# Radiation in transition of charged particles through rough interfaces

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A theory of transition radiation is presented for the rough interface having sufficiently sloping and smooth inhomogeneities with large curvature radius in each point of the surface. Characteristic size of roughness slowly varies over the distance of the emitted quantum wavelength. The advantage of this method is that no limitation is imposed on the dielectric constants of media. The general case of an interface with two-dimensional roughness is considered. The physical picture of the radiation from a rough surface is determined by both longitudinal and transverse effects. Angular and spectral distribution of the intensity are obtained under penetration of a charge into the target at arbitrary angle, both for the forward and backward radiation. For periodically distributed roughness, specific examples of expansion coefficients are considered, characterizing the type of surface. For random surfaces, statistical parameters of the radiation field are found (averaged over the ensemble of surfaces). Particularly, for the two-dimensional Gaussian distribution of roughness, the average intensity of radiation is obtained. All our expressions transform into known ones when the interface is plane. Numerical calculations were carried out whose results agree with experimental data. The research is practically important, since its results may be used for diagnostics of surfaces.

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## I. INTRODUCTION

Interaction of uniformly moving charged particles with realistic interfaces between media, depending on physical conditions under which it takes place leads to such radiative effects as, e.g., transition radiation, Vavilov-Cherenkov radiation, diffraction radiation (see, e.g., Refs. [1,2]), radiation on surface roughness [3,4]. Under certain conditions an interference between these radiations occurs in spite of the fact that one or another radiation dominates.

The very first experiments (see, e.g., Ref. [5]) have indicated that polarization essentially depends on the surface treatment quality. Roughness of the interface was shown to strongly affect the intensity and polarization of the transition radiation. No description of this roughness effect was given before. Particularly, Ginzburg and Tsytovich have observed in their monograph [2] that they are leaving aside the important for applications issues of transition radiation in the presence of various external structures. The same authors also noted that transition radiation from regular and statistically rough interfaces still needs to be described.

All phenomena caused by interface roughness are important and must be studied, as the investigation of nonideal surfaces is one of the rapidly developing directions in optics and solid state physics. Study of this problem is of practical interest too, as it can promote appearance of one more way to determine the surface purity; one can at least hope that dependences of various parameters of radiation from a rough interface will turn out to be useful for comparison of the theory with experimental data (see, e.g., Refs. [5,6]).

The physical picture of this radiation has been considered in detail by the author (see, e.g., Ref. [3,7]).

Analysis of the problem of deviation of an interface from an ideal plane is complicated from the theoretical point of view, so the study of radiation from rough surfaces is performed by various approximate methods depending on the nature of inhomogeneities.

In recent years we realized a common mathematical approach to problems of emission at interaction of charged particles with rough interfaces in the approximation where dielectric constants of media differ from each other insignificantly (see, e.g., Refs. [7,8]). An advantage of this approach is that no restrictions are imposed on the typical sizes of interface roughness.

In the present paper we develop a theory of radiation of charged particles [9] in the case of sufficiently sloping and smooth roughness of interfaces with large curvature radius at every point of surface when typical sizes of roughness vary little at distances of the order of the wavelength of emitted photons. An essential advantage of this approach is the absence of restrictions on dielectric constants of media.

## II. STARTING EXPRESSIONS FOR RADIATION FIELDS

Let us consider transition radiation from interfaces between media with arbitrary dielectric constants by means of generalization of the Kirchhoff approximation (see, e.g., Ref. [10]) known in the light scattering theory.

The interface between two media with dielectric constants $\varepsilon_1$ and $\varepsilon_2$ (magnetic permeabilities are assumed to be equal to unity) is determined by an equation $z=f(x,y)$, where the function $f(x,y)$ may be either periodic or a statistical function of the surface coordinates. Without violating generality, we can choose the rectangular coordinate system with the $z$ axis directed from the first medium into the second so that the plane $z=0$ is coincident with the mean level of the rough surface $z=f(x,y)$ [$f(x,y)$ being the deviations of surface points from the mean plane $z=0$], and the particle velocity $\mathbf{v}$ directed from the first medium into the second is in the plane $(x,z)$ making an angle $\psi$ with the $z$ axis.

The essence of the approximation consists in representa-

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tion of a sufficiently sloping interface with roughness of horizontal size $l$ (exceeding the heights of roughness $f$) and curvature radius at every point of surface much longer than the pseudophoton wavelength $\lambda$ by a surface with differently oriented plane areas replacing the roughness. Hence, the surface roughness vary little over the distances of the order of the pseudophoton wavelength.

The effects of shadowing and multiple scattering are not taken into account, meaning that surface roughness must be sloping enough. This limitation is equivalent to the requirement that correlation between the deviations of rough surface from its average plane must be high enough. We assume this requirement fulfilled, and also demand that roughness correlation radius is much smaller than the surface dimensions. In addition, as the particle field in contrast to that of a plane electromagnetic wave depends on the distance from the trajectory, it is necessary that the transversal dimension $\rho_{eff}$ of the particle field (see, e.g., Refs. [1,7]) be less than the typical size $l$ of the roughness, while the coherent length $l_{coh}$ (see,e.g., Refs. [1,7]) exceeds the height of the roughness, $f$.

