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Correlations and pairing in nuclear matter within the Nozières–Schmitt-Rink approach

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Abstract. The influence of correlations on the critical temperature and density for the onset of superfluidity in nuclear matter is investigated within the scheme of Nozières and Schmitt-Rink [1]. For symmetric nuclear matter a smooth transition from Bose-Einstein condensation (BEC) of deuteron-like bound states at low densities and low temperatures to Bardeen-Cooper-Schrieffer (BCS) pairing at higher densities is described. Compared with the mean field approach a lowering of the critical temperature is obtained for symmetric nuclear matter as well as for pure neutron matter. The Mott transition in symmetric nuclear matter is discussed. Regions in the temperature–density plane are identified where correlated pairs give the main contribution to the composition of the system, so that approximations beyond the quasi-particle picture are requested.

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Superfluidity and superconductivity are macroscopic quantum phenomena occuring for fermion systems with attractive interaction. A usual framework for its microscopic description is the BCS theory [2]. The BCS theory is a meanfield approach and describes the occurence of pairing correlations forming the condensate. This pairing becomes at the critical temperature identical with the special case of zero momentum bound states in the low density limit [1]. However, in the normal phase a meanfield theory is in general not capable to describe two-particle correlations, in particular bound states with finite momentum. This general problem applies also to nuclear matter. Nuclear matter has been treated within the BCS approach by several authors, see e.g., [3–5]. Of special interest with regard to two-particle correlations are the pairing in the $^3S_1$ channel (see, e.g., [6, 7]) and the deuteron formation (see, e.g., [8]). In the normal phase there are works on the improvement of the meanfield approach including bound states and on the composition of nuclear matter (see, e.g., [9, 10]).

An attempt to include the influence of correlations on the critical temperature was proposed by Nozières and Schmitt-Rink for electron-hole systems [1]. The authors calculate the onset of superfluidity as a function of the coupling strength in the framework of BCS theory. The results are combined with a simple extension of the virial expansion to obtain a density formula including correlations in the normal phase. A similar density formula for nuclear matter is given by Schmidt et al. [10]. In the weak coupling limit Nozières and Schmitt-Rink find the ordinary BCS critical temperature. The other limit, the strong coupling, is characterized by the formation of non-interacting bosonic bound states which can undergo a Bose-Einstein condensation with the corresponding critical temperature. A smooth transition from strong to weak coupling is obtained. The open question how to treat correlations (fluctuations) and pairing (superfluidity) consistently has recently been addressed in a number of publications [11–13].

Within this paper we want to study the relevance of the considerations of Nozières and Schmitt-Rink applied to non-relativistic nuclear matter below saturation density. We consider a system of nucleons (protons, neutrons) interacting via a bare nucleon-nucleon interaction. The problem of the transition from strong to weak coupling is focussed in connection with the question of the validity of the quasi-particle picture. For symmetric nuclear matter as well as for neutron matter the occurence of correlated nucleons in the temperature-density plane is calculated to decide in which region a meanfield approach is justified and in which not. As a special topic the composition of the system along the critical temperature is considered.

First we give a set of basic formulas we used in our model calculation and discuss the results afterwards. For convenience we put $\hbar, k_B = 1$. Two–particle properties in medium are described by a thermodynamic T–matrix governed by a Bethe–Salpeter equation. The T–matrix can be represented in a partial wave decomposition. For the sake of simplicity we consider only uncoupled scattering channels α. Introducing relative momentum k and total momentum K for two interacting fermions the two-particle T–matrix in the respective channel reads

$$T_\alpha(kK, k'K'; z) = V_\alpha(k, k') + \sum_{k'',K''} V_\alpha(k, k'') \times G_2^0(k''K''; z)\, T_\alpha(k''K'', k'K'; z) \qquad (1)$$

with the two-particle thermodynamic Green's function in quasi-particle approximation

$$G_2^0(kK;z) = \frac{Q(kK)}{z - E_1 - E_2} .$$

The medium effects enter via the Fermi function $f(E_1) = (\exp((E_1 - \mu_1)/T) + 1)^{-1}$ in the angle-averaged Pauli blocking $Q(kK) = \int \frac{d\Omega}{4\pi} [1 - f(E_1) - f(E_2)]$ and via the self energy shifts v of the quasi-particle energies $E_{1,2} = (\mathbf{k} \pm \mathbf{K}/2)^2/2m + v_{1,2}(\mathbf{kK})$.

