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Physics Letters A 372 (2008) 735–740
PHYSICS LETTERS A
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Photovoltaic spatial soliton pairs in two-photon photorefractive materials

Noushin Asif [a], S. Shwetanshumala [b], S. Konar [a,*]

[a] Department of Applied Physics, Birla Institute of Technology, Mesra, Ranchi-835215, Jharkhand, India
[b] Department of Physics, A.N. College, Patna-800 013, Bihar, India

Received 25 July 2007; accepted 8 August 2007

Available online 10 August 2007

Communicated by V.M. Agranovich

Abstract

We report the existence of incoherently coupled bright–bright steady state photovoltaic soliton pairs in two-photon photorefractive material under open circuit conditions. Based on WKBJ method and paraxial ray approximation, we have obtained coupled equations describing dynamical evolution of spatial soliton pairs. In the steady state regime, the present analysis leads to the identification of existence equation of bright–bright solitons, which captures a plethora of soliton pairs. We have undertaken linear stability analysis which shows that these solitons are stable.

1. Introduction

Photorefractive spatial solitons have been subject of active research both theoretically and experimentally since their theoretical prediction by Segev et al. [1–12]. Though solitons are ubiquitous in various branches of physics, the self trapped photorefractive spatial solitons (PRSS) possess some very attractive features which make them potentially useful for various applications like all optical switching and routing, interconnects, parallel computing, optical storage, etc. [7–9]. In addition, since a photorefractive crystal (PR) has the ability to create optical solitons at very low optical power, of the order of microwatts [7], it is the most promising and prospective media for experimental verification of theoretical models. PRSS can be operated at telecommunication wavelength [9] (∼ 1.5 μm) at very low power (∼ μw) with fast response [11], response time being of the order of microsecond. A large number of review articles are now available highlighting applications of photorefractive spatial solitons [12,13].

Three different types of steady state photorefractive solitons have been predicted till 2000. The one which was identified first is the screening soliton. In the steady state, both bright and dark screening solitons (SS) are possible when an external bias voltage is applied to a non-photovoltaic photorefractive crystal [2,3]. The second kind is the photovoltaic soliton [6–8], the formation of which, however, requires an unbiased PR crystal that exhibits photovoltaic effect, i.e., generation of dc current in a medium illuminated by a light beam. Recently, a third kind of photorefractive soliton has been introduced [5,14], which arises when an electric field is applied to a photovoltaic photorefractive crystal. These solitons owe their existence to both photovoltaic effect and spatially non-uniform screening of the applied field and are also known as screening photovoltaic solitons (SP). It has been shown that if the bias field is much stronger than the photovoltaic field, then the screening photovoltaic solitons are just like screening solitons. On the other hand, if the applied field is absent, then they degenerate into photovoltaic solitons in the closed circuit condition.

The mechanism of three types of photorefractive solitons as elucidated above, are due to single photon photorefractive effect. Recently, a new model was introduced by Castro-Camus [15], which involves two-photon photorefractive effect. This model includes a valance band (VB), a conduction band (CB) and an intermediate allowed level (IL). A gating beam is used to maintain a fixed quantity of excited electrons from the valance band, which are then excited to the conduction band by the signal beam. The signal

* Corresponding author.
E-mail addresses: swakonar@yahoo.com, skonar@bitmesra.ac.in.

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2007.08.009

beam induces a charge distribution that is identical to its intensity distribution, which in turn gives rise to a nonlinear change of refractive index through space charge field. Based on this model [16], several authors have not only shown the existence of screening solitons, but dark and bright solitons as well [17]. Our goal in this paper is to demonstrate the existence of a new very large family of two-component bright–bright steady state photovoltaic spatial soliton pair in two photon photorefractive crystals under open circuit conditions. In addition, we reveal several important properties of these two-component composite solitons. The paper is organized as follows. The mathematical model for (1 + 1) D two-component screening photovoltaic spatial solitons has been developed in Section 2. In this section, we have employed the paraxial ray approximation method to derive the dynamical equations of spatial widths of these composite solitons. In Section 3, we have investigated stationary points of these dynamical equations which lead to identification of broad parameter region where stationary composite solitons exist. In Section 4, we have performed linear stability analysis and numerical experiment to show that these solitons are stationary. We have added a brief conclusion in Section 5.