The conditions that the transversal field dimensions of a moving particle are much smaller than the transversal characteristic size of roughness, and coherence length exceeds the heights of roughness, certainly impose limitations on the combination of parameters:

$$\left|\frac{\lambda\beta\sqrt{\varepsilon}}{l\sqrt{1-\beta^2\varepsilon}}\right|<1,$$

$$\left|\frac{\lambda\beta\sqrt{\varepsilon}\cos\psi}{f(1-\beta\sqrt{\varepsilon}\cos\psi\cos\theta)}\right|>1, \tag{1}$$

where $\beta=v/c$, where $c$ is the speed of light and $\theta$ is the angle of radiation. These limitations are taken into account in obtaining the numerical results as given in the Sec. V, at various incidence angles and observations in the plane of incidence of a charged particle.

The field at each point of the rough surface is assumed to be the same as on the tangent plane drawn at this point. For every area of the tangent plane, as for a section of an infinite plane surface, one can write the field distribution taking into account the orientation of the area with respect to the direction of propagation of pseudophotons. Starting from this distribution, using Green's vector formula [11], we can obtain, by means of integration over the surfaces of all areas, the radiation field in the point of observation $\mathbf{R}$ above any point of the rough surface,

$$\mathbf{E}(R)=\mp\frac{1}{4\pi}\int_f\left[i\frac{\omega}{c}(\mathbf{n}\times\mathbf{H})\Phi+(\mathbf{n}\times\mathbf{E})\times\mathrm{grad}\Phi\right.$$
$$\left.+(\mathbf{n}\cdot\mathbf{E})\mathrm{grad}\Phi\right]df, \tag{2}$$

where $\mathbf{n}=(n_x=-\gamma_x/\sqrt{1+\gamma_x^2+\gamma_y^2},\ n_y=-\gamma_y/\sqrt{1+\gamma_x^2+\gamma_y^2},\ n_z=1/\sqrt{1+\gamma_x^2+\gamma_y^2})$ is the unit normal to the surface $f$ in an arbitrary point, $\gamma_x=\partial f/\partial x$, and $\gamma_y=\partial f/\partial y$; $\mathbf{n}$ makes an acute angle with the $z$ axis. $\mathbf{E}$ and $\mathbf{H}$ are the radiation fields on the surface $f$, $\Phi=e^{ikR}/R$, $R$ being the distance between the point of observation and the current point of the surface, $\mathbf{k}$ is the wave vector of emitted quantum $(k=\omega\sqrt{\varepsilon}/c)$, $\omega$ is the frequency of the emitted photon; differentiation in grad$\Phi$ is performed over the surface points. In order to simplify the formulas we omitted the time factor $\exp(-i\omega t)$ and the subscripts 1 and 2 of the fields and the characteristics of the first (the upper sign before the integral) and the second (the lower sign before the integral) medium, respectively. Using the relation $df=dxdy\sqrt{1+\gamma_x^2+\gamma_y^2}$, we pass in Eq. (2) to the integration over the underlying plane.

The Green formula gives a correct result for either backward (into the first medium) or forward (into the second medium) emission on whether we close the rough, on an average plane, surface in infinity at the left $(z<0)$ or at the right $(z>0)$. Because of the approximation made, the field at every point of the rough surface is the same as in the infinite plane replacing in the given point a section of the rough surface; this means that it is the known radiation fields $\mathbf{E}$ and $\mathbf{H}$ for a planar interface that enter the integrand of formula (2) (see, e.g., Chap. 4 in Refs. [1] and [12]).

## III. INTERFACE WITH PERIODIC ROUGHNESS

Let us consider transition radiation in the case where the interface is described by a function $f(X,Y)$, periodic with respect to both variables $X$ and $Y$ with the periods $l_x$ and $l_y$, respectively. Coordinates of points of the rough surface are denoted by $X$, $Y$, $Z$. With use of the Weyl expansion of the scalar Green function in plane waves,

$$\Phi=\frac{i}{2\pi}\int_{-\infty}^{\infty}e^{ik_x(x-X)+ik_y(y-Y)+ik_z|z-Z|}\frac{dk_xdk_y}{k_z} \tag{3}$$

and having in mind that $\mathbf{H}(k)=(c/\omega)[\mathbf{k}\times\mathbf{E}(\mathbf{k})]$, we obtain

$$\mathbf{E}_{1,2}(R)=\pm\frac{1}{8\pi^2}\int_{-\infty}^{\infty}\mathbf{L}'_{1,2}e^{i(k'_x-k_x)X+i(k'_y-k_y)Y+i(k'_z-k_z)f(X,Y)}$$
$$\times e^{(ik_xx+ik_yy+ik_zz)}dXdYdk'_xdk'_ydk_xdk_y, \tag{4}$$

where

$$\mathbf{L}'_{1,2}=\frac{\sqrt{1+\gamma_x^2+\gamma_y^2}}{2k'_zk_z}\{\mathbf{n}\times[\mathbf{k}'\times\mathbf{E}_{1,2}(\mathbf{k}')]-\mathbf{k}[\mathbf{n}\cdot\mathbf{E}_{1,2}(\mathbf{k}')]$$
$$+\mathbf{k}\times[\mathbf{n}\times\mathbf{E}_{1,2}(\mathbf{k}')]\}. \tag{5}$$

Here $\mathbf{E}_{1,2}(\mathbf{k}')$ are the Fourier components of radiation fields from a planar boundary in, respectively, the first and second medium (for simplification the wave vector subscripts denoting media are omitted).

