\tau\{\hat{a}_{\rm in}^\dagger(\tau)\exp[i(\phi_L-\omega_0\tau)]-\hat{a}_{\rm in}(\tau)\exp[-i(\phi_L-\omega_0\tau)]\}+f_L^{1/2}\int_0^{T_0}d\tau\int d\omega\ \exp[i(\omega-\omega_0)\tau-i\phi_L]$$
$$\times[r^*\hat{F}_a(\omega)-t^*\hat{F}_b(\omega)]+\text{H.c.}, \tag{3.25}$$

where f_L is the photon flux of the local oscillator of frequency ω_0 and phase ϕ_L. The detection operator consists of two parts, one of which depends on the electric field of the input, the other on the noise operators.

The input field is now assumed to be in a squeezed vacuum state, with expectation values

$$\langle\hat{a}_{\rm in}^\dagger(\omega)\hat{a}_{\rm in}(\omega')\rangle=\sinh^2 s(\omega)\,\delta(\omega-\omega'),$$
$$\langle\hat{a}_{\rm in}(\omega)\hat{a}_{\rm in}(\omega')\rangle=\exp(i\theta)\cosh s(\omega)\sinh s(\omega)\,\delta(\omega-2\omega_0+\omega'). \tag{3.26}$$

The expectation value of the homodyne operator (3.25) vanishes and its normally ordered variance is

$$\langle:\hat{O}^2:\rangle-\langle\hat{O}\rangle^2=4[\text{Im}(t^*r)]^2f_LT_0\left[e^{-2s}\cos^2\left(\phi_L-\frac{1}{2}\theta\right)+e^{2s}\sin^2\left(\phi_L-\frac{1}{2}\theta\right)-1\right], \tag{3.27}$$

where $s=s(\omega_0)$, and we have assumed that the squeezing bandwidth is large compared with $1/T_0$. The expression in the large brackets on the right-hand side vanishes for a coherent signal with $s=0$, while it takes negative values for a squeezed signal with $s>0$ and appropriate values of the local oscillator phase. The prefactor of $4[\text{Im}(t^*r)]^2$ in Eq. (3.27) equals unity for a lossless beam splitter with reflection and transmission coefficients of equal magnitude, but the factor is smaller than unity in the presence of loss and it represents a reduction in the observed squeezing with respect to the lossless case. If Eq. (2.4) is satisfied as an equality, then this factor is the square of the one-photon absorption probability and we recover the reduction in detected squeezing noted by Lai *et al.* [19]. Of course the magnitude of the negative variance is reduced further if the squeezing covers a range of frequencies for which the Langevin noise fields are excited.

IV. DISCRETE-MODE MODEL

It is often sufficient to work with single discrete modes rather than the full continuum used in the preceding sections. This simplification is especially useful if the mirror transmission and reflection coefficients may be approximated by frequency-independent quantities and the input modes are perfectly overlapping. The quantum statistics associated with discrete-mode models have been discussed in some detail [19] and we present only an outline of the theory. The discrete-mode model is obtained from the continuum description by suppressing the frequency dependences in the transmission and reflection coefficients and in the annihilation and creation operators, and by replacing the continuum commutation relations by their discrete analogues. In particular, the incident and output mode operators satisfy the commutation relations

$$[\hat{a}_j,\hat{a}_j^\dagger]=1=[\hat{b}_j,\hat{b}_j^\dagger],$$
$$[\hat{a}_j,\hat{b}_j^\dagger]=0=[\hat{b}_j,\hat{a}_j^\dagger], \tag{4.1}$$

with j representing either in or out. The relationships between the operators for the output and incident modes are the natural analogues of equations (2.5) so that

$$\hat{a}_{\rm out}=t\hat{a}_{\rm in}+r\hat{b}_{\rm in}+\hat{F}_a,$$
$$\hat{b}_{\rm out}=t\hat{b}_{\rm in}+r\hat{a}_{\rm in}+\hat{F}_b. \tag{4.2}$$

The remaining operator properties may be obtained from Eqs. (2.6) to (2.11) by suppressing the frequency dependence and replacing frequency delta functions by unity. These properties are sufficient to calculate all the statistical properties of the two output modes for any given state of the two input modes.

