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Slow light in a dielectric slab waveguide with a negative refractive index photonic crystal substrate

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ABSTRACT

Slow light effect in a dielectric slab waveguide with a negative refractive index photonic crystal substrate is numerically investigated. The guided modes are confined in the core layer by total internal reflection. Dispersion and wave propagating properties are explored in details. The result demonstrates that light speed can approach zero at two adjacent frequencies, which is also verified by finite-difference time-domain (FDTD) simulation. Calculated *Q*-factors both exceed several thousand. The structure has the potential to be used as high-*Q* open cavity for dual-wavelength lasers or traveling wave amplifiers.

1. Introduction

The research of slow light has captured significant interests from many researchers of physics and optical engineering in recent years. There have been several ways to achieve the purpose of slowing down the speed of light, e.g., electromagnetically induced transparency (EIT) [1], coherent population oscillations (PO) [2,3], stimulated Brillouin scattering [4], coupled resonator optical waveguides [5,6], and photonic crystals [7–11]. Moreover, theoretical research shows that a layered structure with left-handed materials (LHM) can both support negative giant Goos–Hänchen shift [12,13] and slow light propagation [14,15]. However, realization of LHM in visible frequency range with reasonable loss is still a big challenge. Previously, we achieved a negative giant Goos–Hänchen shift by replacing the LHM in the layered structure with a photonic crystal of negative effective refractive index [16]. The structure is made of a dielectric slab layer and a negative refractive index photonic crystal substrate. The negative giant Goos–Hänchen shift of the structure is mainly determined by the energy flux of a backward wave along the slab layer. Actually, by choosing the incident angle and slab width, both forward and backward waves along the slab layer can be excited and controlled simultaneously, which results in the change of the total energy flux along the slab layer. If this kind of structure is used for waveguiding purpose, it may achieve slow light effect when the forward and backward waves are balanced, but with much lower loss compared with its counterpart with LHM.

In this paper, a slow light waveguide made of a dielectric slab with a negative effective refractive index photonic crystal substrate is proposed and numerically investigated. Two adjacent frequencies with zero group velocity are obtained, and their resonant properties are demonstrated with FDTD simulation.

2. Structure, dispersion and slow light analysis

The structure of a dielectric slab waveguide with a negative refractive index photonic crystal substrate is shown in Fig. 1a. The whole structure is constituted of two parts. The upper part is a homogeneous GaAs slab layer with width *w*. The lower part is a two dimensional (2D) photonic crystal which is the same as the one investigated by Notomi [17]. It is a hexagonal lattice of air holes in a GaAs background. The radius of the holes is $r = 0.4a$, where a is the lattice constant. The top interface of the photonic crystal is truncated in a direction perpendicular to the $\Gamma-\mathrm{M}$ direction. The whole structure is surrounded by air. The refractive indices of air and GaAs are $n_{\mathrm{air}} = 1$ and $n_{\mathrm{GaAs}} = 3.6$, respectively.

In our calculation, only transverse electric (TE, in a direction parallel to the axes of air holes) polarization is considered. The effective refractive index of the photonic crystal substrate (n_s) is negative and nearly isotropic in a frequency range from $0.29 \times 2\pi c/a$ to $0.345 \times 2\pi c/a$, which varies from -1.2 to -0.2 (see Fig. 1b). Our analysis here will be confined in this frequency range. The slab layer forms the core layer of the waveguide, of which the refractive index is $n_{\mathrm{core}} = n_{\mathrm{GaAs}}$. Considering $n_{\mathrm{core}} > n_{\mathrm{air}}$ and $n_{\mathrm{core}} > |n_s|$, if self-consistent condition in z direction is satisfied, guided modes propagating along x direction and localized mainly in this layer can be found. The confinement is achieved by total internal reflections at the core-substrate and core-air interfaces, which is similar to an asymmetric LHM waveguide [14], but different from a conventional photonic crystal waveguide.