We take the quasi-particle self energy $v(\mathbf{kK})$ in the rigid shift approximation as a constant $\bar{v}$ (see also [1] and [10]) which can be incorporated in an effective chemical potential $\mu^* = \mu - \bar{v}$. Since we consider symmetric nuclear matter or pure neutron matter and fermions of the same nucleon mass m we do not need to introduce different chemical potentials for the species.

In case of separable interaction the T-matrix can be written in an analytic form

$$T_\alpha(kK, kK; z) = \frac{V_\alpha(k,k)}{1 - J_\alpha(K,z)} , \quad (2)$$

with $\quad J_\alpha(K,z) = \sum_k V_\alpha(k,k)\, G_2^0(kK;z)$,

and the angle averaged Pauli blocking in the rigid shift approximation

$$Q(kK) = \frac{2mT}{kK} \ln\left[\frac{\exp(((\frac{K}{2} + k)^2/2m - \mu^*)/T) + 1}{\exp(((\frac{K}{2} - k)^2/2m - \mu^*)/T) + 1}\right] - 1 . \quad (3)$$

A possible choice for the nucleon-nucleon interaction is a separable represention of the Paris potential [14]. To produce first results we use a more simple rank=1 potential parametrized by Yamaguchi [15] which is separabel and attractive only. It takes into account S–wave scattering ($\alpha = {}^1S_0, {}^3S_1$) and depends on the relative momenta of the incoming and outgoing two particles and the coupling strength in the respective channel:

$$V_\alpha(k,k') = -\lambda_\alpha v(k) v(k')$$

with the formfactor

$$v(k) = \frac{1}{k^2 + \beta^2} , \quad (4)$$

where $\beta = 1.4488\ \mathrm{fm}^{-1}$ is the inverse potential range, $\lambda_{{}^1S_0}$=2994 MeV fm^{-1} and $\lambda_{{}^3S_1}$=4264 MeV fm^{-1} is the coupling strength in the singlet and in the triplet channel, respectivly. The parameters are fitted to the empirical nucleon-nucleon scattering phase shifts and the vacuum binding energy of the deuteron ($E_b^0 = -2.225$ MeV) which occurs in the triplet channel. The coupling of the ${}^3S_1$ to the ${}^3D_1$ channel is neglected.

From the denominator of (2) we investigate the pole structure of the T–matrix. At real energies below the continuum edge of scattering states, given in rigid shift approximation as $E_{\mathrm{cont}}(K) = K^2/4m + 2\bar{v}$, a pole in the triplet channel is obtained which corresponds to an in-medium binding energy $E_b = E_b(\mu^*, T, K)$ of a deuteron-like bound state. Measuring E_b relative to the continuum edge the pole condition reads: $1 - \mathrm{Re}\, J_{{}^3S_1}(K, z = E_b + E_{\mathrm{cont}}(K)) = 0$, or

$$1 = \lambda_{{}^3S_1} \sum_k \frac{v(k)v(k)\, Q(kK)}{k^2/m - E_b} . \quad (5)$$

Since the rigid shift self energies of the continuum energy are compensated by the respective shifts of the quasi-particle energies in the denominator of G_2^0 in (5) we drop them for the calculations below. Due to the density and temperature dependent Pauli blocking the in-medium binding energy is shifted towards the continuum edge with increasing density until it merges the continuum edge ($E_b \to 0$). This is interpreted as a break up of the deuteron-like bound states [9]. The break up of bound states with total momentum $K = 0$ defines a Mott density $n_{\mathrm{mott}}(T)$ (see Mott line in Fig. 2a) which is determined from the condition: $1 - \mathrm{Re}\, J_{{}^3S_1}(K = 0, z = 0) = 0$, and the density formula (7) (see below).