The discrete-mode normally ordered characteristic function is

$$\chi_k(\xi,\eta)=\langle \exp(\xi\hat{a}_k^\dagger)\exp(\eta\hat{b}_k^\dagger)\exp(-\xi^*\hat{a}_k)\exp(-\eta^*\hat{b}_k)\rangle, \tag{4.3}$$

where k denotes either in or out. We can express the normally ordered characteristic function for the output modes in terms of that for the input modes as

$$\chi_{\rm out}(\xi,\eta)=\chi_{\rm in}(t^*\xi+r^*\eta, t^*\eta+r^*\xi), \tag{4.4}$$

which is analogous to Eq. (3.20) for the continuum modes. The characteristic function provides a complete description of the state and so Eq. (4.4) allows us to calculate any desired property of the output modes if we know the characteristic function for the input modes. As a simple example consider the state

$$|\Psi\rangle=\hat{a}_{\rm in}^\dagger\hat{b}_{\rm in}^\dagger|0\rangle, \tag{4.5}$$

in which each input mode contains precisely one photon. The normally ordered characteristic function for this state is

$$\chi_{\rm in}(\xi,\eta)=1-|\xi|^2-|\eta|^2+|\xi\eta|^2. \tag{4.6}$$

The photon-number factorial moments for the output modes can be found by using equation (4.4) to obtain the characteristic function for the output modes and then calculate derivatives [12]. The nonzero factorial moments are

$$\begin{gathered}
\langle\hat{a}_{\rm out}^\dagger\hat{a}_{\rm out}\rangle=-\frac{\partial^2}{\partial\xi\partial\xi^*}\,\chi_{\rm out}(\xi,\eta)\bigg|_{\xi=\eta=0}=|t|^2+|r|^2,\\
\langle\hat{b}_{\rm out}^\dagger\hat{b}_{\rm out}\rangle=-\frac{\partial^2}{\partial\eta\partial\eta^*}\,\chi_{\rm out}(\xi,\eta)\bigg|_{\xi=\eta=0}=|t|^2+|r|^2,\\
\langle\hat{a}_{\rm out}^\dagger\hat{b}_{\rm out}^\dagger\hat{b}_{\rm out}\hat{a}_{\rm out}\rangle=|t^2+r^2|^2,\\
\langle\hat{a}_{\rm out}^{\dagger 2}\hat{a}_{\rm out}^2\rangle=4|tr|^2=\langle\hat{b}_{\rm out}^{\dagger 2}\hat{b}_{\rm out}^2\rangle.
\end{gathered} \tag{4.7}$$

These are of the same form as the factorial moments calculated in Appendix B using the continuum modes with perfect overlap between the two input photons so that $I=1$.

V. CONCLUSION

The electric-field amplitude transmission and reflection coefficients of a lossless beam splitter at a given frequency are constrained by the requirements of unitarity, or equivalently energy conservation between the output and input beams, to satisfy the relations given in Eq. (1.3). These relations show that $t(\omega)$ and $r(\omega)$ are orthogonal numbers in the complex plane and that they form two sides of a right-angled triangle with unit hypotenuse.

We have calculated the effects of loss in a beam splitter on the transmission and reflection coefficients and on the relations between the output and input fields, expressed in terms of the continuous-mode frequency-dependent output and input annihilation and creation operators. The loss causes reductions in the magnitudes of the transmission and reflection coefficients below their values for a lossless beam splitter, so that the sum of their square moduli is less than unity. The requirement that $t(\omega)$ and $r(\omega)$ should be orthogonal complex numbers is also removed. The presence of loss is associated with the existence of noise sources in the beam-splitter material, which are here modeled by Langevin operators. The output fields thus acquire Langevin noise components in addition to the transmitted and reflected contributions from the input fields. The magnitudes of the output noise components are such as to maintain the necessary free-space values of the commutators of the output field operators, which would otherwise be reduced by the removal of intensity from the input fields.

