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Perfect lenses and corners for flexural waves

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ABSTRACT

The paper presents numerical models describing transformation based perfect lenses and corners for flexural waves propagating in thin elastic plates. We show that complementary media can be designed to cancel out the elastic space, in a way similar to what Pendry and Ramakrishna (2003) [1] proposed for the optical space.

1. Introduction

Transformation based electromagnetic media allow for an unprecedented control of the propagating and evanescent components of the electromagnetic field [1–4]. However, the Navier equations which govern the propagation of elastic waves do not generally retain their form under geometric transforms: for instance elastic cloaks generally require metamaterials beyond Newton's laws [5], as the pressure and shear waves are inherently coupled [6]. Nevertheless, in the specific case of thin elastic plates, flexural waves are governed by the biharmonic equation which behaves nicely under geometric transforms [7].

2. Geometric transform and biharmonic equation

The equation governing the propagation of bending waves in (possibly anisotropic and heterogeneous) thin-plates involves a fourth order differential equation [8]. The main assumption is that the working wavelength λ is supposed to be large enough compared to the thickness of the plate h and small compared to its in-plane dimension L, i.e. $h \ll \lambda \ll L$. With all the above assumptions, the out-of-plane displacement $\mathbf{u} = (0, 0, U(x_1, x_2))$ along the vertical direction x_3 satisfies

$$\lambda \nabla(\underline{\underline{\zeta}}^{-1} \nabla(\lambda \nabla(\underline{\underline{\zeta}}^{-1} \nabla U))) - \beta_0^4 U = 0, \tag{1}$$

with $\beta_0^4 = \omega^2 \rho_0 h / D_0$, where D_0 is the flexural rigidity of the plate, ρ_0 its density and h its thickness. $\underline{\underline{\zeta}}$ is a diagonal rank-2 tensor describing and λ is a varying coefficient of the material, a case encompassed in Ref. [8].

Importantly, physical considerations based on dimensional analysis show that this equation retains its form under geometric transform provided that

$$\underline{\underline{\zeta}}^2 = \underline{\underline{E}}^{-1} \quad \text{and} \quad \lambda^2 = \rho^{-1}, \tag{2}$$

where $\underline{\underline{E}}$ has the physical dimension of a Young modulus and ρ is a scalar density as shown in Ref. [7].

2.1. Perfect lenses and complementary media

The original 'perfect lens' presupposed a uniform slab of isotropic material with dielectric permittivity $\varepsilon = -1$ and magnetic permeability $\mu = -1$. However, focussing will occur under more general conditions [1]. Any system for which

$$\varepsilon_1 = +\varepsilon(x_2), \quad \mu_1 = +\mu(x_2), \quad -d < x_1 < 0,$$

$$\varepsilon_2 = -\varepsilon(x_2), \quad \mu_2 = -\mu(x_2), \quad 0 < x_1 < d \tag{3}$$

will show identical focussing. Focussing will always occur irrespective of the medium in which the lens is embedded. This is true for any medium which is mirror antisymmetric about a plane, such as checkerboards [5]. A negatively refracting medium is complementary to an equal thickness of vacuum and optically 'cancels' its presence. The compensating action extends to both the evanescent and the propagating modes [1]. It can be shown using geometric transforms that such complementary media fold the optical space onto itself [9,10]. However, the biharmonic wave equation retains its form under geometric transform, in a way similar to the harmonic wave equation [7]. This property forms the basis of the following discussion.

In the case of thin-elastic plates, complementary media are such that

$$\zeta_1 = +\zeta(x_2), \quad \lambda_1 = +\lambda(x_2), \quad -d < x_1 < 0,$$

$$\zeta_2 = -\zeta(x_2), \quad \lambda_2 = -\lambda(x_2), \quad 0 < x_1 < d. \tag{4}$$

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If we now consider a heterogeneous anisotropic thin plate described by

$$\underline{\underline{\zeta_1}} = \begin{pmatrix} \zeta_{11} & \zeta_{12} \\ \zeta_{21} & \zeta_{22} \end{pmatrix}(x_2), \quad \lambda_1 = +\lambda(x_2), \quad -d < x_1 < 0, \tag{5}$$

then the resulting complementary medium is given by

$$\underline{\underline{\zeta_2}} = \begin{pmatrix} -\zeta_{11} & +\zeta_{12} \\ +\zeta_{21} & -\zeta_{22} \end{pmatrix}(x_2), \quad \lambda_2 = -\lambda(x_2), \quad 0 < x_1 < d. \tag{6}$$

2.2. *Perfect corners arranged in a checkerboard fashion*

Among the large class of optical systems built by sticking together complementary media are corner lenses, or perfect corner reflectors. In two dimensions, they are obtained by mapping a 1D photonic crystal onto a chessboard like structure via a change from Cartesian to polar coordinates, as first demonstrated by Pendry and Ramakrishna in Ref. [1]. These represent very singular situations (including plasmons at the interface between complementary media associated with divergent series [11]) whereby the optical space can be folded back onto itself in all three dimensions, as shown in Ref. [3]. Similar physics actually applies to thin elastic plates made of complementary media, as we now illustrate with numerics.