2. Mathematical model

To start with, we consider a pair of optical beams which are propagating in a photorefractive material along z-direction with two-photon photorefractive effect. They are of same frequency but mutually incoherent. The crystal here is taken to be Cu:KNSBN with its optical c-axis oriented along the x coordinate. The two optical beams are allowed to diffract only along the x-direction and for the sake of simplicity the photorefractive material is assumed to be loss less. The two optical beams can be obtained by splitting a laser beam by a polarizing beam splitter. Making their optical path difference greatly exceed the coherence length of the laser can make these two beams mutually incoherent at the input face of the crystal. Thus, no stationary interference grating can be formed within a time scale comparable with the response time of the crystal. These two beams act as soliton-forming beams and note that a similar arrangement has been previously employed [18] to experimentally and theoretically investigate incoherently coupled soliton pairs in photorefractive crystals. We assume that the incident soliton forming optical beams are polarized along x-direction and the photorefractive crystal is illuminated by a separate gating beam. The optical fields are expressed in the form $\vec{E}_1 = \hat{x}\phi_1(x,z)\exp(ikz)$ and $\vec{E}_2 = \hat{x}\phi_2(x,z)\exp(ikz)$, where $k = k_0 n_e = (2\pi/\lambda_0)n_e$, n_e is the unperturbed extraordinary index of refraction and λ_0 is the free space wavelength; ϕ_1 and ϕ_2 are slowly varying envelopes of two optical fields respectively. It can be readily shown that the slowly varying envelopes of two interacting spatial solitons inside the photovoltaic PR crystal are governed by the following evolution equations

$$i\frac{\partial\phi_1}{\partial z} + \frac{1}{2k}\frac{\partial^2\phi_1}{\partial x^2} - \frac{k_0 n_e^3 r_{33} E_{\mathrm{sc}}}{2}\phi_1(x,z) = 0, \tag{1}$$

$$i\frac{\partial\phi_2}{\partial z} + \frac{1}{2k}\frac{\partial^2\phi_2}{\partial x^2} - \frac{k_0 n_e^3 r_{33} E_{\mathrm{sc}}}{2}\phi_2(x,z) = 0, \tag{2}$$

where r_{33} is the electro-optic coefficient, and E_{sc} is the space charge field. In the photorefractive material the nonlinearity originates due to induced space charge field, which perturbs the refractive index through the Pockel's effect. Assuming the diffusion effect to be small, which is true in photovoltaic medium, the expression for space charge field [15,17] is given by

$$E_{\mathrm{sc}} = -E_p\frac{s_2 I_2(I_2 + I_{2d} + \gamma_1 N_A/s_2)}{(s_1 I_1 + \beta_1)(I_2 + I_d)}, \tag{3}$$

where $E_p = \frac{\kappa\gamma N_A}{e\mu}$ is the photovoltaic field, $I_d = \beta_2/s_2$ is the so called dark irradiance, κ is the photovoltaic constant, N_A is the acceptor or trap density, γ is the recombination factor of the conduction to valence band transition, γ_1 is the recombination factor for intermediate allowed level to valence band transition, and β_1 is the thermo ionization probability constant for transition from valence band to intermediate level. I_1 is the intensity of the gating beam, which is kept constant; $I_2 = (n_e/2\eta_0)(|\phi_1|^2 + |\phi_2|^2)$ is the intensity of the two mutually incoherent soliton forming beams, $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$. Substituting expression (3) into Eqs. (2) and (1), we derive the following dimensionless dynamical equations for two optical beams

$$i\frac{\partial U}{\partial\xi} + \frac{1}{2}\frac{\partial^2 U}{\partial s^2} + \alpha\eta\sigma U + \alpha\eta\big(|U|^2 + |V|^2\big)U - \frac{\alpha\eta\sigma U}{(1 + |U|^2 + |V|^2)} = 0, \tag{4}$$

$$i\frac{\partial V}{\partial\xi} + \frac{1}{2}\frac{\partial^2 V}{\partial s^2} + \alpha\eta\sigma V + \alpha\eta\big(|U|^2 + |V|^2\big)V - \frac{\alpha\eta\sigma V}{(1 + |U|^2 + |V|^2)} = 0, \tag{5}$$