Let the expansion of the product of periodic functions entering the integral (4) into a double Fourier series be known as

$$\mathbf{L}'_{1,2}e^{i(k'_z-k_z)f(X,Y)}=\sum_{m,s=-\infty}^{\infty}\mathbf{L}'_{ms1,2}(\mathbf{k})e^{impX+istY}, \tag{6}$$

where $p=2\pi/l_x$, $t=2\pi/l_y$, and $\mathbf{L}'_{ms1,2}(\mathbf{k})$ are the expansion coefficients depending on the shape of surface, with $m,s=0,\pm1,\pm2,\ldots$. Substituting Eq. (6) into Eq. (4) and using the definition of $\delta$ function, we obtain for Fourier components of the radiation field the following expression:

$$\mathbf{E}_{1,2}(\mathbf{k})=\frac{1}{2}\sum_{m,s}\mathbf{L}'_{ms1,2}(\mathbf{k}), \tag{7}$$

where $k_x=k'_x+mp$, $k_y=k'_y+st$. If the coefficients of the subsidiary Fourier expansion

$$e^{i(k'_z-k_z)f(X,Y)}=\sum_{m,s}L_{ms1,2}e^{impX+istY} \tag{8}$$

are known, the coefficients $L'_{ms1,2}$ can be found in terms of $L_{ms1,2}$ by differentiating with respect to $X$ and $Y$ (8) and composing the left-hand side of expression (6).

Let us mark out the polarization planes as in the case of a planar interface. These are the parallel polarization ($\|$) with the electric vector lying in the radiation plane (containing the wave vector of the emitted quantum and the normal to the interface) and the perpendicular one ($\perp$) with the electric vector perpendicular to the radiation plane. It should be noted that when determining the projections of the electric field on the tangential plane the problem of transition radiation in a rectangular coordinate system has been incidentally solved and the connection between these formulas and conventional fields in an oblique-angled coordinate system has been found (formulas in the rectangular coordinate system are symmetric and relatively simpler). For a planar interface this problem was solved in Ref. [13].

So, we obtain the following expressions for the components of the field vectors parallel and perpendicular to the radiation plane:

$$E^{\|}_{t1,2}=\frac{c}{2\omega\sqrt{\varepsilon_{1,2}}\cos\theta_{1,2}}\sum_{m,s}L_{ms1,2}\Gamma_{ms1,2},$$

$$E^{\perp}_{t1,2}=\frac{c}{2\omega\sqrt{\varepsilon_{1,2}}\cos\theta_{1,2}}\sum_{m,s}L_{ms1,2}N_{ms1,2}, \tag{9}$$

where

$$\Gamma_{ms1,2}=\frac{c}{2\omega\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2}}\Bigg\{[E_{q'1,2}\cos(\varphi-\varphi')+E_{x1,2}\cos\varphi]P_{ms1,2}-\frac{\sin\theta'_{1,2}}{\cos\theta'_{1,2}}(E_{q'1,2}+E_{x1,2}\cos\varphi')\times(A_{ms1,2}\cos\varphi+B_{ms1,2}\sin\varphi)\Bigg\},$$

$$N_{ms1,2}=\frac{c}{2\omega\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2}}\Bigg\{[E_{q'1,2}\sin(\varphi-\varphi')+E_{x1,2}\sin\varphi]P_{ms1,2}-\frac{\sin\theta'_{1,2}}{\cos\theta'_{1,2}}(E_{q'1,2}+E_{x1,2}\cos\varphi')\times(A_{ms1,2}\sin\varphi-B_{ms1,2}\cos\varphi)\Bigg\},$$

$$P_{ms1,2}=\pm\frac{\omega^2 l_{coh}}{c^2\beta_z}\sqrt{\varepsilon_{1,2}}(\cos\theta'_{1,2}+\cos\theta_{1,2})\times(1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2}\pm\beta_z\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2}),$$

$$A_{ms1,2}=-mp\frac{\omega l_{coh}}{c\beta_z}(1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2}\pm\beta_z\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2}),$$

$$B_{ms1,2}=-st\frac{\omega l_{coh}}{c\beta_z}(1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2}\pm\beta_z\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2}),$$

$$l_{coh}=\frac{c\beta_z}{\omega}(1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2}\pm\beta_z\sqrt{\varepsilon_{1,2}}\cos\theta_{1,2})^{-1}, \tag{10}$$

where $E_{q'1,2}$ and $E_{x1,2}$ are the Fourier components of the electric field from a planar interface in an oblique-angled coordinate system,

$$E_{q'1,2}=\frac{ie\beta_z}{\pi^2 c}\sin\theta'_{1,2}\cos\theta'_{1,2}\frac{(\varepsilon_{2,1}-\varepsilon_{1,2})(1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2}\mp\beta_z\sqrt{\varepsilon_{2,1}-\varepsilon_{1,2}\sin^2\theta'_{1,2}})}{(1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2})^2-\beta_z^2(\varepsilon_{2,1}-\varepsilon_{1,2}\sin^2\theta'_{1,2})}\times\frac{(1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2}\pm\beta_z\sqrt{\varepsilon_{2,1}-\varepsilon_{1,2}\sin^2\theta'_{1,2}}-\beta_z^2\varepsilon_{1,2})\cos\theta'_{1,2}\pm\beta_x\beta_z\varepsilon_{1,2}\cos\theta'_{x1,2}}{[(1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2})^2-\beta_z^2\varepsilon_{1,2}\cos^2\theta'_{1,2}](\varepsilon_{2,1}\cos\theta'_{1,2}+\sqrt{\varepsilon_{1,2}\varepsilon_{2,1}-\varepsilon_{1,2}^2\sin^2\theta'_{1,2}})},$$

