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Nonlinear optical rectification in parabolic quantum dots in the presence of electric and magnetic fields

Bin Li *, Kang-Xian Guo, Zuo-Lian Liu, Yun-Bao Zheng

Department of Physics, College of Physics and Electronic Engineering, Guangzhou University, Guangzhou 510006, PR China

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Abstract

The optical rectification (OR) coefficient in a parabolic quantum dots (QDs) subject to applied electric and magnetic fields is theoretically investigated in the framework of the compact-density-matrix approach and an iterative method. The confined wave functions and energies of electrons in the QDs are calculated in the effective-mass approximation. Numerical results are presented for typical GaAs/AlGaAs parabolic QDs. These results show that the OR coefficient strongly depends on the radius of QDs and the magnitude of electric and magnetic fields. And the peak shifts to the aspect of high energy when considering the influence of electric and magnetic fields.

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Keywords: Parabolic quantum dot; Nonlinear optical rectification; Electric and magnetic fields

1. Introduction

Due to the remarkable advances in nanofabrication techniques in recent years, the low-dimensional nanostructures such as quantum wells, quantum-well wires and quantum dots (QDs) can be viewed as natural candidates for optoelectronics devices, which have been extensively studied in different situations such as with the present magnetic and electric fields [1–18]. Significant efforts have recently been put into understanding the electronic and optical properties of QDs in which the motion of the carriers is confined in all spatial directions. In particular, the application of a magnetic field is equivalent to introducing an additional confining potential, which modifies the transport and optical properties of electron in QD and an introduced electric field gives rise to a polarization of carrier distribution and to an energy shift of the quantum states, which may be used to control and modulate the intensity of optoelectronic devices. Consequently, it is worthwhile to study the influence of the electric and magnetic fields on carriers in the QD.

Much attention has been paid to the nonlinear optical properties of low-dimensional semiconductor structures in the past few decades [1–18]. Many works take the effects of an electric field or a magnetic field into account in studying the nonlinearities of the quantum wells, quantum wires, or QDs [10–18]. The purpose of this Letter is to study the confinement and the electric and magnetic fields effects on the OR coefficient in QDs with parabolic confining potential. In Section 2, the eigenfunctions and eigenenergies of electron states are obtained using the effective-mass approximation, and the analytical expression for OR coefficient is derived by means of the compact-density-matrix approach and an iterative method. In Section 3, the numerical results and discussions are presented for GaAs/AlGaAs parabolic QDs. In the present work, we found that the effects of electric and magnetic fields on OR in QDs depends strongly upon the size of spherical QD. The numerical results show that OR coefficient is greatly enhanced when the effects of electric and magnetic fields in QDs are considered. A brief summary is given in Section 4.

* Corresponding author.
E-mail addresses: libin-ping@163.com (B. Li), kxguo@gzhu.edu.cn (K.-X. Guo).

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2. Theory

2.1. Electronic structure

Electrons in a spherical QD confined by a radial potential of the form $\frac{1}{2}m^*\omega_0^2 r^2$ in the external electric and magnetic fields along z direction can be described in cylindrical coordinates by the effective-mass Hamiltonian [19,20]

$$\hat{H} = -\frac{\hbar^2}{2m^*}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{\partial^2}{\partial z^2} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right] + \frac{1}{2}\omega_c\hat{l}_z + \frac{m^*\Omega^2\rho^2}{8} - eFz + \frac{m^*\omega_0^2 z^2}{2}, \tag{1}$$

where $\hat{l}_z$ is the projection of the angular momentum onto the magnetic field direction, m^* is the effective mass, F is the electric field, $\omega_c = eB/m^*c$ is the cyclotron frequency, and $\Omega = \sqrt{\omega_c^2 + 4\omega_0^2}$.