Bound states with finite total momentum survive up to higher densities since the Pauli blocking is less effective. From the pole condition, $1 - \mathrm{Re}\, J_{{}^3S_1}(K = K_{\mathrm{mott}}, z = K_{\mathrm{mott}}^2/4m) = 0$, one can define a so-called Mott momentum K_{mott} which is the lowest total momentum for the existence of a bound state at densities higher than the Mott density.

For real energies above the continuum edge the scattering of two nucleons in a dense medium can be expressed by means of in-medium scattering phase shifts δ_α (for derivation see [10])

$$\cot \delta_\alpha(K, T, \mu^*, E) = \frac{1 - \mathrm{Re}\, J_\alpha(K, K^2/4m + E + i0)}{\mathrm{Im}\, J_\alpha(K, K^2/4m + E + i0)} . \quad (6)$$

Effective binding energy, Mott momentum and in-medium scattering phase shifts are ingredients for the equation of state for the normal phase which is applicable until the critical temperature for pairing given by (9) is reached. We use a density formula of a form given by Schmidt et al. [10] which was derived within a rigorous quantum statistical approach by Matsubara Green's functions. In this so-called generalized Beth-Uhlenbeck formula [10] the total nucleon density splits into contributions due to free quasi-nucleons and correlated nucleons

$$n_{\mathrm{tot}}(\mu, T) = n_{\mathrm{free}}(\mu, T) + 2 n_{\mathrm{corr}}(\mu, T) . \quad (7)$$

This density formula goes beyond the usual quasi-particle equation of state since it explicitly takes into account the formation of two–nucleon correlations. For symmetric nuclear matter the correlation contribution can be decomposed in a bound state density and a contribution due to scattering states, which reads for uncoupled channels [10]

$$n_{\mathrm{free}} = 4 \sum_{k_1} f(E_1) ,$$

$$\begin{aligned} n_{\mathrm{corr}} &= n_{\mathrm{bound}} + n_{\mathrm{scatt}} \\ &= c_{{}^3S_1} \sum_{K > K_{\mathrm{mott}}} g(E_b(K, \mu^*, T) + K^2/4m) \\ &\quad - c_{{}^3S_1} \sum_{K > K_{\mathrm{mott}}} g(K^2/4m) \\ &\quad - \sum_K \int_0^\infty \frac{dE}{\pi} \left[\frac{d}{dE} g(E + K^2/4m)\right] \end{aligned}$$

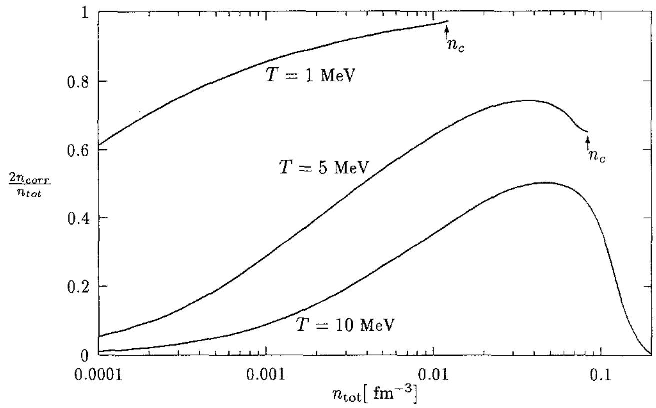

Fig. 1. Composition of symmetric nuclear matter: percentage of correlated nucleons versus total density n_{tot} at different temperatures. $n_c(T)$ denotes the critical density where superfluidity sets in and the T-matrix approach breaks down

$$\times \sum_{\alpha} c_\alpha (\delta_\alpha - \frac{1}{2}\sin(2\delta_\alpha)) \qquad (8)$$

with $g(E) = (\exp((E - 2\mu^*)/T) - 1)^{-1}$ the Bose distribution function. Spin and isospin degeneration factors are explicitly given ($c_\alpha = 3$ for S–wave channels).