where

$$U = \sqrt{\frac{n_e}{2\eta_0 I_d}}\phi_1, \qquad V = \sqrt{\frac{n_e}{2\eta_0 I_d}}\phi_2, \qquad \xi = \frac{z}{k_0 n_e x_0^2}, \qquad s = x/x_0,$$

$$\alpha = (k_0 x_0)^2\big(n_e^4 r_{33}/2\big)E_p, \qquad \eta = \frac{\beta_2}{s_1 I_1 + \beta_1}, \qquad \sigma = \frac{\gamma_1 N_A}{s_2 I_d} = \frac{\gamma_1 N_A}{\beta_2},$$

and x_0 is an arbitrary spatial width taken for scaling. Depending upon the value of different system parameters, the above set of two equations is suitable for bright–bright, dark–dark and bright–dark solitons. Eqs. (4) and (5) are the coupled modified nonlinear Schrödinger equations whose solutions will yield solitons through the nonlinear photorefractive media. These two equations are nonintegrable in nature and may be solved by various approximation methods which include Vlasov's moment method [19], variational method due to Anderson [20] and paraxial theory of Akhmanov et al. [21,22]. All these methods are approximate, yet results predicted have good agreement with experimental values. Therefore, we will employ paraxial theory and assume the solutions of the form $U(\xi,s)=A_1(\xi,s)e^{-i\Omega_1(\xi,s)}$ and $V(\xi,s)=A_2(\xi,s)e^{-i\Omega_2(\xi,s)}$. Upon substituting these ansatz into (4) and (5), we obtain a set of four equations as follows

$$\frac{\partial\Omega_1}{\partial\xi}A_1-\frac{1}{2}\left(\frac{\partial\Omega_1}{\partial s}\right)^2A_1+\frac{1}{2}\frac{\partial^2A_1}{\partial s^2}+\alpha\eta\sigma A_1+\alpha\eta\left(|A_1|^2+|A_2|^2\right)A_1-\alpha\eta\sigma A_1\left(1+|A_1|^2+|A_2|^2\right)^{-1}=0, \tag{6}$$

$$\frac{\partial A_1}{\partial\xi}-\frac{\partial\Omega_1}{\partial s}\frac{\partial A_1}{\partial s}-\frac{1}{2}\frac{\partial^2\Omega_1}{\partial s^2}A_1=0, \tag{7}$$

$$\frac{\partial\Omega_2}{\partial\xi}A_2-\frac{1}{2}\left(\frac{\partial\Omega_2}{\partial s}\right)^2A_2+\frac{1}{2}\frac{\partial^2A_2}{\partial s^2}+\alpha\eta\sigma A_2+\alpha\eta\left(|A_1|^2+|A_2|^2\right)A_2-\alpha\eta\sigma A_2\left(1+|A_1|^2+|A_2|^2\right)^{-1}=0, \tag{8}$$

$$\frac{\partial A_2}{\partial\xi}-\frac{\partial\Omega_2}{\partial s}\frac{\partial A_2}{\partial s}-\frac{1}{2}\frac{\partial^2\Omega_2}{\partial s^2}A_2=0. \tag{9}$$

The first two terms in Eqs. (6) and (8) determine the behavior of the eikonal Ω_1 and Ω_2 respectively and hence the convergence or divergence properties of the two beams. The third term in these equations determines the diffraction of the beam. Fifth and sixth terms represent nonlinear refraction of the beams. The fourth, fifth and sixth terms together in (6) and (8) represent contribution from the photovoltaic properties of the material. Evolution of the beam envelopes A_1 and A_2 are described by Eqs. (7) and (9). Solutions of these equations can be tackled with a very popular ansatz, the Gaussian one. Though the Gaussian profile for pulse envelope is only approximate, handling the mathematics of nonlinear equations with them becomes simple as well as results obtained agrees with numerical simulation results. Therefore, the solutions of above equations are taken to be Gaussian with amplitude and phase of the form