The effective temperatures of the noise sources can be taken equal to zero for experiments with visible light, and the output noise does not contribute to normally ordered expectation values of the output field operators. Such expectation values are, however, modified by the changes in transmission and reflection caused by the loss. Loss is particularly important in experiments that detect nonclassical properties of light, where beam splitters usually play crucial roles in the measurements. We have evaluated the reductions caused by beam-splitter loss in the detectable squeezing and in the photon-number factorial moments tha determine the photon antibunching, although the ratio of moments that occurs in the degree of second-order coherence is unchanged by beam-splitter transmission.

The most striking loss-related phenomena occur, however, in the two-photon interference effect. The orthogonality of $t(\omega)$ and $r(\omega)$ in the lossless beam splitter is sufficient to produce this well-known interference, in which a pair of photons incident in different input arms can only leave the beam splitter in the same output arm. The removal of the orthogonality constraint in the presence of loss allows a more varied range of interference effects to occur. Thus the standard two-photon interference survives if the transmission and reflection coefficients remain orthogonal even after the introduction of loss. However, it is also possible in principle for the coefficients of a lossy beam splitter to be equal or opposite, and we have shown that an apparent nonlinear behavior can then occur, in which both photons are absorbed or neither is absorbed. The beam splitter thus acts as an effective two-photon absorber, despite the linear optical properties assumed in its construction, and the observation of this predicted effect would add a new kind of interference experiment to the range of measured quantum-optical phenomena.

ACKNOWLEDGMENTS

This work was supported by the UK Engineering and Physical Sciences Research Council and by the European Community Human Capital and Mobility Programme.

APPENDIX A: DIELECTRIC SLAB AND DELTA-FUNCTION MIRROR

We indicate how the general beam-splitter formalism derived in the present paper applies to the quantized normally incident electromagnetic fields derived previously [9] (this paper and its equations are identified by the abbreviation MLBJ) for the absorbing dielectric slab illustrated in Fig. 1(b). The notation used here differs from that of MLBJ, but the annihilation operator input-output relations in MLBJ (5.11) and (5.15) are essentially the same as those given here in Eq. (2.5), except that the output noise operators $\hat{F}_a(\omega)$

and $\hat{F}_b(\omega)$ in the former are expressed in terms of spatial integrals over the slab thickness $2l$. The integrands in these output noise operators include the complex refractive index $n(\omega)$ of the slab material, functions $V(\omega)$ and $W(\omega)$ that describe the multiply reflected fields in the slab, and a distributed Langevin noise operator $\hat{f}(x,\omega)$, whose commutation relation is given by MLBJ (3.10) as

$$[\hat{f}(x,\omega),\hat{f}^{\dagger}(x',\omega')]=\delta(x-x')\,\delta(\omega-\omega'). \tag{A1}$$

MLBJ verified that their output operators satisfy the commutation relations (1.1) and it is straightforward to show that the output noise operators also have the ground-state expectation values given by Eq. (2.11). The dielectric slab quantization of MLBJ thus conforms fully with the general beam-splitter theory presented here.

A useful special case of the dielectric slab is that of the "delta-function mirror," obtained by letting the slab thickness tend to zero as its complex refractive index tends to infinity

$$2l\to 0,$$

$$n(\omega)\to\infty \tag{A2}$$

such that $2 l n(\omega)^2$ is finite. Such slabs have been used to model the mirrors of Fabry-Pérot cavities for normally incident light beams [35] and the theory has been extended to oblique incidence for photonic crystal structures made from arrays of delta-function mirrors [36]. However, the refractive index is assumed to be real in this previous work. For a lossy delta-function mirror with normal incidence, the limits in Eq. (A2) are readily taken in the expressions of MLBJ. The resulting input-output relations are the same as in Eq. (2.5) with the identifications

$$t(\omega)=\frac{1}{1-i\mu(\omega)},\quad r(\omega)=\frac{i\mu(\omega)}{1-i\mu(\omega)} \tag{A3}$$

and

$$\hat{F}_a(\omega)=\hat{F}_b(\omega)=\frac{i}{1-i\mu(\omega)}\left(\frac{\mathrm{Im}\ \mu(\omega)}{l}\right)^{1/2}\int_{-l}^{l}dx\ \hat{f}(x,\omega), \tag{A4}$$

where

$$\mu(\omega)=\omega\ l n(\omega)^2/c \tag{A5}$$

is a dimensionless parameter that characterizes the optical properties of the lossy dielectric slab. The transmission and reflection coefficients in this case satisfy