The corresponding solutions can be written as

$$\Psi = f(\rho,\varphi)\chi(z), \tag{2}$$

$$f(\rho,\varphi) = \frac{1}{a^{1+|m|}}\sqrt{\frac{(|m|+n)!}{2\pi 2^{|m|}n!}}\frac{1}{|m|!}e^{im\varphi}e^{-\frac{\rho^2}{4a^2}} \times \rho^{|m|}F\left(-n, |m|+1, \frac{\rho^2}{2a^2}\right), \tag{3}$$

$$\chi(z) = \left(\frac{m^*\omega_0}{\pi\hbar}\right)^{\frac{1}{4}}\frac{1}{\sqrt{2^{n_z}n_z!}}\exp\left(-\frac{m^*\omega_0}{2\hbar}\left(z - \frac{eF}{m^*\omega_0^2}\right)^2\right) \times H_n\left[\sqrt{\frac{m^*\omega_0}{\hbar}}\left(z - \frac{eF}{m^*\omega_0^2}\right)\right], \tag{4}$$

where $a = \sqrt{\hbar/m^*\Omega}$ is the effective length scale, $F(a,b,x)$ is the confluent hypergeometric function, n is the radial quantum number, m is the magnetic quantum number, $H_n(x)$ is the Hermite polynomial and n_z is the quantum number, respectively.

The electronic eigenenergies E_{n,m,n_z} are given by

$$E = \hbar\Omega\left(n + \frac{1+|m|}{2}\right) + \frac{m\hbar\omega_c}{2} + \hbar\omega_0\left(n_z + \frac{1}{2}\right) - \frac{e^2F^2}{2m^*\omega_0^2}. \tag{5}$$

2.2. OR coefficients

Next, the formula of OR in spherical QD can be obtained by the compact-density-matrix method and the iterative procedure. Let us consider the system which is excited by an incidence light $E(t) = \tilde{E}e^{i\omega t} + \tilde{E}e^{-i\omega t}$. The evolution of density matrix is given by the time-dependent Schrödinger equation

$$\frac{\partial\varrho_{ij}}{\partial t} = \frac{1}{i\hbar}\left[H_0 - qzE(t), \varrho\right]_{ij} - \Gamma_{ij}\left(\varrho - \varrho^{(0)}\right)_{ij}. \tag{6}$$

Eq. (6) is calculated by the following iterative method [16]

$$\varrho(t) = \sum_n \varrho^{(n)}(t), \tag{7}$$

with

$$\frac{\partial\varrho_{ij}^{(n+1)}}{\partial t} = \frac{1}{i\hbar}\left\{\left[H_0, \varrho^{(n+1)}\right]_{ij} - i\hbar\Gamma_{ij}\varrho_{ij}^{(n+1)}\right\} - \frac{1}{i\hbar}\left[qz, \varrho^{(n)}\right]_{ij}E(t). \tag{8}$$

The electric polarization of the QD due to $E(t)$ can be expressed as

$$P(t) = \left(\varepsilon_0\chi^{(1)}\tilde{E}e^{-i\omega t} + \varepsilon_0\chi_0^{(2)}|\tilde{E}|^2 + \varepsilon_0\chi_{2\omega}^{(2)}\tilde{E}^2e^{-i2\omega t} + \varepsilon_0\chi_\omega^{(3)}|\tilde{E}|^2e^{-i\omega t} + \varepsilon_0\chi_{3\omega}^{(3)}\tilde{E}^3e^{-i3\omega t}\right) + \text{c.c.}, \tag{9}$$

where $\chi^{(1)}$, $\chi_0^{(2)}$, $\chi_{2\omega}^{(2)}$, $\chi_\omega^{(3)}$, $\chi_{3\omega}^{(3)}$ are the linear susceptibility, optical rectification, second-harmonic generation, third-order and third-harmonic generation susceptibilities, respectively. ε_0 is the vacuum dielectric constant. The electronic polarization of the nth order is given as

$$P^{(n)}(t) = \frac{1}{V}\,\text{Tr}\left(\varrho^{(n)}ez\right), \tag{10}$$

where V is the volume of interaction and Tr denotes the trace or summation over the diagonal elements of the matrix $\varrho^{(n)}ez$.

In our Letter, the OR coefficient per unit volume is given as

$$\chi_0^{(2)} = \frac{4e^3\sigma_v}{\varepsilon_0\hbar^2}M_{01}^2\delta_{01} \times \frac{\omega_{01}^2(1+\frac{T_1}{T_2}) + (\omega^2 + \frac{1}{T_2^2})(\frac{T_1}{T_2} - 1)}{[(\omega_{01}-\omega)^2 + \frac{1}{T_2^2}][(\omega_{01}+\omega)^2 + \frac{1}{T_2^2}]}, \tag{11}$$

where σ_v is the density of electrons in the spherical QD, $\delta_{01} = |M_{11} - M_{00}|$, $\omega_{ij} = (E_i - E_j)/\hbar$ is the transition frequency, and $M_{ij} = |\langle\Psi_j|z|\Psi_i\rangle|$ is the off-diagonal matrix element.