This expression corresponds to a density formula given by Nozières and Schmitt-Rink ([1], (23)) after integration by parts. However, (8) differs in the occurrence of an additional term $\frac{1}{2}\sin(2\delta_\alpha)$ in the scattering contribution. The origin of this term is extensively discussed by Zimmermann and Stolz for the electron-hole system in excited semiconductors [16]. They point out that this term is needed to avoid double counting of the quasi-particle energy shift. Furthermore, at high densities the expression given by Nozières and Schmitt-Rink diverges as soon as the chemical potential becomes positive. This singularity is compensated if the quasi-particle picture is applied (see [10]). To avoid restriction to the low density case where the more simple density formula of [1] is applicable we use (8) instead which has been derived by Zimmermann parallel to the work of Nozières and Schmitt-Rink and applied to nuclear matter by Schmidt et al. [10]. In contrast to a summation of ring diagrams for the scattering process of two free nucleons [1] the latter approach includes a propagation of nucleonic quasi-particles.

From (8) the composition of nuclear matter, particularly the concentration of correlated nucleons, can be determined. Figure 1 shows for isothermes an increase of correlated nucleons with the density until a maximum is reached. Going to higher densities correlations are suppressed due to Pauli blocking without abrupt changes in the thermodynamic variables.

Plotting this results of the normal phase in the temperature–density plane we find the area with equal concentration of correlated nucleons (dashed lines in Fig. 2a). For each temperature a maximum of correlations can be found. Since bound states with finite total momentum and scattering correlations survive up to higher densities, the maximal percentage of correlated nucleons is reached for densities above the Mott density (see Mott line in Fig. 2a). For densities below Mott density and temperatures near to the critical temperature correlation contributions predominate the total density.