$$\begin{aligned}1-|t(\omega)|^2-|r(\omega)|^2&=-t(\omega)r^*(\omega)-r(\omega)t^*(\omega)\\&=\frac{2\ \mathrm{Im}\ \mu(\omega)}{1+2\ \mathrm{Im}\ \mu(\omega)+|\mu(\omega)|^2},\end{aligned} \tag{A6}$$

which is zero only if the imaginary part of the dielectric constant, $\varepsilon(\omega)=n^2(\omega)$, is zero, corresponding to no absorption. It is easy to verify with the use of Eq. (A1) that the noise operators of Eq. (A4) satisfy the commutation relations in Eq. (2.7), and Eq. (A6) clearly reduces to Eq. (1.3) in the absence of loss.

APPENDIX B: CALCULATION OF PHOTON-NUMBER FACTORIAL MOMENTS

In our discussion of two-photon interference effects we needed the form of the first and second photon-number factorial moments for the two output fields given the input states (3.4) and (3.10). These may be evaluated by using the relationship between the input and output fields (2.5), the action of the annihilation operators on the vacuum or ground state (2.8), and the commutation relations (1.1). Consider, for example, the action of the operator $\hat{a}_{\mathrm{out}}(\omega)$ on the state $|\phi\rangle$ defined in Eq. (3.4):

$$\begin{aligned}\hat{a}_{\mathrm{out}}(\omega)|\phi\rangle&=\{t(\omega)\hat{a}_{\mathrm{in}}(\omega)+r(\omega)\hat{b}_{\mathrm{in}}(\omega)+\hat{F}_a(\omega)\}\int_0^{\infty}d\omega_a\int_0^{\infty}d\omega'_a\phi(\omega_a,\omega'_a)\hat{a}^{\dagger}_{\mathrm{in}}(\omega_a)\hat{a}^{\dagger}_{\mathrm{in}}(\omega'_a)|0\rangle\\&=t(\omega)\int_0^{\infty}d\omega_a\int_0^{\infty}d\omega'_a\phi(\omega_a,\omega'_a)\{\hat{a}^{\dagger}_{\mathrm{in}}(\omega_a)\hat{a}_{\mathrm{in}}(\omega)+\delta(\omega-\omega_a)\}\hat{a}^{\dagger}_{\mathrm{in}}(\omega'_a)|0\rangle\\&=t(\omega)\int_0^{\infty}d\omega_a\int_0^{\infty}d\omega'_a\phi(\omega_a,\omega'_a)\{\delta(\omega-\omega_a)\hat{a}^{\dagger}_{\mathrm{in}}(\omega'_a)+\delta(\omega-\omega'_a)\hat{a}^{\dagger}_{\mathrm{in}}(\omega_a)\}|0\rangle\\&=2t(\omega)\int_0^{\infty}d\omega_a\phi(\omega_a,\omega)\hat{a}^{\dagger}_{\mathrm{in}}(\omega_a)|0\rangle.\end{aligned} \tag{B1}$$

It then follows that the expectation value of the continuum number operator $\hat{N}_a$ is

$$\langle\hat{N}_a\rangle=\int_0^{\infty}d\omega\langle\phi|\hat{a}^{\dagger}_{\mathrm{out}}(\omega)\hat{a}_{\mathrm{out}}(\omega)|\phi\rangle=4\int_0^{\infty}d\omega|t(\omega)|^2\int_0^{\infty}d\omega_a|\phi(\omega_a,\omega)|^2. \tag{B2}$$

If we consider cases where the transmission coefficient is approximately constant over the range of frequencies for which $|\phi(\omega_a,\omega)|$ is significant, then this first moment reduces with the use of Eq. (3.5) to $\langle\hat{N}_a\rangle=2|t|^2$.