$$E_{x1,2}=\mp\frac{ie\beta_x\beta_z^2\sqrt{\varepsilon_{1,2}}}{\pi^2 c}\cos\theta'_{1,2}\frac{(\varepsilon_{2,1}-\varepsilon_{1,2})(1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2}\mp\beta_z\sqrt{\varepsilon_{2,1}-\varepsilon_{1,2}\sin^2\theta'_{1,2}})}{(1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2})^2-\beta_z^2(\varepsilon_{2,1}-\varepsilon_{1,2}\sin^2\theta'_{1,2})}\times\{[(1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2})^2-\beta_z^2\varepsilon_{1,2}\cos^2\theta'_{1,2}](\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2}+\sqrt{\varepsilon_{2,1}-\varepsilon_{1,2}^2\sin^2\theta'_{1,2}})\}^{-1}, \tag{11}$$

where $e$ is the electron charge. The upper signs in these expressions correspond to the field to the left of the interface, while the lower signs correspond to the field in the second medium. The direction of incidence of the particle is determined by the quantities $\beta_x=(v/c)\sin\psi$, $\beta_z=(v/c)\cos\psi$, and the direction of emission is determined by directing cosines of the wave vector $\mathbf{k}$ ($\cos\theta_x=\sin\theta\cos\varphi$, $\cos\theta_y=\sin\theta\sin\varphi$, $\cos\theta_z=\cos\theta$) with $\varphi$ being the angle between the $x$ axis and the tangential component of the wave vector $\mathbf{q}$ of the emitted quantum. The angles $\theta,\varphi$ are related to the angles $\theta',\varphi'$ as follows:

$$\cos\theta'_x=\cos\theta_x-mp\frac{c}{\omega\sqrt{\varepsilon}},$$

$$\cos\theta'_y=\cos\theta_y-st\frac{c}{\omega\sqrt{\varepsilon}},$$

$$\cos^2\theta'=\cos^2\theta-\frac{c^2}{\omega^2\varepsilon}[(mp)^2+(st)^2]+2\frac{c}{\omega\sqrt{\varepsilon}}(mp\cos\theta_x+st\cos\theta_y). \tag{12}$$

Let us now calculate the energy flux of the transition radiation through the planes $z\to\pm\infty$. The Pointing vector of radiation with different polarizations has, by definition, the form

$$\mathbf{S}^{\parallel}_{1,2}=(\mathbf{E}^{\parallel}_{t1,2}\times\mathbf{H}^{\perp}_{t1,2}),$$
$$\mathbf{S}^{\perp}_{1,2}=(\mathbf{E}^{\perp}_{t1,2}\times\mathbf{H}^{\parallel}_{t1,2}). \tag{13}$$

Making use of these formulas, we obtain the following expression for the spectral densities of the radiation energy in the frequency range $d\omega$ into the solid angle $d\Omega$,

$$\frac{dI^{\parallel}_{\omega,\mathbf{k}1,2}}{d\omega d\Omega}=\frac{\pi^2c^3}{4\omega^2\sqrt{\varepsilon_{1,2}}\cos^2\theta_{1,2}}\sum_{m,s}\left|\frac{L_{ms1,2}}{\cos\theta'_{1,2}}\left\{[E_{q'1,2}\cos(\varphi-\varphi')+E_{x1,2}\cos\varphi]P_{ms1,2}-\frac{\sin\theta'_{1,2}}{\cos\theta'_{1,2}}(E_{q'1,2}+E_{x1,2}\cos\varphi')(A_{ms1,2}\cos\varphi+B_{ms1,2}\sin\varphi)\right\}\right|^2,$$

$$\frac{dI^{\perp}_{\omega,\mathbf{k}1,2}}{d\omega d\Omega}=\frac{\pi^2c^3}{4\omega^2\sqrt{\varepsilon_{1,2}}}\sum_{m,s}\left|\frac{L_{ms1,2}}{\cos\theta'_{1,2}}\left\{[E_{q'1,2}\sin(\varphi-\varphi')+E_{x1,2}\sin\varphi]P_{ms1,2}-\frac{\sin\theta'_{1,2}}{\cos\theta'_{1,2}}(E_{q'1,2}+E_{x1,2}\cos\varphi')(A_{ms1,2}\sin\varphi-B_{ms1,2}\cos\varphi)\right\}\right|^2. \tag{14}$$

These formulas determine the intensities of transition radiation backward and forward. They are significantly simplified if one observes the emission in the plane of incidence of the particle, i.e., if $\varphi=0$. At normal incidence of the particle when $\beta_x=0$ and $\beta_z=\beta$, expressions (14) take relatively simple form. If one considers in addition a surface wave in only one direction, say $f=f(Y)$, then at observation of the emission in vacuum ($\varepsilon_2=1$, $\varepsilon_1=\varepsilon$) one obtains

$$\frac{dI^{\parallel}_{\omega,\mathbf{k}}}{d\omega d\Omega}=\frac{e^2\beta^2|1-\varepsilon|^2\sin^2\theta}{4\pi^2c|1-\beta\cos\theta|^2\cos^2\theta}\sum_s\left|\frac{L_s(1-\beta\sqrt{\varepsilon-\sin^2\theta'}-\beta^2)(\cos\theta+\cos\theta')\cos\theta'}{(1+\beta\cos\theta')(1-\beta\sqrt{\varepsilon-\sin^2\theta'})(\varepsilon\cos\theta'+\sqrt{\varepsilon-\sin^2\theta'})}\right|^2,$$