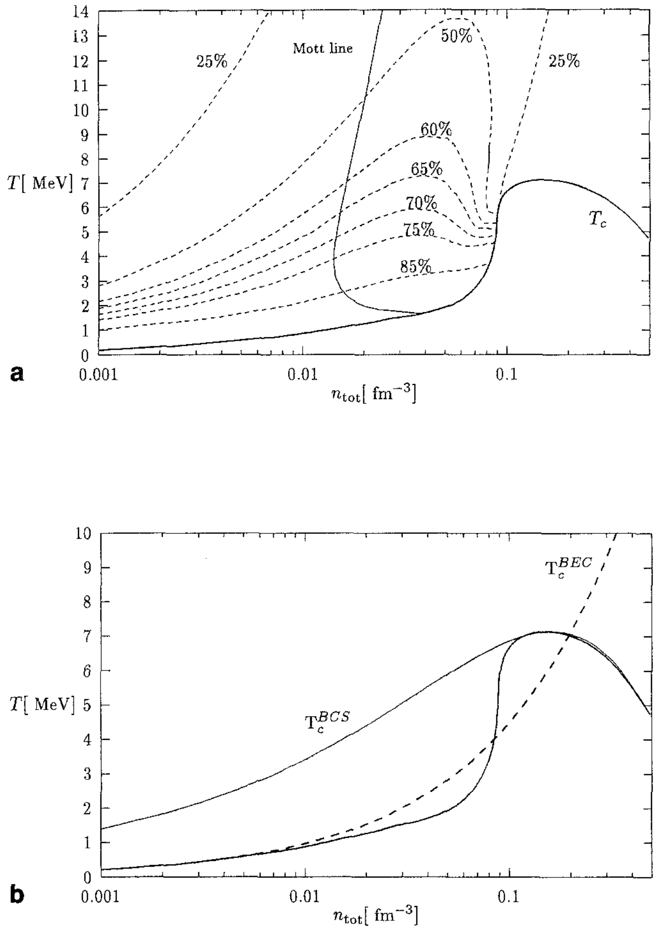

Fig. 2. a Temperature-density plane of symmetric nuclear matter showing lines of equal concentration of correlated nucleons $2n_{corr}/n_{tot}$ (*dashed*). The Mott line indicates the break up of the deuteron-like bound states with $K = 0$. The critical temperature in the $^3S_1$ channel (*bold*) marks the onset of superfluidity. **b** The critical temperature in the $^3S_1$ channel (*bold*) is compared with the BCS estimate (*thin*) and the Bose-Einstein condensation curve (*dashed*)

The equation of state (8) is valid only for temperatures above the critical temperature for the onset of superfluidity in nuclear matter. To find the critical temperature in dependence of the total density we use the Thouless criterion first and put the result into (8). The critical temperature T_c for the onset of pairing is given by the Thouless criterion if a pole at $z = 2\mu^*$ arises for the two-particle T-matrix at $K = 0$. For the T-matrix this pole condition reads: $1 - \mathrm{Re}\, J_\alpha(K = 0, T = T_c, z = 2\mu^*) = 0$, or

$$1 = \lambda_\alpha \sum_k \frac{v(k)v(k)\tanh(\frac{1}{4T_c}(k^2/m - 2\mu^*))}{k^2/m - 2\mu^*} . \qquad (9)$$

If the pole corresponds to a two-body bound state [$2\mu^* = E_b$, see (5)], the imaginary part of J_α vanishes and the Thouless criterion holds in given form. Furthermore, at this particular energy $z = 2\mu^*$ the imaginary part of J_α vanishes when it lies in the continuum of scattering states, because the Pauli blocking factor contained in G_2^0 in (1) vanishes in this case. Thus, the pole condition (9) for the onset of superfluidity holds for positive as well as negative values of $2\mu^*$. It coincides with the solution of the BCS theory for vanishing gap (for further discussion and influence of k–dependent Hartree-Fock self energy see [17, 18]).

In our calculation the effective chemical potential μ^* is related to the total density via the equation of state (8). The bold line in Fig. 2a shows the critical temperature of the $^3S_1$ channel in symmetric nuclear matter plotted versus total density separating the normal and the superfluid phase.

Figure 2b compares the critical temperature with two limiting cases. At higher densities we obtain coincidence with the BCS-limit. The BCS estimate of the critical temperature can be obtained by taking only n_{free} in (7). At lower densities, especially below the Mott density, the critical temperature according to (9) is shifted towards the limit of Bose-Einstein condensation until it coincides with the critical temperature of an ideal boson gas of deuteron-like bound states with mass $2m$ and density of the bosonic bound states divided by the degeneracy factor $c_{^3S_1} = 3$

$$T_c^{\mathrm{BEC}} = \left(\frac{\pi}{m}\right)\left(\frac{n_{\mathrm{bound}}}{3}\frac{1}{\zeta(3/2)}\right)^{2/3} .$$

Therefore, we consider the limit $n_{\mathrm{tot}} \to 0$ as the strong coupling limit while for high densities correlations are suppressed and the weak coupling or BCS-limit is approached. In the intermediate region a smooth transition between both limits is obtained.

Superfluidity in asymmetric nuclear matter is expected to be realized in the interior of neutron stars. There are several papers considering the problem of superfluidity in neutron stars (see, e.g., [5, 19]) or the influence of the neutron excess in isospin asymmetric nuclear matter on the critical temperature [17].

