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Chirality in Swiss Roll metamaterials

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ABSTRACT

A numerical investigation is presented for the electromagnetic behaviour of Swiss Roll metamaterials that show considerable agreement with analytical theory. Their chiral design exhibits enormous chirality, at least 100 times larger than other chiral structures previously reported in the literature. This property ensures the change of a wave polarization by 90° in less than a wavelength. Hence, chiral Swiss Rolls support a negative refractive band for one-wave polarization, which is predicted analytically and calculated numerically.

1. Introduction

The electromagnetic properties of conventional materials arise from their molecular composition, since macroscopic electromagnetic fields are the averages of their microscopic constituents. Recently, this idea was extended to a new class of artificial media, 'metamaterials'. Metamaterials' sub-units are of a much larger scale than molecules. However, they can be considered as homogeneous media for frequencies where the wave is too myopic to resolve their sub-unit's geometry. Therefore, their electromagnetic response can be described by the effective electric permittivity (ε_{eff}) and magnetic permeability (μ_{eff}). Sub-units of metamaterials can be specifically designed in order to manufacture artificial media with electromagnetic properties usually not seen in nature, like negative refraction and perfect lensing.

Veselago in 1968 [1] realized that a medium exhibits negative refracting index if both the real parts of the electric permittivity (ε) and magnetic permeability μ are simultaneously negative:

$$n = \begin{cases} +\sqrt{\varepsilon(\omega)\mu(\omega)} & \text{for } Re(\varepsilon) \geq 0 \text{ and } Re(\mu) \geq 0 \\ -\sqrt{\varepsilon(\omega)\mu(\omega)} & \text{for } Re(\varepsilon) < 0 \text{ and } Re(\mu) < 0 \end{cases} \tag{1}$$

a property not found in nature, due to restrictions of molecular composition. However, metamaterials can be specifically designed that have both $Re(\varepsilon_{eff})$ and $Re(\mu_{eff})$ negative for a frequency range.

The magnetic response of conventional materials tails off even for few GHz. However, artificial media can be constructed from entirely conducting elements that are macroscopically magnetic, such as split-ring and Swiss Roll resonators [2]. In this paper, non-chiral Swiss Rolls are initially investigated numerically, where significant agreement was found with analytical work. An alternative way to negative refraction is through the addition of chiral elements in a resonant medium, where a negative band is obtained for one-wave polarization [3]. Therefore, the chiral version of Swiss Rolls is investigated and its band structure, electromagnetic and chirality parameters are discussed.

2. Swiss Roll metamaterials

A Swiss Roll (non-chiral) metamaterial is an artificial magnetic medium, constructed by a wrapped thin insulated conducting sheet around a dielectric mandrel, as shown in Fig. 1(a) [2]. When a wave is incident on a Swiss Roll with the magnetic field along the roll, surface currents are induced on the conducting elements, which give rise to an electromotive force opposing the applied magnetic field [2]. Therefore, it macroscopically seems that 'magnetic' monopoles are flowing up and down the cylinder.

The effective magnetic permeability along the roll is given by [2]

$$\mu_z^{eff} = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\gamma\omega} \tag{2}$$

where $F = \pi R^2/a^2$ is the filling factor, $\gamma = 2\rho/[\mu_0 R(N-1)]$ accounts for resistivity losses of the conducting material and the resonant frequency is given by

$$\omega_0 = c_0\sqrt{\frac{d}{2\pi^2 R^3 \varepsilon_d (N-1)}} = \sqrt{1-F}\omega_{mp} \tag{3}$$

where a is the lattice constant, N number of spiral turns, ε_d the dielectric constant of the material inside the gap, d and R as defined in Fig. 1(a) [2]. Eq. (2) takes negative values for frequencies $\omega_o < \omega < \omega_{mp}$ and is plotted in Fig. 2(b) (black solid line).