$$\frac{dI^{\perp}_{\omega,\mathbf{k}}}{d\omega d\Omega}=\frac{e^2\beta^2|1-\varepsilon|^2}{4\pi^2c|1-\beta\cos\theta|^2}\sum_s\left|\frac{s\dfrac{\lambda}{l_y}L_s(1-\beta\sqrt{\varepsilon-\sin^2\theta'}-\beta^2)(1+\cos\theta\cos\theta')}{(1+\beta\cos\theta')(1-\beta\sqrt{\varepsilon-\sin^2\theta'})(\varepsilon\cos\theta'+\sqrt{\varepsilon-\sin^2\theta'})}\right|^2, \tag{15}$$

where

$$\cos^2\theta'=\cos^2\theta-\left(s\frac{\lambda}{l_y}\right)^2,$$

$$\lambda=\frac{2\pi c}{\omega}. \tag{16}$$

So for complete solution of the problem one needs to know only the coefficients $L_{ms}$ of the subsidiary expansion in which the shape of surface is taken into account. Consideration of specific examples of expansion coefficients is given in Ref. [7]. Here it can only be noted that for an interface sinusoidal in both directions $f(X,Y)=a\cos(pX)+b\cos(tY)$, these coefficients are expressed in a simple manner in terms of the Bessel functions:

$$L_{ms1,2}=i^{m+s}J_m\left(\frac{a}{l_{coh}}\right)J_s\left(\frac{b}{l_{coh}}\right). \tag{17}$$

In the case of one-dimensional sinusoidal roughness, assuming $f$ to depend only on $Y$, we obtain

$$L_{ms1,2} \rightarrow L_{s1,2} = i^s J_s(\rho_{1,2}), \tag{18}$$

where $J_s(\rho_{1,2})$ is the Bessel function of the $s$th order and its argument

$$\rho_{1,2} = \frac{b}{l_{coh}} = \frac{b\omega}{c\beta_z}(1 - \beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2} \pm \beta_z\sqrt{\varepsilon_{1,2}}\cos\theta_{1,2}) \tag{19}$$

is the roughness parameter representing the ratio of the sinusoid amplitude to the coherence length. For small $\rho$ noticeable values are obtained with first terms of the series (amplitudes of spectra decrease rapidly with increasing number, and for calculation one can retain small numbers $m$, $s$). For large values of $\rho$ the first terms are small while the series converges more slowly. At some values of $\rho$ the emission vanishes in certain directions. Equation (19) shows that the effect of large roughness at small $\psi$ is the same as the effect of small roughness at large $\psi$. In expressions for spectral densities of the emission energy the term of the series with $m=s=0$ gives the intensity of transition radiation modified by the presence of interface roughness. It differs from the conventional planar interface formula by the factor $J_0^2(\rho)$ which reduces the intensity of the transition radiation. The formulas pass to the expressions for a planar surface at $\rho=0$.

It should be noted that the expressions for spectral densities of the emission energy could have been obtained if one used instead of Eq. (9) the fields at long distances $|\mathbf{R}_0| = |\mathbf{r} + \mathbf{R}|$, where $\mathbf{R}_0$ is the position vector drawn from the origin to the point of observation and $\mathbf{r}$ is a point of the surface.

## IV. INTERFACE WITH STATISTICAL ROUGHNESS

Let us consider the radiation from a statistically rough interface of two media described by an equation $z=f(x,y)$, where $f$ is a random stationary differentiable function of coordinates, values of which range about the plane $z=0$. Here the solution of the problem of transition radiation differs somewhat from the calculation of radiation fields on a regular interface. Actually, if a periodic boundary is given, it determines unambiguously either the shape of the surface or the radiation field, whereas a statistically rough surface can be given by some parameters of deviations of surface points from the mean plane (by distribution probability density for these deviations, by correlation function). Such setting of the surface determines an infinite ensemble of various samples of surfaces similarly described statistically. Each sample of an ensemble gives a certain emission pattern which does not, generally, coincide with the emission pattern of an other sample. So, one needs to find the statistical characteristics of an ensemble of emission patterns provided that the statistical parameters of the surface shape are given. According to the statement of the problem, it is the mean value and the average intensity of radiation fields that are of basic interest here, while it is assumed the averages over the surface coincide with averages over the ensemble.

To determine the radiation fields we use formula (4), where $f(X,Y)$ is now a function of randomly rough surface. This expression may be transformed so that the shape of surface enters only the power of the exponential factor,

$$\mathbf{E}_{1,2}(R) = \pm\frac{1}{16\pi^2}\int_{-\infty}^{\infty}\{(\boldsymbol{\varkappa}\cdot\mathbf{E}_{1,2})(\mathbf{k}'-\mathbf{k}) + \boldsymbol{\varkappa}(\mathbf{k}\cdot\mathbf{E}_{1,2}) - \mathbf{E}_{1,2} \times[\boldsymbol{\varkappa}\cdot(\mathbf{k}'+\mathbf{k})]\}\frac{\exp[i(\boldsymbol{\varkappa}_\perp\cdot\mathbf{r}) + i\varkappa_z f(\mathbf{r})]}{k'_z k_z \varkappa_z} \times e^{(ik_x x + ik_y y + ik_z z)} dk'_x dk'_y dk_x dk_y d\mathbf{r}, \tag{20}$$