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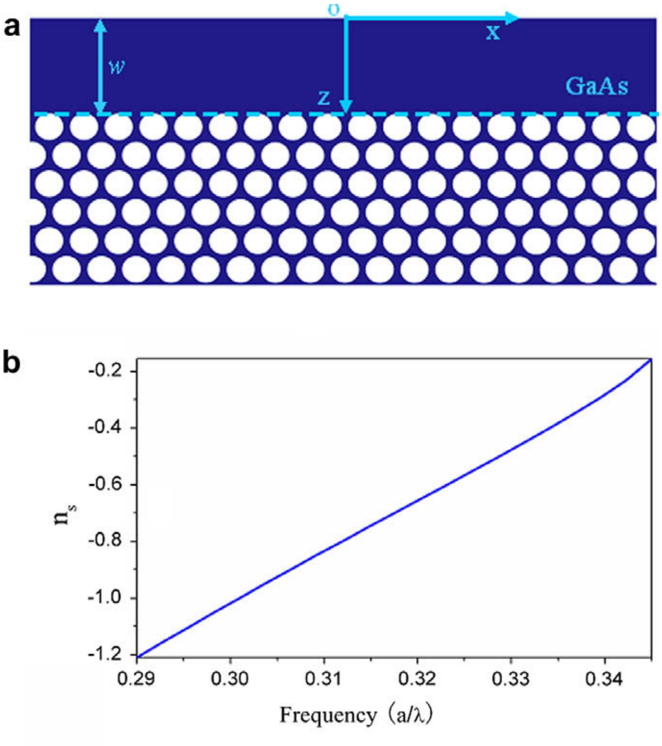

Fig. 1. (a) Schematic diagram of the waveguide structure with slab width *w*. A photonic crystal with hexagonal pattern of air holes is used as the substrate. The radius of the air holes is 0.4*a*, where *a* is the lattice constant. (b) Effective refractive index of the photonic crystal substrate in the considered frequency range.

A two dimensional layer Korringa–Kohn–Rostoker (2D layer-KKR) method [18] with interface boundary conditions described in [19] is used to find the guided modes of the structure with different slab widths. The mode dispersion curves for a structure with width *w* = 2.9*a* are given out in Fig. 2, they show characteristics of a mini-stop band, which should be a result of refractive periodic distribution of photonic crystal along the light propagating direction [20]. Furthermore, the four curves all have some regions with a relative flat slope, which means a relative slow speed of light propagation, it makes the structure possible to be used as slow light waveguide.

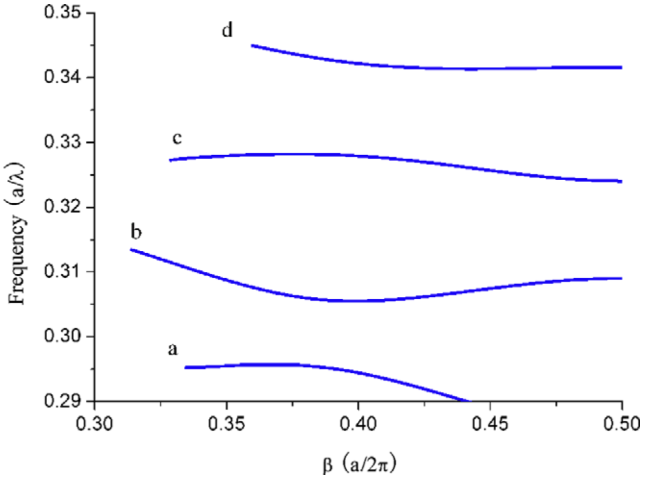

Fig. 2. Dispersion diagram of guiding modes for a slab width of 2.9*a*, both of the frequencies and propagation constants are normalized.