Higher-order moments can be calculated by the same method. Consider, for example, the action of the operators $\hat{a}_{\rm out}(\omega)$ and $\hat{a}_{\rm out}(\omega')$ on the state $|\psi\rangle$ defined in Eq. (3.10):

$$\begin{aligned}\hat{a}_{\rm out}(\omega')\hat{a}_{\rm out}(\omega)|\psi\rangle &= \{t(\omega)r(\omega')\hat{a}_{\rm in}(\omega)\hat{b}_{\rm in}(\omega') \\ &\quad + t(\omega')r(\omega)\hat{a}_{\rm in}(\omega')\hat{b}_{\rm in}(\omega)\}|\psi\rangle \\ &= \{t(\omega)r(\omega')\psi(\omega,\omega') \\ &\quad + t(\omega')r(\omega)\psi(\omega',\omega)\}|0\rangle, \end{aligned} \tag{B3}$$

where only those terms making a nonzero contribution have been retained in the expansion of $\hat{a}_{\rm out}(\omega')\hat{a}_{\rm out}(\omega)$. It follows that the second factorial moment of $\hat{N}_a$ is

$$\begin{aligned}\langle \hat{N}_a(\hat{N}_a-1)\rangle &= \int_0^\infty d\omega \int_0^\infty d\omega' \langle\psi|\hat{a}^\dagger_{\rm out}(\omega) \\ &\quad \times \hat{a}^\dagger_{\rm out}(\omega')\hat{a}_{\rm out}(\omega')\hat{a}_{\rm out}(\omega)|\psi\rangle \\ &= \int_0^\infty d\omega \int_0^\infty d\omega' |t(\omega)r(\omega')\psi(\omega,\omega') \\ &\quad + t(\omega')r(\omega)\psi(\omega',\omega)|^2. \end{aligned} \tag{B4}$$

If we again specialize to situations for which the transmission and reflection coefficients may be approximated by constant values then this second moment reduces to $2|tr|^2(1+I)$, with I given by Eq. (3.13). The complete set of nonzero factorial moments used in Sec. III B is

$$\begin{aligned}&\langle\hat{N}_a\rangle = 2|t|^2, \quad \langle\hat{N}_b\rangle = 2|r|^2, \\ &\langle\hat{N}_a\hat{N}_b\rangle = 2|t|^2|r|^2, \\ &\langle\hat{N}_a(\hat{N}_a-1)\rangle = 2|t|^4, \quad \langle\hat{N}_b(\hat{N}_b-1)\rangle = 2|r|^4\end{aligned} \tag{B5}$$

for $|\phi\rangle$ and

$$\begin{aligned}&\langle\hat{N}_a\rangle = |t|^2+|r|^2 = \langle\hat{N}_b\rangle, \\ &\langle\hat{N}_a\hat{N}_b\rangle = |t|^4+|r|^4+[t^2r^{*2}+t^{*2}r^2]I, \\ &\langle\hat{N}_a(\hat{N}_a-1)\rangle = 2|tr|^2[1+I] = \langle\hat{N}_b(\hat{N}_b-1)\rangle\end{aligned} \tag{B6}$$

for $|\psi\rangle$.

[1] R. Hanbury Brown, and R. Q. Twiss, Nature (London) **177**, 27 (1956).

[2] H. J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett. **39**, 691 (1977).

[3] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1994).

[4] C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987).

[5] P. Grangier, G. Roger, and A. Aspect, Europhys. Lett. **1**, 173 (1986).

[6] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 1995).

[7] H. Khosravi and R. Loudon, Proc. R. Soc. London, Ser. A **433**, 337 (1991); **436**, 373 (1992).

[8] L. Allen and S. Stenholm, Opt. Commun. **93**, 253 (1992).

[9] R. Matloob, R. Loudon, S. M. Barnett, and J. Jeffers, Phys. Rev. A **52**, 4823 (1995); R. Matloob and R. Loudon, Phys. Rev. A **53**, 4567 (1996).

[10] B. J. Dalton, E. S. Guerra, and P. L. Knight, Phys. Rev. A **54**, 2292 (1996).

[11] K. J. Blow, R. Loudon, S. J. D. Phoenix, and T. J. Shepherd, Phys. Rev. A **42**, 4102 (1990).

[12] S. M. Barnett and P. M. Radmore, *Methods in Theoretical Quantum Optics* (Oxford University Press, Oxford, 1997).