where $\boldsymbol{\varkappa}_\perp = (k'_x - k_x, k'_y - k_y)$, $\mathbf{r}=(X,Y)$, and $\varkappa_z = [(\omega - k'_x v_x)/v_z] - k_z$. Let us now average Eq. (20) over all realizations of $f(\mathbf{r})$. Denoting the averaging by a line, we obtain

$$\overline{\mathbf{E}_{1,2}(R)} = \pm\frac{1}{2}\int_{-\infty}^{\infty}\mathbf{E}_{1,2}e^{ik_x x + ik_y y + ik_z z}h(\varkappa_z)\frac{dk_x dk_y}{k_z}, \tag{21}$$

where

$$h(\varkappa_z) = \int e^{i\varkappa_z f(\mathbf{r})}W(f)df = \overline{e^{i\varkappa_z f(\mathbf{r})}}, \tag{22}$$

$W(f)$ is the function of distribution of surface points over heights, and $h(\varkappa_z)$ is the corresponding characteristic function (see, e.g., Ref. [10]). In obtaining Eq. (21) the definition of the $\delta$-function has been used. It should be noted that the mean value of the radiation field is represented as a superposition of plane waves, each of them with its coefficient $h(\varkappa_z)$.

For radiation fields at long distances $R_0$ we have in the first and second media, respectively,

$$\mathbf{E}_{1,2}(R_0) = \mp\frac{ie^{ikR_0}}{8\pi R_0}\int_{-\infty}^{\infty}\{(\boldsymbol{\varkappa}\cdot E_{1,2})(\mathbf{k}'-\mathbf{k}) + \boldsymbol{\varkappa}(\mathbf{k}\cdot\mathbf{E}_{1,2}) - \mathbf{E}_{1,2}[\boldsymbol{\varkappa}\cdot(\mathbf{k}+\mathbf{k}')]\}\frac{1}{k'_z\varkappa_z}e^{i(\boldsymbol{\varkappa}_\perp\cdot\mathbf{r})}e^{i\varkappa_z f(\mathbf{r})}dk'_x dk'_y d\mathbf{r}, \tag{23}$$

while for the average radiation fields we obtain

$$\overline{\mathbf{E}_{1,2}(R_0)} = \mp i\pi\frac{e^{ikR_0}}{R_0}\mathbf{E}_{1,2}(\mathbf{k}')h(\varkappa_z). \tag{24}$$

They differ from the fields in the case of a planar interface by a factor $h(\varkappa_z)$.

So, the average radiation fields from randomly rough interfaces can be calculated if either distribution function for the quantity $f$ [the density of probability $W(f)$] or the corresponding characteristic function is known.

When the transition radiation from a statistically rough boundary is considered, the mean intensity of the field (mean value of squared fields) is much more interesting. The radiation energy in the frequency range $d\omega$ into a solid angle $d\Omega$ is

$$dI_{\omega,\mathbf{k}1,2} = c\sqrt{\varepsilon_{1,2}}|\mathbf{E}_{1,2}(R_0)|^2 R_0^2 d\omega d\Omega. \tag{25}$$

We will perform averaging of this expression over samples of surfaces by means of a binary distribution function [a probability $W(f,f')$ that at two points determined by two-dimensional position vectors $\mathbf{r}$ and $\mathbf{r}'$ the heights of the surface will be $f$ and $f'$]. For a very general case of a two-dimensional normal distribution of deviation of surface points in height $f$ from the mean plane $z=0$ (see, e.g., Ref. [10]).

$$W(f,f')=\frac{1}{2\pi f_0^2\sqrt{1-F^2}}\exp\left[-\frac{f^2-2Fff'+f'^2}{(1-F^2)f_0^2}\right], \tag{26}$$

all the information on statistical properties of the surface is determined by the mean-square deviation $\overline{f}^2=f_0^2$ and the correlation coefficient $F$ of heights in two different points $\mathbf{r}$ and $\mathbf{r}'$ of the surface. The correlation coefficient, in the general case of spatially uniform surfaces ($F$ depends on the difference of arguments), is determined by the relation

$$\overline{f(\mathbf{r})f'(\mathbf{r}')}=f_0^2F\left(\frac{X-X'}{l_x},\frac{Y-Y'}{l_y}\right). \tag{27}$$

Here $l_x$ and $l_y$ are the correlation radii, i.e., typical distances at which the correlation coefficient varies essentially. If $l_x=l_y=l$, the surface is statistically isotropic and $F$ depends on $|\boldsymbol{\zeta}|/l$, where $\boldsymbol{\zeta}=r-r'$. Correlation coefficient is equal to unity when its argument is zero and drops to zero when $|\boldsymbol{\zeta}|$ exceeds the correlation radius $l$.