The OR coefficient has a resonant peak for $\omega = \omega_{01}$ obtained by

$$\chi_{0,\max}^{(2)} = 2\frac{T_1T_2e^3\sigma_v}{\varepsilon_0\hbar^2}M_{01}^2\delta_{01}. \tag{12}$$

3. Results and discussions

In this section, we will discuss the OR coefficient $\chi_0^{(2)}$ in GaAs/AlGaAs spherical QDs under the external electric and magnetic fields. In the calculation we use the following parameter values [18]: $m^* = 0.067m_0$ (m_0 is the mass of a free electron), $T_1 = 1$ ps, $T_2 = 0.2$ ps, $\sigma_v = 5\times10^{24}$ m^{-3}. In the case of QD with parabolic confinement, the energy of electron is $\hbar\omega_0 = \hbar^2/m^*R^2$ [21].

The product $M_{01}^2\delta_{01}$ as a function of the applied electric field F for $B = 0$ T, $B = 10$ T with the radius of the QD for different values is displayed in Fig. 1. From Fig. 1, we can observe that the product $M_{01}^2\delta_{01}$ vanishes in the absence of the electric field and subsequently increases, reaches a somewhat maximum and decreases gradually when the electric field F increases. It can be further noted that the larger the radius of

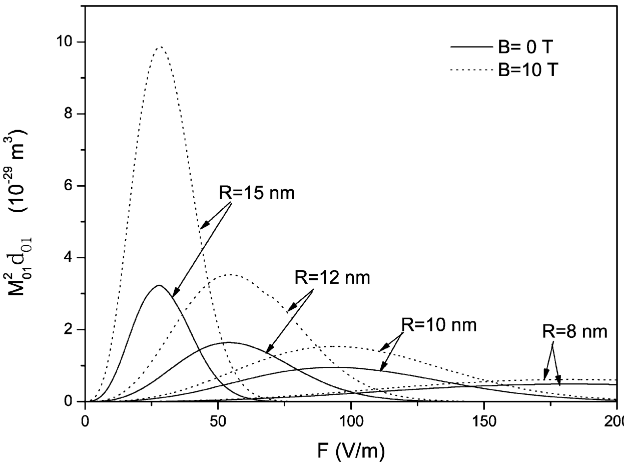


Fig. 1. The product $|\chi_0^{(2)}|$ as a function of the applied electric field F for $B = 0$ T, $B = 10$ T with the radii of the quantum dot for $R = 8, 10, 12, 15$ nm, respectively.