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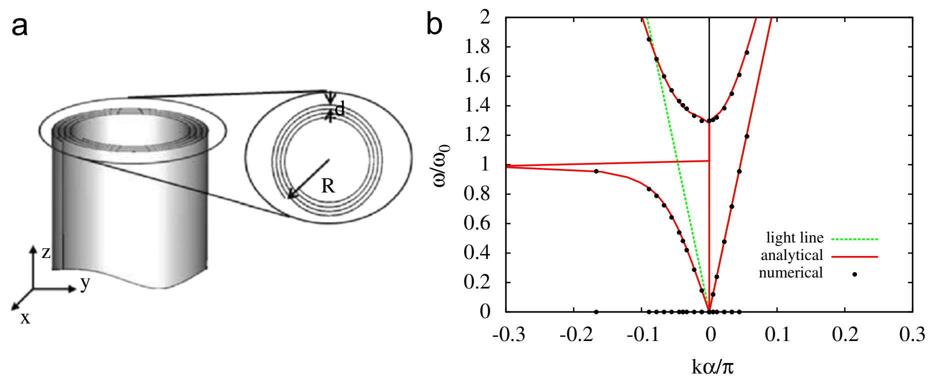

Fig. 1. (a) Swiss Roll resonator, where R is the outer radius and d is the gap between the conducting sheets. (b) The band structure of Swiss Rolls in a square lattice with $N=2$, $d=0.1$ mm, $x=0.05$ mm, $R=2$ mm and $a=5$ mm: analytic prediction (red line) and numerical results (dots). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

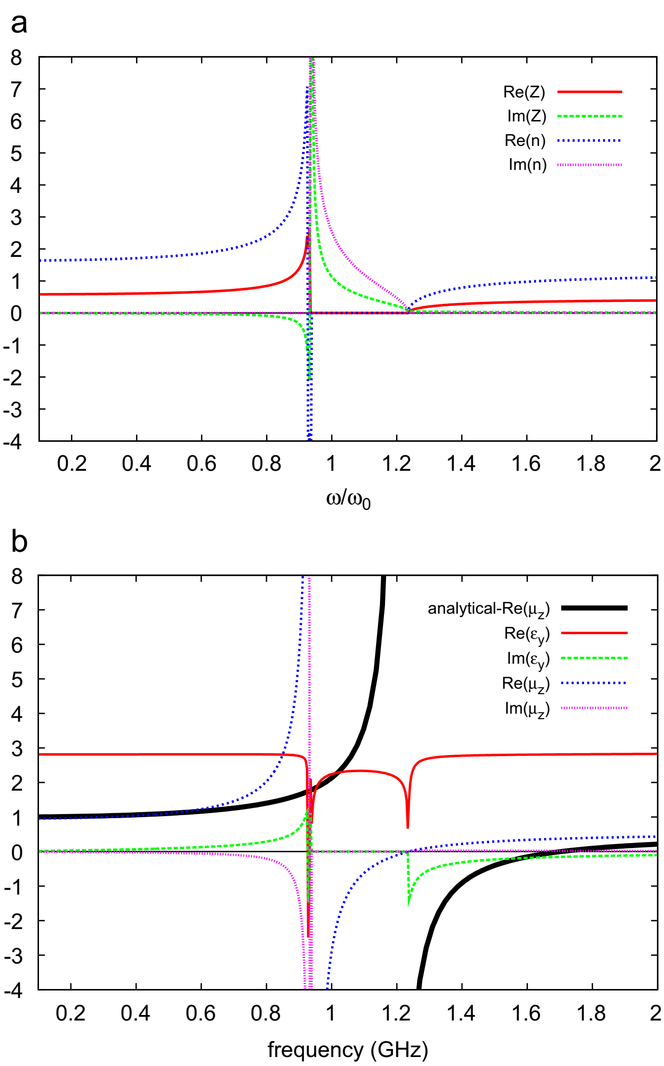

Fig. 2. Retrieved electromagnetic parameters from numerical S-parameter-results: real and imaginary parts of (a) refractive index (n) and impedance (Z) (b) effective permittivity across the roll (ε_y) and effective permeability along the roll (μ_z) for Swiss Rolls with $N=2$, $d=0.1$ mm, $x=0.05$ mm, $R=2$ mm and $a=5$ mm. The bold black lines are a plot of the analytical prediction of Eq. (2).

The effective electric permittivity along the roll obeys Drude model, since E_z sees just a thick conducting rod and expressed by $\varepsilon_z^{eff}=1-\omega_p^2/(\omega^2+i\omega\Gamma)$, where Γ represents conductivity losses and ω_p is the plasma frequency. However, $\omega_p \gg \omega_0$ since the wave sees a thick conducting rod, which means that when the structure is magnetically active, is also electrically inactive for frequencies where $\omega < \omega_p$. Therefore, no degeneracy is expected for the band structure. The electromagnetic parameters across the roll are approximately constant and are given by $\varepsilon_x^{eff}=\varepsilon_y^{eff}\approx 2/\sqrt{1-F}$ and $\mu_x^{eff}=\mu_y^{eff}\approx\sqrt{1-F}/2$ [4].