$$A_1(\xi,s)=\frac{\sqrt{P_1}}{\sqrt{a(\xi)}}e^{\frac{-s^2}{2r_1^2a^2(\xi)}},\quad \Omega_1(\xi,s)=\frac{1}{2}s^2\Theta_1(\xi)+\psi_1(\xi)\quad\text{and}\quad\Theta_1(\xi)=-\frac{d}{d\xi}\ln a, \tag{10}$$

$$A_2(\xi,s)=\frac{\sqrt{P_2}}{\sqrt{b(\xi)}}e^{\frac{-s^2}{2r_2^2b^2(\xi)}},\quad \Omega_2(\xi,s)=\frac{1}{2}s^2\Theta_2(\xi)+\psi_2(\xi)\quad\text{and}\quad\Theta_2(\xi)=-\frac{d}{d\xi}\ln b, \tag{11}$$

where P_1 and P_2 are respectively the normalized peak power of two soliton forming beams. $a(\xi)$ and $b(\xi)$ are two variable beam width parameters, r_1 and r_2 are two positive constants; $r_1a(\xi)$ and $r_2b(\xi)$ are the spatial widths of two solitons respectively. Adopting standard method [22], the dynamical equations describing the evolution of two soliton forming beams are obtained as

$$\dot{a}=\Omega=F(a,b,\Omega,\Psi), \tag{12}$$

$$\dot{b}=\Psi=G(a,b,\Omega,\Psi), \tag{13}$$

$$\dot{\Omega}=\frac{1}{r_1^4a^3}-2\alpha\eta\left(\frac{P_1}{r_1^2a^2}+\frac{P_2}{r_2^2b^2}\right)-2\alpha\eta\sigma\frac{\frac{P_1}{r_1^2a^2}+\frac{P_2a}{r_2^2b^2}}{(1+\frac{P_1}{a}+\frac{P_2}{b})^2}=X(a,b,\Omega,\Psi), \tag{14}$$

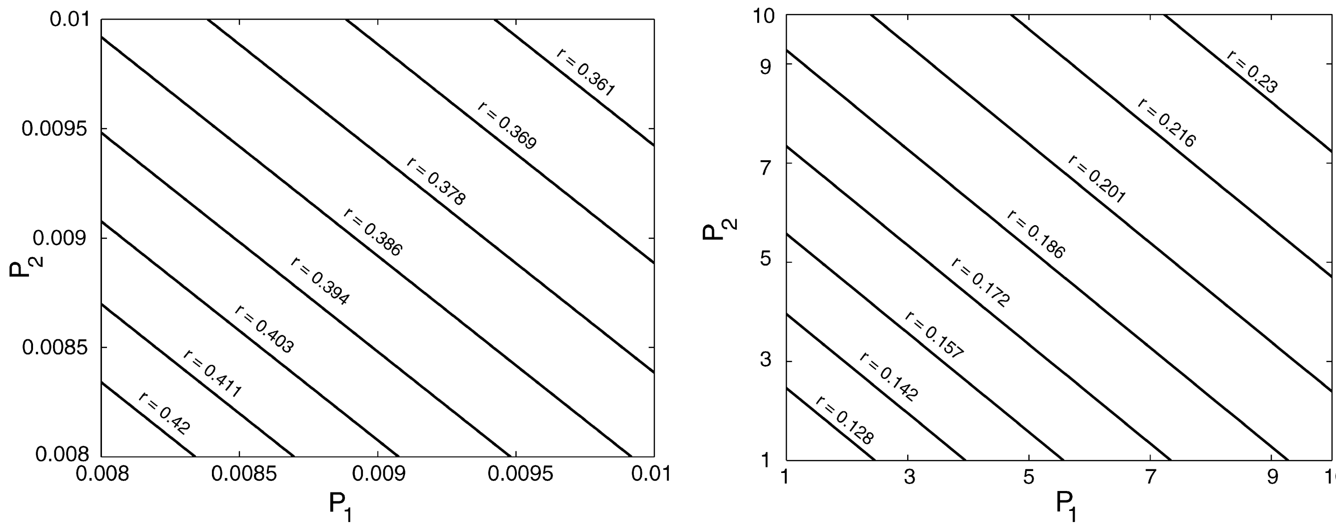
$$\dot{\Psi}=\frac{1}{r_2^4b^3}-2\alpha\eta\left(\frac{P_1}{r_1^2a^2}+\frac{P_2}{r_2^2b^2}\right)-2\alpha\eta\sigma\frac{\frac{P_1b}{r_1^2a^3}+\frac{P_2}{r_2^2b^2}}{(1+\frac{P_1}{a}+\frac{P_2}{b})^2}=Y(a,b,\Omega,\Psi), \tag{15}$$

where the prime over a quantity signifies derivative with respect to ξ. Eqs. (12)–(15) describe the evolution of widths of two solitons with distance of propagation.