[13] The coefficients for nonreciprocal and asymmetric beam splitters are constrained only by the requirements of unitarity, and it is possible for the phases of the transmission and reflection coefficients for the two incident beams to be different. See A. Zeilinger, Am. J. Phys. **49**, 882 (1981).

[14] S. Prasad, M. O. Scully, and W. Martienssen, Opt. Commun. **62**, 139 (1987).

[15] Z. Y. Ou, C. K. Hong, and L. Mandel, Opt. Commun. **63**, 118 (1987).

[16] H. Fearn and R. Loudon, Opt. Commun. **64**, 485 (1987); R. A. Campos, B. E. A. Saleh, and M. C. Teich, Phys. Rev. A **40**, 1371 (1989).

[17] F. Wooten, *Optical Properties of Solids* (Academic Press, New York, 1972).

[18] B. Huttner and S. M. Barnett, Phys. Rev. A **46**, 4306 (1992).

[19] W. K. Lai, V. Buzek, and P. L. Knight, Phys. Rev. A **43**, 6323 (1991); J. Janszky, C. Sibilia, and M. Bertolotti, J. Mod. Opt. **38**, 2467 (1991).

[20] S. M. Barnett, C. R. Gilson, B. Huttner, and N. Imoto, Phys. Rev. Lett. **77**, 1739 (1996).

[21] A. Bandilla, J. Mod. Opt. **36**, 435 (1989).

[22] J. Jeffers and S. M. Barnett, Phys. Rev. A **47**, 3291 (1993).

[23] J. Jeffers and S. M. Barnett, J. Mod. Opt. **41**, 1121 (1994).

[24] H. Fearn and R. Loudon, J. Opt. Soc. Am. B **6**, 917 (1989).

[25] R. Loudon, *The Quantum Theory of Light* (Oxford University Press, Oxford, 1983).

[26] D. Falkoff, in *Quantum Optics*, Proceedings of the International School of Physics ''Enrico Fermi,'' Course 42, edited by R. J. Glauber (Academic Press, New York, 1969), p. 147; U. M. Titulaer, *ibid.* p. 153.

[27] An exception is provided by the squeezed states, which for special choices of squeezing parameters and of reflection and transmission coefficients can lead to unentangled outputs. See G. Yeoman and S. M. Barnett, J. Mod. Opt. **40**, 1497 (1993).

[28] J. Brendel, S. Schütrumpf, R. Lange, W. Martienssen, and M. O. Scully, Europhys. Lett. **5**, 233 (1988).

[29] R. Lange, J. Brendel, E. Mohler, and W. Martienssen, Europhys. Lett. **5**, 619 (1988).

[30] J. G. Rarity and P. R. Tapster, in *Photon Localization, Detection, Amplification and Antibunching*, edited by E. R. Pike and H. Walther (Hilger, Bristol, 1988), p. 122.

[31] R. Loudon, in *Coherence and Quantum Optics VI*, edited by J. H. Eberly, L. Mandel, and E. Wolf (Plenum, New York, 1990), p. 703.

[32] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965); L. H. Ryder, *Quantum Field Theory* (Cambridge University Press, Cambridge, 1985); R. J. Rivers, *Path Integral Methods in Quantum Field Theory* (Cambridge University Press, Cambridge, 1987).

[33] S. M. Barnett, Phys. Rev. A **43**, 6418 (1991).

[34] A. Gatti, H. Wiedemann, L. A. Lugiato, I. Marzoli, G.-L. Oppo, and S. M. Barnett, Phys. Rev. A **56**, 877 (1997).

[35] M. B. Spencer and W. E. Lamb, Phys. Rev. A **5**, 884 (1972); R. Lang, M. O. Scully, and W. E. Lamb, Phys. Rev. A **7**, 1788 (1973); B. Baseia and H. M. Nussenzweig, Opt. Acta **31**, 39 (1984); J. C. Penaforte and B. Baseia, Phys. Rev. A **30**, 1401 (1984); Phys. Lett. **107A**, 250 (1985); M. Ley and R. Loudon, J. Mod. Opt. **34**, 227 (1987).

[36] P. M. Vissier and G. Nienhuis, Opt. Commun. **136**, 470 (1997); T. J. Shepherd, P. J. Roberts, and R. Loudon, Phys. Rev. E **55**, 6024 (1997).