The dispersion equation for a metamaterial consisted of Swiss Rolls in a square lattice is given by [5]

$$\omega = c_0\sqrt{\frac{k_x^2}{\varepsilon_y\mu_z}+\frac{k_y^2}{\varepsilon_x\mu_z}+\frac{k_z^2}{\varepsilon_y\mu_x}} \tag{4}$$

and plotted for k_x and k_z propagation in Fig. 1(b) (red solid line) together with numerical results (dots) calculated using CST Microwave Studio[1] for Swiss Rolls with $N=2$, $d=0.1$ mm, $x=0.05$ mm- thickness of conducting sheet, $R=2$ mm and $a=5$ mm. The agreement between analytical and numerical results is $\sim 95\%$. Also, S-parameters for a three-unit-cell slab were calculated numerically, from which the electromagnetic parameters are retrieved, using the method described in Ref. [10] and are plotted in Fig. 2. In Fig. 2(b), Eq. (2) is plotted together with the retrieved μ_{eff}, where both have similar behaviour, with small disagreement ($\sim 10\%$) on the value of ω_0. Finally, at higher frequencies flat equally spaced modes appear, identified as waveguide modes trapped inside the spiral gap [4,6].

3. Chiral Swiss Roll metamaterials

In the view of the fact that Swiss Rolls are well described by analytical work, the more complicated design of chiral Swiss Rolls is now investigated. Chiral Swiss Rolls can be constructed by winding a conducting-insulated sheet around a cylindrical mandrel, creating an overlapping helix, as shown in Fig. 3(a). Generally, the electromagnetic fields in a chiral medium are written as [3]

$$\begin{bmatrix}\mathbf{D}\\ \mathbf{B}\end{bmatrix}=\begin{bmatrix}\chi_{EE} & \chi_{EH}\\ \chi_{HE} & \chi_{HH}\end{bmatrix}\begin{bmatrix}\mathbf{E}\\ \mathbf{H}\end{bmatrix} \tag{5}$$

where $\mathbf{D}$ is the electric displacement vector, $\mathbf{E}$ the electric field intensity, $\mathbf{B}$ the magnetic induction field and $\mathbf{H}$ the magnetic field intensity, χ_{EE} and χ_{HH} are the electric and magnetic parameters of the medium, respectively, and χ_{EH} and χ_{HE} are the chirality parameters.

[1] CST GmbH, Darmstadt, Germany.

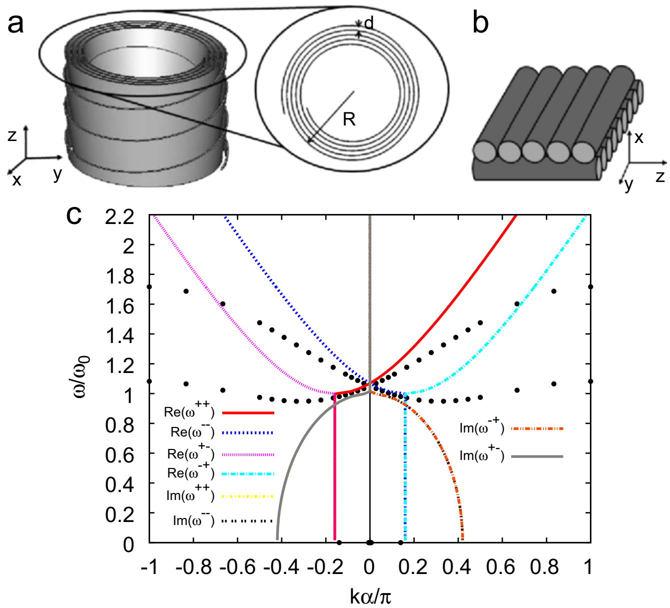

Fig. 3. (a) Chiral Swiss Roll, where R and d are the radius and the gap between the conducting sheet, respectively, (b) a 2D chiral Swiss Roll, (c) real and imaginary parts of the analytic prediction for the band structure of a 2D chiral Swiss Roll metamaterial (lines) plotted with numerical results (dots) for Swiss Rolls with $N=2$, $R=1$ mm, $x=0.05$ mm, $d=0.35$ mm, $\theta=21.7^{\circ}$ and $a=5$ mm and aligned with the y- and z-axes.