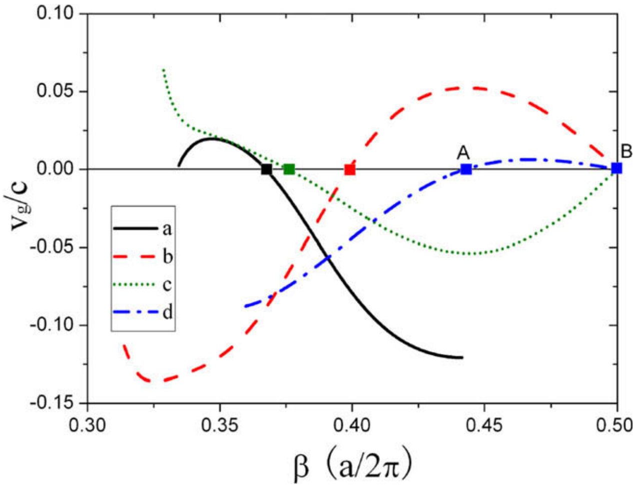

Fig. 3. Group velocity vs normalized propagation constant for a slab width of *w* = 2.9*a*.

Light speed can be described by group velocity, and it is given as $v_g = d\omega/d\beta$, where ω is angular frequency, β is propagation constant. Corresponding to the four dispersion curves in Fig. 2, we plot the calculated group velocity as a function of propagation constant in Fig. 3. Except curve a, which is not complete because part of its frequency is out of the considered frequency range, there are two zero group velocity points at each curve, one of them is at the edge of Brillouin region. For light of frequencies near these zero group velocity points, its field energy can be trapped effectively by the waveguide structure. Considering curve d, for propagation constant in the range of $0.443 \times 2\pi/a$ and $0.5 \times 2\pi/a$, group velocity does not exceed a value of *c*/160. This level of light speed is enough for devices like optical buffer or delay line.

We also calculated the dispersion characteristics for structures with different slab widths near 2.9*a*, and find out that the curves are all very similar compared with that of width 2.9*a*, but shift to a lower position with the increase of *w*. Thus the light frequency with zero group velocity can be easily tuned by changing the slab width, but the difference of the two zero group velocity frequencies is not sensitive to it.

The two points at curve d denoted by *A* and *B* in Fig. 3 correspond to the two zero group velocity frequencies of $0.34138 \times 2\pi c/a$ and $0.34161 \times 2\pi c/a$, respectively. Propagating light energy of the guided mode at these two frequencies can be stopped and trapped effectively. The structure may be used as an open cavity with two resonant frequencies. FDTD simulation will be used to verify the predication and show energy trapping ability of the structure.

In our FDTD calculation with perfectly matched layer (PML) boundary condition, a point Gaussian pulse expressed as Eq. (1) is used to excite the light field. It is positioned at the center of the core dielectric layer, which is marked as *P* in Fig. 4a.

$$E(t) = E_0 e^{-t^2/\tau^2} \sin(\omega_0 t) \qquad (1)$$

where angular frequency ω_0 is $0.34148 \times 2\pi c/a$ between the two frequencies with zero group velocity, pulse width τ is 200*T* ($T = 2\pi/\omega_0$). A monitor at the same position as the source is used to record the electric field as a function of time. The whole calculating time is 60,000*T*. In Fig. 4a, we give out a slice of electric field distribution after 36,000*T*, it is clear that the light field is trapped by the waveguide. Moreover, the light energy is mainly confined in the uniform dielectric layer, while a small part of energy extending into the photonic crystal is localized in the first two layers of holes. There is no reflection at the two ends of the waveguide, so the trap