After substitution of Eq. (23) into Eq. (25), transformation to new variables $\boldsymbol{\zeta}$, $\mathbf{r}'$, and integration over $\mathbf{r}'$, one obtains for the mean value of the radiation intensity for parallel and perpendicular polarizations, respectively,

$$dI^{\parallel,\perp}_{\omega,\mathbf{k}1,2}=\frac{c\sqrt{\varepsilon_{1,2}}}{4}\int_{-\infty}^{\infty}D^{\parallel,\perp}_{1,2}e^{i(\boldsymbol{\varkappa}_\perp\cdot\boldsymbol{\zeta})}\times\exp\left\{-\varkappa_z^2f_0^2\left[1-F\left(\frac{|\boldsymbol{\zeta}|}{l}\right)\right]\right\}dk_x'dk_y'd\boldsymbol{\zeta}, \tag{28}$$

where

$$D^{\parallel}_{1,2}=\left|\frac{1}{2k_z'\varkappa_z}\left(\hat{\mathbf{q}}-\hat{\mathbf{z}}\frac{q}{k_z}\right)(G_{x1,2}E_{x1,2}+G_{q'1,2}E_{q'1,2})\right|^2,$$

$$D^{\perp}_{1,2}=\left|\frac{1}{2k_z'\varkappa_z}(Q_{x1,2}E_{x1,2}+Q_{q'1,2}E_{q'1,2})\right|^2,$$

$$G_{x1,2}=\frac{1}{q}[k_xC_{1,2}+(k_x'k_x+k_y'k_y-q^2)k_x'V_{1,2}],$$

$$G_{q'1,2}=\frac{1}{q}\left[\frac{k_x'k_x+k_y'k_y}{q'}C_{1,2}+(k_x'k_x+k_y'k_y-q^2)q'V_{1,2}\right],$$

$$Q_{x1,2}=\frac{1}{q}[k_yC_{1,2}+(k_x'k_y-k_y'k_x)k_x'V_{1,2}],$$

$$Q_{q'1,2}=\frac{1}{q}(k_x'k_y-k_y'k_x)\left(\frac{C_{1,2}}{q'}+q'V_{1,2}\right)$$

$$C_{1,2}=q^2-q'^2-(k_z'+k_z)\varkappa_z,$$

$$V_{1,2}=1-\frac{1}{k_z'}\left(\frac{\omega-k_x'v_x}{v_z}\right). \tag{29}$$

Here, for simplification, the subscripts 1 and 2 of the wave vectors are omitted. Expressions (28) and (29) determine completely the intensity of emission from a rough interface forward and backward if the function $W$ is known. By passing in Eq. (28) to new variables $\zeta=\sqrt{\zeta_x^2+\zeta_y^2}$, $\chi$, using the formula

$$\int_0^{2\pi}\exp[i(\boldsymbol{\varkappa}_\perp\cdot\boldsymbol{\zeta})]\zeta d\zeta d\chi=2\pi\int_0^\infty J_0(\varkappa_\perp\zeta)\zeta d\zeta, \tag{30}$$

and denoting $\zeta/l$ by $\eta$, one arrives at

$$\frac{dI^{\parallel,\perp}_{\omega,\mathbf{k}1,2}}{d\omega d\Omega}=\frac{\pi c\sqrt{\varepsilon_{1,2}}l^2}{2}\int_{-\infty}^{\infty}dk_x'dk_y'\int_0^\infty D^{\parallel,\perp}_{1,2}J_0(\varkappa_\perp l\eta)\times e^{-\varkappa_z^2f_0^2[1-F(\eta)]}\eta d\eta. \tag{31}$$

When the emission is observed in the plane of incidence of the particle, $\varphi=0$ must be taken in all these formulas. At $\eta=0$ the correlation coefficient is equal to unity and Eq. (31) goes to the expression for a planar interface. When $\eta\to\infty$, the correlation coefficient equals zero and we obtain

$$\frac{dI^{\parallel,\perp}_{\omega,\mathbf{k}1,2}}{d\omega d\Omega}=\frac{dI^{\parallel,\perp}_{\omega,\mathbf{k}1,2}}{d\omega d\Omega}(\textit{from planar interface})e^{-\varkappa_z^2f_0^2}, \tag{32}$$

while $k_x'=k_x$ in $\varkappa_z$. The expression

$$\int_0^\infty J_0(\varkappa_\perp l\eta)e^{-\varkappa_z^2f_0^2[1-F(\eta)]}\eta d\eta \tag{33}$$

entering the formulas (31) can be represented as

$$e^{-\varkappa_z^2f_0^2}\sum_{n=0}^{\infty}\frac{(\varkappa_zf_0)^{2n}}{n!}\int_0^\infty J_0(\varkappa_\perp l\eta)[F(\eta)]^n\eta d\eta. \tag{34}$$

By taking $F(\eta)=e^{-\eta^2}$ and using the relation

$$\int_0^\infty J_0(gu)e^{-w^2u^2}udu=\frac{1}{2w^2}\exp\left(-\frac{g^2}{4w^2}\right),\quad \mathrm{Re}w^2>0, \tag{35}$$

the following result is obtained:

$$\frac{dI^{\parallel,\perp}_{\omega,\mathbf{k}1,2}}{d\omega d\Omega}=\frac{dI^{\parallel,\perp}_{\omega,\mathbf{k}1,2}}{d\omega d\Omega}\ (\textit{from planar interface})\, e^{-\varkappa_z^2 f_0^2} + \frac{\pi c\sqrt{\varepsilon_{1,2}}l^2}{2}\int_{-\infty}^{\infty} D^{\parallel,\perp}_{1,2} e^{-\varkappa_z^2 f_0^2} \times\sum_{n=1}^{\infty}\frac{(\varkappa_z f_0)^{2n}}{2n!n}\exp\left[-\frac{(\varkappa_\perp l)^2}{4n}\right]dk_x' dk_y'. \quad (36)$$

For small heights of roughness, $f_0 \ll 1/\varkappa_z = l_{coh}$, one can limit the sum in the second term of Eq. (36) by the first term.