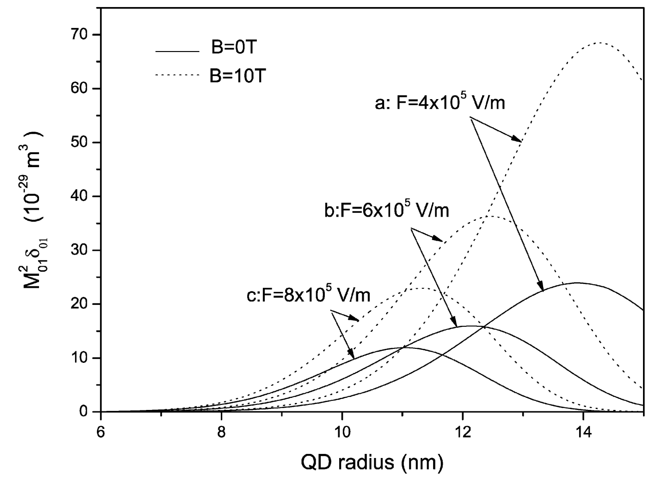
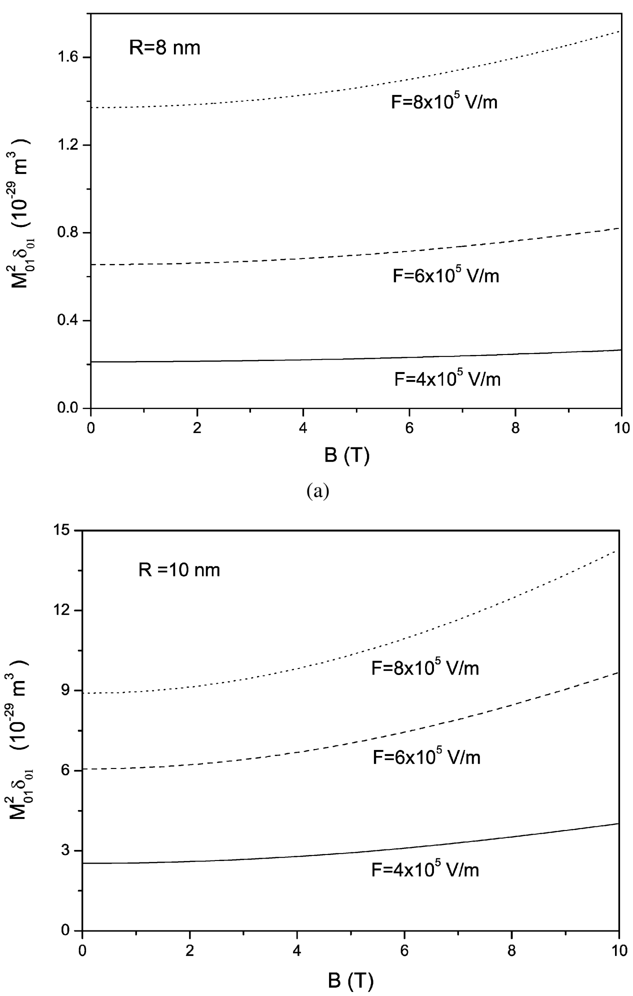


Fig. 3. Dependence of the product $|\chi_0^{(2)}|$ on the radius of the quantum dot for different values of the electric and magnetic fields: (a) $F = 4 \times 10^5$ V/m, (b) $F = 6 \times 10^5$ V/m, (c) $F = 8 \times 10^5$ V/m.



(a)

(b)

Fig. 2. The product $|\chi_0^{(2)}|$ as a function of the applied magnetic field B with the applied electric field $F = 4 \times 10^5$ V/m (solid line), 6×10^5 V/m (dash line), 8×10^5 V/m (dot line) for a fixed radius of quantum dot $R = 8$ nm (a) and $R = 10$ nm (b).

QD is, the more prominent will be. And in the final analysis, the product of matrix elements with $R = 8$ nm will decrease with the higher electric field F. This is due to the fact that the parabolic potential will be by degrees translated into the semi-parabolic potential in z direction with a increase in F [10]. Because the increasing F makes the wave function of electron spread to wider space and this enhances the overlap of the wave function. But the product of transition elements will decrease subsequently due to the decrease of the effective scope of the electronic wave function in the spherical QD. Fig. 1 also shows that the application of the magnetic field increases the sensitivity of the product of matrix elements to the applied field F.

We plot the product $M_{01}^2\delta_{01}$ versus the applied magnetic field B with different values of applied electric field for the two cases: $R = 8$ nm in Fig. 2(a) and $R = 10$ nm in Fig. 2(b). From Fig. 2(a) or Fig. 2(b), for a given radius of QD, the product of matrix elements increases with increasing magnitude of magnetic field B for different values of F. In Figs. 2(a) and 2(b), we can see that the bigger the applied electric field is, the higher value of the product $M_{01}^2\delta_{01}$ is. When comparing Figs. 2(a) and 2(b), we can see that for a fixed electric field F the bigger the QD radius is, the higher magnitude of $M_{01}^2\delta_{01}$ will be.

Fig. 3 illustrates the dependence of the product $M_{01}^2\delta_{01}$ on the radius R for different values of electric and magnetic fields. Fig. 3 shows that the product of matrix elements initially increases, reaches a somewhat maximum and subsequently decreases monotonously when the QD radius R increases. We can see that the smaller magnitude of the applied electric field is, the sharper the peak will be and the bigger the peak intensity will be. And the results clearly show that the application of the magnetic field increases the sensitivity of the product of matrix elements to the radius R.