3. Existence of stationary solitons

The steady state solution can be obtained from Eqs. (12)–(15) by setting all the four derivatives to zero. Defining steady state values of four variables a, b, Ω, and Ψ as a_s, b_s, Ω_s, and Ψ_s respectively, the conditions for steady propagation is thus obtained as follows

$$\frac{1}{r_1^4a_s^3}-2\alpha\eta\left(\frac{P_1}{r_1^2a_s^2}+\frac{P_2}{r_2^2b_s^2}\right)-2\alpha\eta\sigma\left(1+\frac{P_1}{a_s}+\frac{P_2}{b_s}\right)^{-2}\left(\frac{P_1}{r_1^2a_s^2}+\frac{P_2a_s}{r_2^2b_s^3}\right)=0, \tag{16}$$



(a) (b)

Fig. 1. Existence curve of spatial solitons. (a) Low power, (b) high power.

$$\frac{1}{r_2^4 b_s^3} - 2\alpha\eta\left(\frac{P_1}{r_1^2 a_s^2} + \frac{P_2}{r_2^2 b_s^2}\right) - 2\alpha\eta\sigma\left(1 + \frac{P_1}{a_s} + \frac{P_2}{b_s}\right)^{-2}\left(\frac{P_1 b_s}{r_1^2 a_s^3} + \frac{P_2}{r_2^2 b_s^2}\right) = 0. \tag{17}$$

At this stage, without any loss of generality we can introduce $a_s = b_s = 1$. Simultaneous validity of Eqs. (16) and (17) will ensure steady state propagation of incoherently coupled photovoltaic soliton pairs, which require $r_1 = r_2 = r$ (say). Therefore, the steady propagation is possible only when both the solitons are of equal spatial width, however, they can possess different power. The existence equation of bright–bright solitons now turns out to be

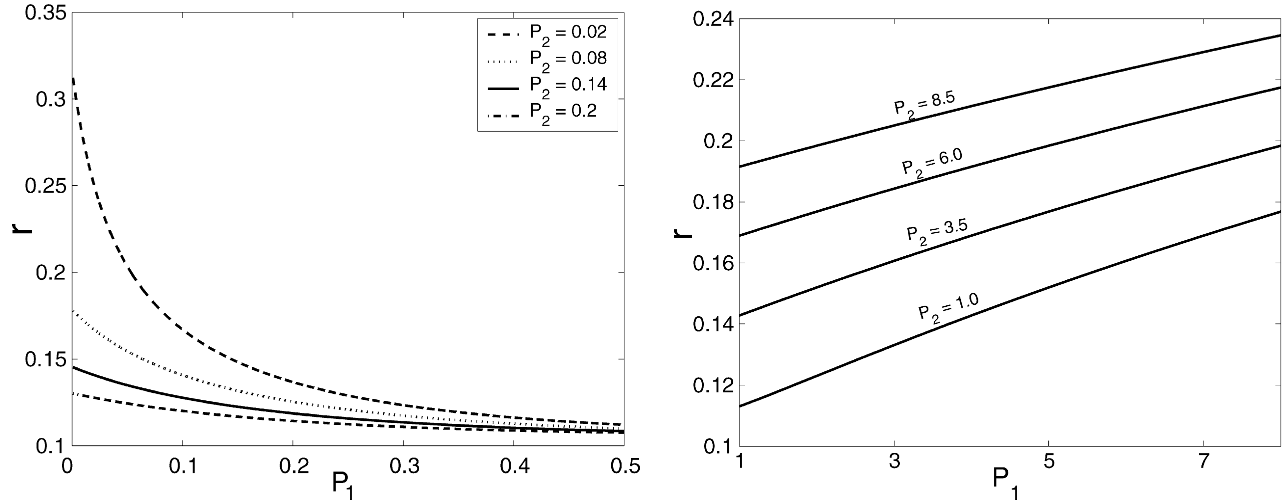
$$r = \left(2\alpha\eta(P_1 + P_2)\left\{1 + \frac{\sigma}{(1 + P_1 + P_2)^2}\right\}\right)^{-1/2}. \tag{18}$$