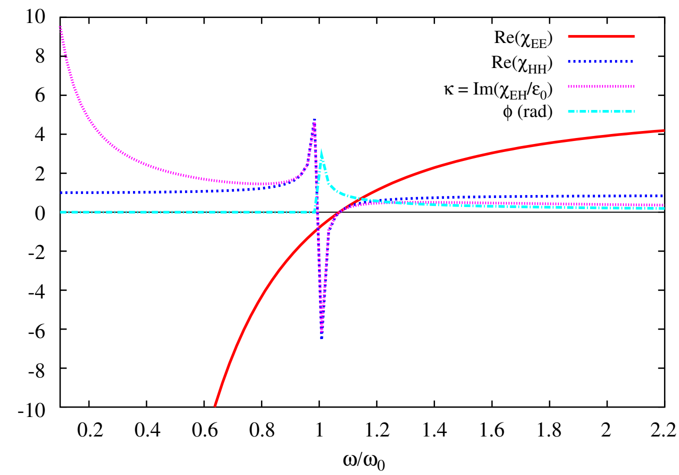

Fig. 4. The real parts of (non-inverse) χ_{EE}, χ_{HH}, the imaginary part of chirality (χ_{EH}) term and the optical rotation (ϕ) in radians for chiral Swiss Rolls with $N=2$, $R=1$ mm, $d=0.35$ mm, $\theta=21.7^{\circ}$ and $a=5$ mm, assuming Swiss Rolls are made of perfect conducting material impended in vacuum.

Chirality in a medium results to the splitting of degenerate modes [3]. Now, when chiral inclusions are added in a resonant (magnetic or electric) medium, a negative band arises for one-wave polarization, due to the mode splitting [3]. Chiral Swiss Rolls are magnetically resonant and exhibit strong chirality due to their helical shape. Therefore, they are expected to have a negative band for one-wave polarization.

The inverse electromagnetic parameters and chirality terms (i.e. inverse of the matrix in Eq. (5)) along a chiral Swiss Roll were derived in Ref. [7] and are given by

$$[\chi^{-1}]_{HH}=\frac{1}{(1-F)}\left(\frac{\omega^2-\omega_0^2+i\Gamma\omega}{\omega^2-\omega_{mp}^2+i\Gamma\omega/(1-F)}\right) \quad (6)$$

$$[\chi^{-1}]_{EE}=G\left(\frac{\omega^2+i\Gamma\omega/(1-F)}{\omega^2-\omega_p^2+i\Gamma\omega/(1-F)}\right) \quad (7)$$

$$\frac{[\chi^{-1}]_{EH}}{\mu_0}=\frac{iR}{2L\tan\theta}\left(\frac{\omega\omega_{mp}^2}{\omega^2-\omega_{mp}^2+i\Gamma\omega/(1-F)}\right)=-\frac{[\chi^{-1}]_{HE}}{\varepsilon_0}=i\kappa^{-1} \quad (8)$$

where θ is the angle between the edge of the conducting foil and the normal to the rotation axes, a the lattice constant and R, d are defined in Fig. 3(a). G is given by

$$G=\frac{a^2 d}{L(8\pi^3R^3(N-1)L\tan^2\theta+a^2d)} \quad (9)$$

the resonant frequencies ω_0, ω_{mp} and ω_p are given by

$$\omega_0=c_0\sqrt{4\pi GL\tan^2\theta/(\varepsilon_d a^2)}=\sqrt{1-F}\omega_{mp}=\sqrt{1-F}\omega_p \quad (10)$$

and $\Gamma=-2\rho/(\mu_0 R)$, $L=1-i\varepsilon_d\varepsilon_0 a^2\rho\omega/(2\pi R)$ account for resistivity losses in metals (where ρ is the resistivity of the conducting sheet per unit area and ε_d is the dielectric constant of the material in the gap given by $\varepsilon_d=\varepsilon'+i\varepsilon''$ and $\varepsilon''=\sigma/(\omega\varepsilon_0)$ and σ is the conductivity accounting for losses in the dielectric material). Note that resistivity losses are negligible, since for MHz and GHz frequencies that Swiss Rolls operate, metals have extremely small resistivities [7]. For Swiss Roll metamaterials (both chiral and not), the most important contributor in losses is the dielectric material in the gap.