Therefore, we apply our approach to the case of pure neutron matter as an approximation for neutron star matter. Since no bound states occur in the considered $^1S_0$ neutron–neutron channel the generalized Beth-Uhlenbeck formula contains only free and scattering contributions

$$n_{\mathrm{tot}}(\mu, T) = 2\sum_{k_1} f(E_1) - 2\sum_K \int_0^\infty \frac{\mathrm{d}E}{\pi}\left[\frac{\mathrm{d}}{\mathrm{d}E} g(E_{\mathrm{cont}} + E)\right] \times (\delta_{^1S_0} - \frac{1}{2}\sin(2\delta_{^1S_0})) . \qquad (10)$$

In Fig. 3 the critical temperature in the $^1S_0$ channel for neutron matter is displayed versus total density. In the region of about 1/1000 to 1/100 of nuclear matter density n_0 we obtain a slight lowering of T_c in comparison to the BCS estimate considering only free density contribution. The percentage of correlated nucleons is generally reduced compared to symmetric nuclear matter. While for symmetric matter correlations are of main importance in the low density and low temperature region (see Fig. 2a) they disappear for neutron matter in the high and in the low density limit. This is due to the lack of bound states in neutron matter and clearly to be seen if we compare the composition at $T = T_c$ in dependence of total density for both cases (Fig. 4).

We calculated the composition and the critical temperature for symmetric nuclear matter and neutron matter consistent with an equation of state which includes two–nucleon correlations. For the sake of simplicity we have considered a simple separable nucleon-nucleon potential. A more sophisticated calculation of correlation and pairing in hot nuclear matter should start from more realistic interaction potentials such as higher rank separable representation of the Paris potential in different channels.

For the determination of the critical temperature we applied the approach given by Nozières and Schmitt-Rink [1] to nuclear matter. In contrast to [1] where the inverse potential strength is varied for different densities we increase the total density for fixed coupling parameters to obtain the smooth transition from the strong coupling limit (BEC) of bosonic deuteron-like bound states to the weak coupling limit (BCS) of quasi-nucleons. To consider higher densities as well it was necessary to use an improved density formula basing on the quasi-particle picture [10, 16].

In conclusion, we point out that at definite regions in the temperature–density plane of nuclear matter two–nucleon correlations are of major importance. The critical temperature for pairing as a function of density is modified if correlations are taken into account. In particular, this means that below the Mott density the BCS estimate is shifted towards the BEC limit. In these regions a quasi-particle treatment is not justified.

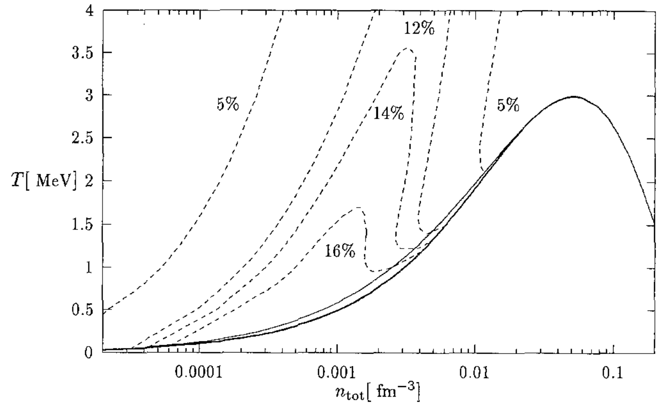

Fig. 3. Temperature-density plane of pure neutron matter showing lines of equal concentration of correlated nucleons (*dashed*). The critical temperature in the $^1S_0$ channel (*bold*) marks the onset of superfluidity. For comparison the BCS estimate (*thin*) is presented

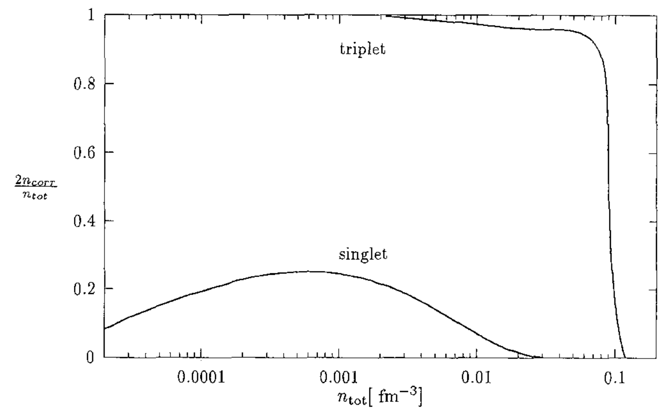

Fig. 4. Percentage of correlated nucleons along the curves of critical temperature (see Figs. 2a, 3) is compared for the triplet channel in symmetric nuclear matter and the singlet channel in neutron matter