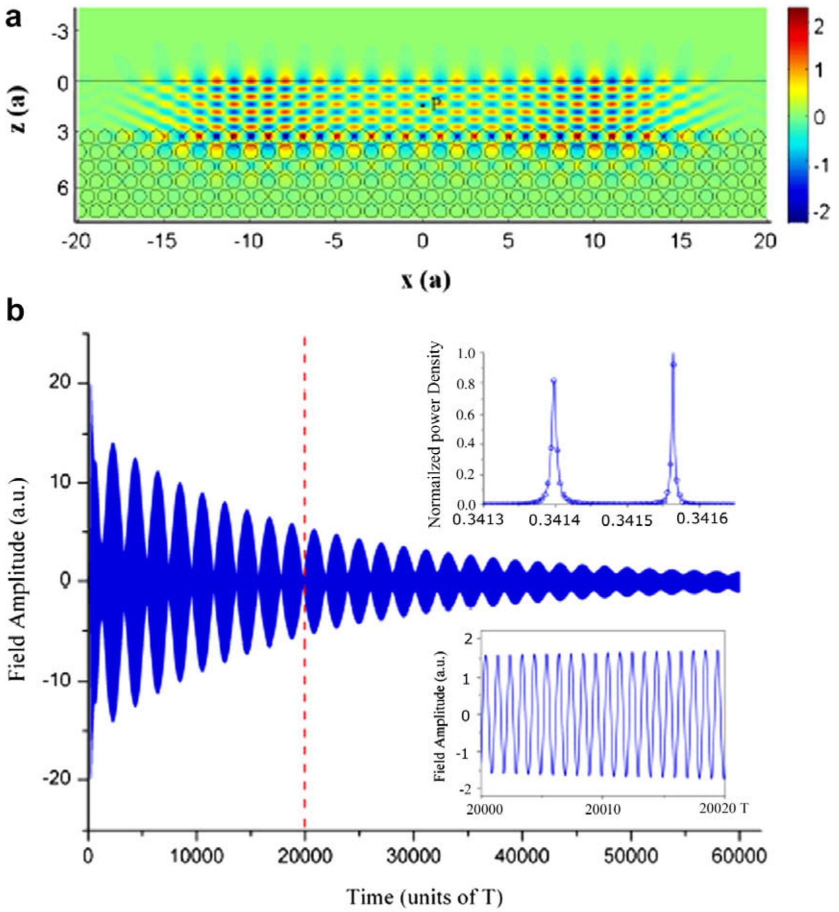

Fig. 4. (a) A slice of transverse electric field distribution after 36,000*T*. (b) Magnitude of the transient electric field (the whole recorded time is 60,000*T*), the lower subplot is a magnification of the signal after 20,000*T*; the upper is the frequency spectrum of power density, shows the two resonant frequencies of the trapped field.

of light should be the slow light effect of our waveguide structure completely.

The evolution of transient electric field at the position of the point source in the whole calculating time range is shown in Fig. 4b. The beginning irregular waveform attributes to the superimposition of source excitation, leaky and confined waves. After 2600*T*, the waveform becomes regular and elapses slowly. The resonant characteristic of the wave envelope seems to be an interference pattern of light wave of two frequencies.

By fast Fourier transform (FFT) of the time-domain signal (exclude the beginning 2600*T* of irregular time), we give out the frequency spectrum of power density in Fig. 4b (normalized to the second peak). Both of the two peaks are in a form of Lorentz type, just as the line shape of resonant frequency of a conventional cavity. Thus, fitting with Lorentz function, we can get the two resonant frequencies of $0.34140 \times 2\pi c/a$, and $0.34156 \times 2\pi c/a$, respectively, both deviate a little from the values calculated by the 2D layer-KKR method. The deviation may be caused by calculating errors in FDTD simulation due to the limited spatial mesh. Also we obtain their linewidth (full width at half maximum, FWHM) of $9.97 \times 10^{-5} \times 2\pi c/a$ and $5.52 \times 10^{-5} \times 2\pi c/a$, respectively. According to equation $Q = f_0/\Delta f$ [21] (where f_0 is the resonant frequency, and Δf is the FWHM of power density distribution.), Q-factors of the two trapped frequencies can be calculated as high as 3424 and 6188, respectively. The structure may be used as open cavities for lasers or traveling wave amplifiers. For the characteristic of two resonant frequencies, it has the potential to be used as cavity for dual-wavelength lasers [22,23], which is an important device for the generation of terahertz wave by photon mixing.

3. Conclusion

Slow light effect in a slab waveguide with a photonic crystal substrate has been studied. In a frequency range where the effective refractive index of the photonic crystal substrate is negative and nearly homogeneous, guided modes are found with a 2D layer-KKR method. The confinement is achieved by total internal reflection similar to an asymmetric LH waveguide, but different from that of a conventional photonic crystal waveguide. Both light stop and slow propagating can be achieved with our structure, and the corresponding frequencies can be easily tuned with the change of core width. Characteristic of two-frequency resonance makes the structure potential as a high-Q open cavity for dual-wavelength lasers.

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