Finally, in Figs. 4(a) and 4(b), we present the OR coefficient $|\chi_0^{(2)}|$ as a function of the incident photon energy $\hbar\omega$ with two different values of radius, $R = 8$ nm, $R = 10$ nm for different values of electric and magnetic fields, respectively. From Fig. 4(a), it can be easily seen that the electric and mag-

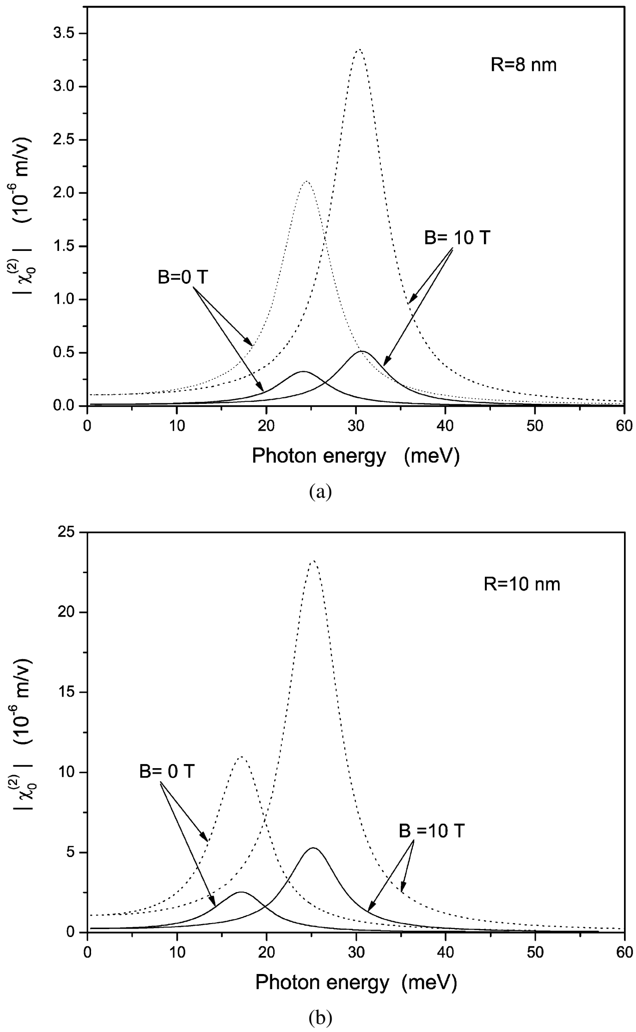


Fig. 4. Dependence of the OR coefficient $|\chi_0^{(2)}|$ on the incident photon energy $\hbar\omega$ for $R = 8$ nm (a) and $R = 10$ nm (b) with $F = 2 \times 10^5$ V/m (solid line), $F = 4 \times 10^5$ V/m (dot line) and different values of magnetic field.

netic fields have a great influence on the OR coefficient $|\chi_0^{(2)}|$, and two resonant peaks occur at $\hbar\omega = 0.024, 0.031$ eV, respectively. We also can see that the resonant peak will move to the right side of the curve with B increase and the bigger value of magnetic field is, the sharper peak will be. From Fig. 4(b), it also can be seen that two resonant peaks occur at $\hbar\omega = 0.017, 0.026$ eV, respectively. From Figs. 4(a) and 4(b), with the fixed values of electric and magnetic fields, we can see that the narrower size of QD is, the smaller value of OR coefficient will be. This is due to the fact that the quantum confinement effect results in the separation of energy levels, and the stronger the confinement effect is, the broader the separation will be. Note that given a fixed B, the resonant peak will move to the higher energy when R decreases, which predicts a strong confinement-induced blueshift in QD.

4. Summary

We present an study of the OR coefficient $|\chi_0^{(2)}|$ for a spherical quantum dot in GaAs/AlGaAs. The calculations mainly focus on the dependence of the OR coefficient on the electric and magnetic fields, the QD radius R, and the incident photon energy. The results display that the theoretical value of the OR coefficient $|\chi_0^{(2)}|$ is enhanced significantly over 10^{-6} m/V due to the applied electric and magnetic fields.

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