3. Numerical illustrations

We have implemented the weak form of Eq. (1) in the Finite Element software COMSOL. We used specially designed perfectly matched layers for flexural waves to model the infinite domain. Our computations reported in Figs. 1 and 2 show that electromagnetic paradigms such as the perfect lens or perfect corners can be also achieved in the context of flexural waves. We note the strong oscillations of the field U on the interface

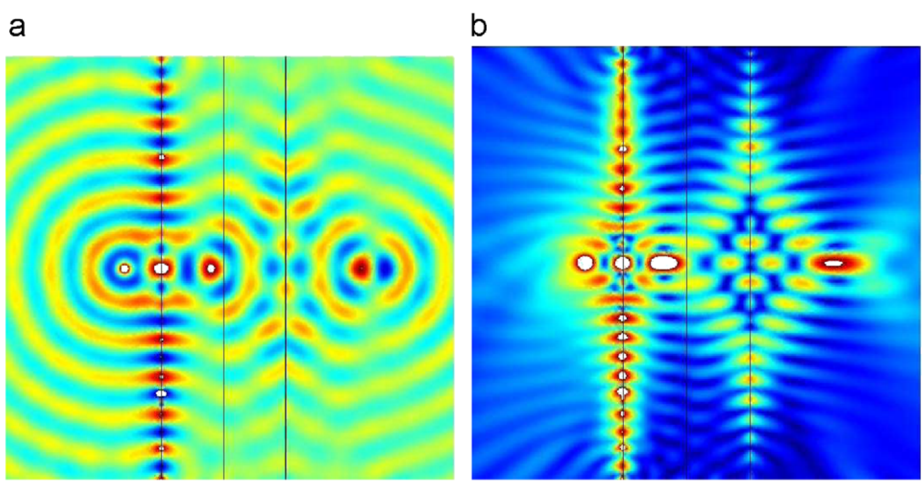

Fig. 1. (Color online) Perfect lens of width 1 symmetric about the middle vertical segment at $x_1 = 0$: a point force generating a concentric flexural wave of wavelength $\lambda = 0.315$ located in the plane $x_1 = -0.8$, displays a ghost image in the plane around $x_1 = -0.2$ and a perfect image close to $x_1 = 1.2$ through a slab lens of Young modulus $E = -0.95$ and density $\rho = -0.95$. The slab lens is surrounded by an infinite elastic material with $E = 0.1$ and $\rho = 1$. (a) Plot of $\Re\{U\}$ (green means zero, and red and blue highest values on that color scale); (b) plot of $|\Re\{U\}|$ (blue means zero and red highest values on that color scale). The plate thickness is $h = 1$.

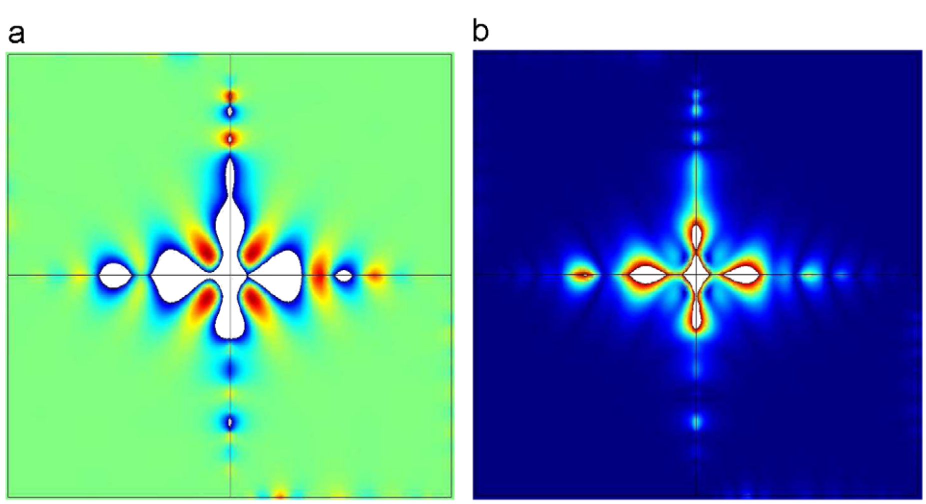

Fig. 2. (Color online) Perfect corner: a point force generating a concentric flexural wave of wavelength $\lambda = 0.315$ located at $(x_1, x_2) = (0.3, 0.6)$, displays a ghost image at $(x_1, x_2) = (-0.3, 0.6)$, a perfect image at $(x_1, x_2) = (-0.3, -0.6)$ and a second ghost image at $(x_1, x_2) = (0.3, -0.6)$ through a corner reflector alternating four elastic regions of sidelength 1.5. The Young modulus and density in the upper right and lower left regions are respectively $E = -0.95$ and $\rho = -0.95$, while in the upper left and lower right regions $E = 0.1$ and $\rho = 1$. (a) Plot of $\Re\{U\}$ (green means zero, and red and blue highest values on that color scale); (b) plot of $|\Re\{U\}|$ (blue means zero and red highest values on that color scale). The plate thickness is $h = 1$.

$x_1 = -0.5$ close to the source. We note that the white color stands for saturated values in Figs. 1 and 2 i.e. appears where the field magnitude lies outside the range of values selected for the color scales.

4. Conclusion

In this paper, we have extended the design of negatively refracting electromagnetic metamaterials to the area of bending waves propagating at the surface of elastic thin-plates with negative density and Young modulus (which could be obtained via homogenization of locally resonant structures). Our proposal is an alternative to the design of transformation based elastic metamaterials within which pressure and shear elastodynamic waves are in general fully coupled [5,7]. We hope it will foster research efforts in these directions.

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