When both components of the pair possess equal power (i.e., $P_1 = P_2 = P$), the above equation takes a more simplified form giving $r = (4\alpha\eta P\{1 + \frac{\sigma}{(1+2P)^2}\})^{-1/2}$. The parameters σ and η are positive, hence, a material with positive value of α only can support bright–bright soliton pairs. The variation of P_1 and P_2 for various values of r has been demonstrated in Figs. 1(a) and 1(b) for low and high powers respectively. Each point on any curve of these two figures represents a stationary composite soliton with a definite spatial width and peak power. It proves the existence of such composite solitons in which the peak power of one component could be only a fraction of the other component. Therefore, we can conclude that due to self and cross phase modulation, a very weak optical spatial soliton of appropriate width could be self trapped and propagate as a stationary spatial soliton with the help of another co-propagating strong spatial soliton. In Fig. 2, we have plotted the variation of the width of bright–bright pair with peak power of one of the component of the composite soliton by keeping the peak power of the other component as constant. This shows that for a given peak power of one of the components, the other component can possess multiple values of peak power. To this end, we take up the special case of equal power of the constituent solitons. The soliton existence curve is depicted in Fig. 3, which clearly shows the existence of a bistable regime, i.e., two sets of soliton pair exist with same spatial width but having different peak power and consequently different peak amplitudes. Kindly note only the degenerate case possesses bistable property.

4. Stability analysis

At this stage it is imperative to examine the stability of steady state solutions since only stable solutions are solitons. Eqs. (12)–(15) are our starting point to establish a stability criterion using the Lyapunov's exponent [23–25]. We write the variables as a sum of steady value and a small perturbation from steady value, i.e., $a = a_s + \hat{a}$, $b = b_s + \hat{b}$, $\Omega = \Omega_s + \hat{\Omega}$, and $\Psi = \Psi_s + \hat{\Psi}$. Linearizing Eqs. (12)–(15), we obtain a set of equations which can be written as

$$\frac{d\Re}{d\xi} = J\Re, \quad \text{where } \Re = (\hat{a} \quad \hat{b} \quad \hat{\Omega} \quad \hat{\Psi})^T \text{ and } J = \begin{pmatrix} F_a & F_b & F_\Omega & F_\psi \\ G_a & G_b & G_\Omega & G_\psi \\ X_a & X_b & X_\Omega & X_\psi \\ Y_a & Y_b & Y_\Omega & Y_\psi \end{pmatrix}. \tag{19}$$



(a) (b)

Fig. 2. Variation of peak power P_1 of one of the component of the composite soliton with spatial width r while the peak power P_2 of the other component is constant. (a) Low power regime, (b) high power regime.

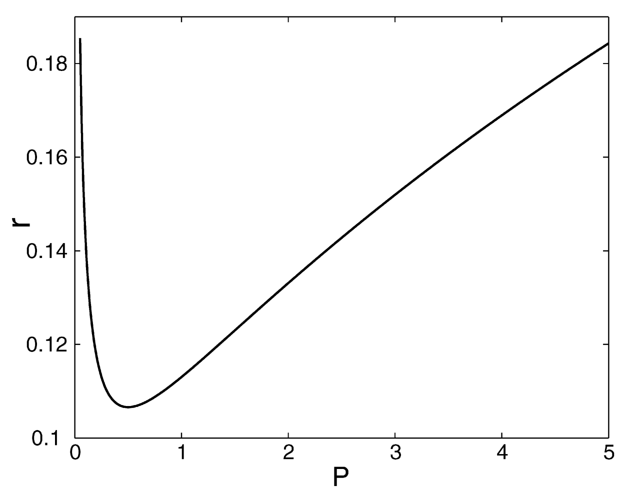


Fig. 3. Variation of peak power P of the degenerate composite soliton with spatial width r. Nature of the curve signifies existence of bistable property.