The electromagnetic and chirality (non-inverse) parameters are plotted in Fig. 4, where losses are ignored. The magnetic behaviour of the structure arises from the same mechanism as for non-chiral Swiss Roll, hence the similar behaviour of Eq. (2) and χ_{HH}. Also, an electric field along the roll sees a thick wire, and therefore χ_{EE} is expected to follow a Drude model. However, along the roll the conducting sheet is interrupted, forcing the electric plasma frequency (ω_p) to take lower values. Hence, in contrary to non-chiral Swiss Rolls, this structure is both magnetically and electrically active. Therefore, two degenerate modes are expected, referring to the two-wave polarizations that propagate in the medium. However, the chirality of the structure splits the two modes spoiling the degeneracy of the two modes.

Furthermore, κ has a resonant behaviour at ω_0 frequency and takes infinity values for $\omega\to 0$, which is justified by noting that the structure discussed here is an infinite roll. The optical rotation of the structure ϕ is given by $\phi=\kappa/\sqrt{\chi_{EE}\chi_{HH}}$ and also plotted in Fig. 4shows rotation of wave polarization of higher than 90° in the region $\omega_0<\omega<\omega_{mp}$. Therefore, it is concluded that chiral Swiss Roll metamaterials exhibit huge chirality, which is at least 100 times higher than other chiral structures discussed in the literature.

Finally, if chiral Swiss Rolls are placed along the y- and z- axes (Fig. 3(b)) and k_x propagation is considered, a 2D isotropic-chiral-metamaterial is realized. Now Eqs. (6)–(8) are valid for electric and magnetic fields along both y- and z-axes. Using Maxwell's equations and helical polarization for electric and magnetic fields, the dispersion equations for an isotropic chiral medium are derived [3]:

$$\omega_{+\pm}=k_{+\pm}(\kappa^{-1}\pm\sqrt{[\chi^{-1}]_{EE}[\chi^{-1}]_{HH}}) \quad (11)$$

$$\omega_{-\pm}=k_{-\pm}(-\kappa^{-1}\pm\sqrt{[\chi^{-1}]_{EE}[\chi^{-1}]_{HH}}) \quad (12)$$

where the first and second subscripts refer to wave-polarization and sign of group velocity, respectively. Eqs. (11) and (12) are plotted in Fig. 3(c) (lines) with numerical results (dots) calculated using CST Microwave Studio for a 2D chiral-Swiss-Roll metamaterial with dimensions $N=2$, $R=1$ mm, $x=0.05$ mm (width of conducting sheet), $d=0.35$ mm, $\theta=21.7^{\circ}$ and $a=5$ mm. For frequencies $\omega<\omega_0$ there is a stop-band as expected, since χ_{HH} is positive and χ_{EE} is negative, Furthermore, there is a negative band for frequencies $\omega_0<\omega<\omega_{mp}$. Note that despite the differences between analytical and numerical band structures, there is a negative band for one-wave polarization for both numerical and analytical results. The analytical and numerical values for ω_0 show an agreement of approximately $\backsim 80\%$. Considering that several assumptions taken analytically are not valid for the simulated structure (i.e. $x\rightarrow 0$ and $d\ll R$), the agreement is acceptable.

For most chiral metamaterials discussed in the literature, the negative band is observed for a less broad frequency range and is usually lost in the stop-bands created by the sharp resonances [8]. It is worth noting that chiral Swiss Rolls have a negative band even for GHz frequencies due to their extreme chirality. The chirality of Swiss Rolls is at least two orders of magnitude higher than other chiral structures previously reported in the literature, such as helical wires [8]. This property establishes chiral Swiss Rolls as ideal candidate structures for various applications, such as magnetic resonance imaging (MRI) and polarization rotation/selection antennas [9].

4. Conclusions

Swiss Rolls are macroscopically magnetic metamaterials. The numerical investigation performed for non-chiral Swiss-Rolls showed significant agreement with analytical predictions. By modifying their design slightly, chiral Swiss Rolls are realized that, except from magnetically resonant structures, are also extremely chiral. Therefore, it was shown that 2D chiral Swiss Roll metamaterials show a negative band for one-wave polarization.

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