The Mott density is characteristic for the region where bound states disappear due to Pauli blocking. In this region, also the transition from Bose-Einstein condensation to BCS-pairing occurs.

Evidently, the approach of Nozières and Schmitt-Rink to evaluate T_c as well as the presented approach is not very adequate in the region near and below the Mott transition where bound state formation is dominant. In addition to the modification of the equation of state, also the T–matrix has to be improved by considering the effect of blocking due to correlated states. Moreover, as denoted in [17] the Thouless criterion can be generalized by considering two–particle propagators G_2^0 including correlations beyond the meanfield approach. A modification of the critical temperature in the region of the transition from strong to weak coupling is expected.

Better approximations to improve the presented approach [1, 10] need to include correlations in the G_2^0–propagator in a self-consistent way, see [11–13]. This can be based on an approach which considers the spectral function as a basic quantity instead of the picture of bound states and quasiparticles with sharp peaked energies [20].

References

1. Nozières, P., Schmitt-Rink, S.: J. Low Temp. Phys. **59**, 195 (1985)
2. Bardeen J., Cooper, L.N., Schrieffer, J.R.: Phys. Rev. **108**, 1175 (1957)
3. Chen, J.M.C., Clark, J.W., Krotschek, E., Smith, R.A.: Nucl. Phys. A **451**, 509 (1986)
4. Su, R.K., Yang, S.D., Kuo, T.T.S.: Phys. Rev. C **35**, 1539 (1987)
5. Baldo, M., Cugnon, J., Lejeune, A., Lombardo, U.: Nucl. Phys. A **515**, 409 (1990)
6. Baldo, M., Bombaci, I., Lombardo, U.: Phys. Lett. B **283**, 8 (1992)
7. Alm, T., Röpke, G., Schmidt, M.: Z. Phys. A 355 (1990)
8. Danielewicz, P., Bertsch, G.F.: Nucl. Phys. A **533**, 712 (1991)
9. Röpke, G., Schmidt, M., Münchow, L., Schulz, H.: Nucl. Phys. A **379**, 536 (1982); **399**, 587 (1983)
10. Schmidt, M., Röpke, G., Schulz, H.: Ann. Phys. (NY) **202**, 57 (1990)
11. Tokumitu, A., Miyake, K., Yamada, K.: Phys. Rev. B **47**, 11988 (1993)
12. Haussmann, R.: Z. Phys. B **91**, 291 (1993)
13. Röpke, G.: Ann. Physik (Leipzig) **3**, 145 (1994)
14. Haidenbauer, J., Plessas, W.: Phys. Rev. C **30**, 1822 (1984)
15. Yamaguchi, Y.: Phys. Rev. **95**, 1628 (1954)
16. Zimmermann, R., Stolz, H.: Phys. Status Solidi B **131**, 151 (1985)
17. Alm, T., Friman, B., Röpke, G., Schulz, H.: Nucl. Phys. A **551**, 45 (1993)
18. Hellmich, A., Röpke, G., Schnell, A., Stein, H.: In: Bose-Einstein condensation. A. Griffin, D.W. Snoke, S. Stringari, (eds.). New York: CUP 1994
19. Baldo, M., Cugnon, J., Lejeune, A., Lombardo, U.: Nucl. Phys. A **536**, 349 (1992)
20. Alm, T., Röpke, G., Schnell, A., Stein, H.: (submitted to Phys. Lett. B)