The subscript on F, G, X, Y denotes partial derivative with respect to that variable evaluated at the steady state. We construct a Jacobian determinant and for nontrivial solutions its value is set to zero, therefore

$$\det(J-\lambda I)=\begin{vmatrix} F_a-\lambda & F_b & F_\Omega & F_\psi \\ G_a & G_b-\lambda & G_\Omega & G_\psi \\ X_a & X_b & X_\Omega-\lambda & X_\psi \\ Y_a & Y_b & Y_\Omega & Y_\psi-\lambda \end{vmatrix}=0. \tag{20}$$

Following Lyapunov, the steady state solutions of Eqs. (12)–(15) are stable if no value of the solution λ of the quadratic equation

$$\lambda^4+\Sigma\lambda^2+\Xi=0 \tag{21}$$

is positive, where $\Sigma=(\frac{\partial X}{\partial a}+\frac{\partial Y}{\partial b})_{a_s,b_s}$, $\Xi=(\frac{\partial X}{\partial a}\frac{\partial Y}{\partial b}+\frac{\partial X}{\partial b}\frac{\partial Y}{\partial a})_{a_s,b_s}$. The four eigen-values of the above equation are given by $\lambda=\pm\frac{1}{\sqrt{2}}[-\Sigma\pm\sqrt{\Sigma^2-4\Xi}]^{\frac{1}{2}}$. All four roots are found to be purely imaginary, indicating stationary solitons. We have undertaken numerical simulation to examine the stability of steady states. Typical behavior in the phase plane under small perturbations has been depicted in Fig. 4, indicating stable propagation.

5. Conclusion

In conclusion, we have investigated the characteristics and stability properties of incoherently coupled bright–bright steady state photovoltaic soliton pairs in two-photon photorefractive material under open circuit condition. We have identified the existence

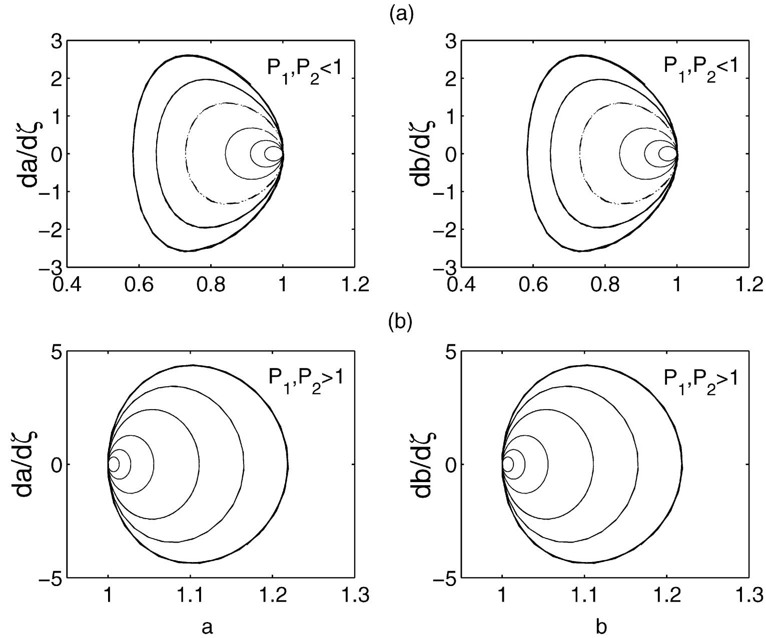


Fig. 4. Phase trajectory of spatial solitons when a and b have been perturbed from their respective stationary value. Amount of perturbation vary from 2.5% to 20%. Closed trajectory signifies solitons are stable against small perturbation. (a) Low power regime, (b) high power regime.

equation of bright–bright solitons, which captures a plethora of soliton pairs. We have shown that composite solitons with different widths cannot propagate as a stationary entity. Linear stability analysis shows that these solitons are stable.

Acknowledgements

This investigation is supported by Defence Research & Development Organization (DRDO), Ministry of Defence, Government of India, through a Grant No. RIP/ER/0403457/M/01 and the support is thankfully appreciated. One of the authors (N.A.) would like to thank DRDO for support through Senior Research Fellow.

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