If we consider the case where the wavelength of emitted quantum is shorter than the typical sizes of boundary roughness, i.e., $\varkappa_\perp l$, $\varkappa_z f_0$ in Eq. (31) are large, the high-frequency Fourier components of the function $W$ must be taken into account. Correspondingly, it is small $\eta$ values of $W$, where the function $F$ is close to unity and $W$ has a prominent maximum at $f=f'$, that will enter effectively the integrand. Within the corresponding small region of the surface the height $f$ is always close to the height $f'$. For example, for a Gaussian distribution the expression

$$\exp\{-\varkappa_z^2 f_0^2[1-F(\eta)]\} \quad (37)$$

vanishes rapidly when $F$ only slightly differs from unity. Hence, one can expand the correlation function in series near the null of its argument and retain the first nonvanishing term

$$F(\eta)\approx 1+\frac{1}{2}F''(0)\eta^2, \quad (38)$$

with $F''(0)<0$ as a result of general properties of the correlation coefficient. By inserting this expansion into Eq. (31) and performing the integration by means of Eq. (35), we obtain

$$\frac{dI^{\parallel,\perp}_{\omega,\mathbf{k}1,2}}{d\omega d\Omega}=\frac{\pi c\sqrt{\varepsilon_{1,2}}l^2}{2f_0^2|F''(0)|}\int_{-\infty}^{\infty} D^{\parallel,\perp}_{1,2}\frac{1}{\varkappa_z^2} \times\exp\left[-\frac{\varkappa_\perp^2 l^2}{2\varkappa_z^2 f_0^2|F''(0)|}\right]dk_x' dk_y'. \quad (39)$$

In the case of normal correlation, when $F(\eta)=e^{-\eta^2}$ we have $|F''(0)|=2$, and hence,

$$\frac{dI^{\parallel,\perp}_{\omega,\mathbf{k}1,2}}{d\omega d\Omega}=\frac{\pi c\sqrt{\varepsilon_{1,2}}l^2}{4f_0^2}\int_{-\infty}^{\infty} D^{\parallel,\perp}_{1,2}\frac{1}{\varkappa_z^2}\exp\left[\left(-\frac{\varkappa_\perp l}{2\varkappa_z f_0}\right)^2\right]dk_x' dk_y'. \quad (40)$$

Note also that the formulas are simplified if we consider one-dimensional roughness, i.e., $f(x,y)\to f(x)$ or $f(y)$.

All the obtained expressions go to conventional formulas of transition radiation when the interface tends to a plane. They determine completely the emission intensity if the function of distribution of deviations of surface points from the mean plane is known. The inverse is also obvious; experimental study of the distribution of emission enables one to get information on correlation characteristics of the surface. So, a connection is established between the shape of the interface and the characteristics of the radiation in a form allowing concrete calculation.

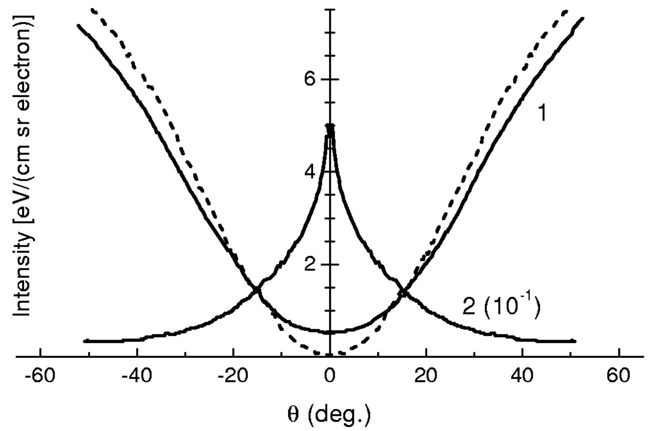


FIG. 1. Angular distribution of radiation. $\lambda=4400$ Å, $\psi=0°$, $f_0=100$ Å, $l/f_0=100$. Ordinates of curves 2 are ten times magnified.

Real interfaces may only be weakly approximated by the surfaces considered. The high sensitivity of radiation to statistical properties of the interface does not allow one to obtain final formulas for the intensity of emission from any surface. A base to construct formalized mathematical schemes served the qualitative notion on appearance of roughness in the process of treatment. Since the process of roughness formation is a consequence of multiple factors exhibiting randomly and to almost the same degree, it has been confined here to consideration of rather general case of normal distribution of heights of roughness with a smooth correlation coefficient of the Gaussian type. So bringing the theory up to formulas for calculations can be successful for only specific models, although the obtained results allow one to understand general regularities of radiation without going into details of the specific structure of the surface.

## V. NUMERICAL RESULTS

The expressions obtained above have been used to evaluate the angular and spectral distribution of the intensity for the transition radiation created by incidence of the electrons having energy 80 keV on the rough interface between vacuum ($\varepsilon_1=1$) and aluminum target. Surface roughness was described either by a periodic function $f(X,Y)=f_0[\cos(2\pi X/l)+\cos(2\pi Y/l)]$, or random function, with correlation coefficient $F(\eta)=e^{-\eta^2}$. The intensity distribution was evaluated numerically in the optical range of spectrum ($\lambda=2800$–$5600$ Å) at various incidence angles and observations in the $(xz)$ plane of incidence of a charged particle.

As an illustration, the distribution curves are shown in Figs. 1 and 2 in the case of periodic roughness. The curves 1 describe the parallel component, while the curves 2 perpendicular component of radiation. For comparison, the polarized component of transition radiation from planar interface is also shown by dotted curves, obtained from the well-

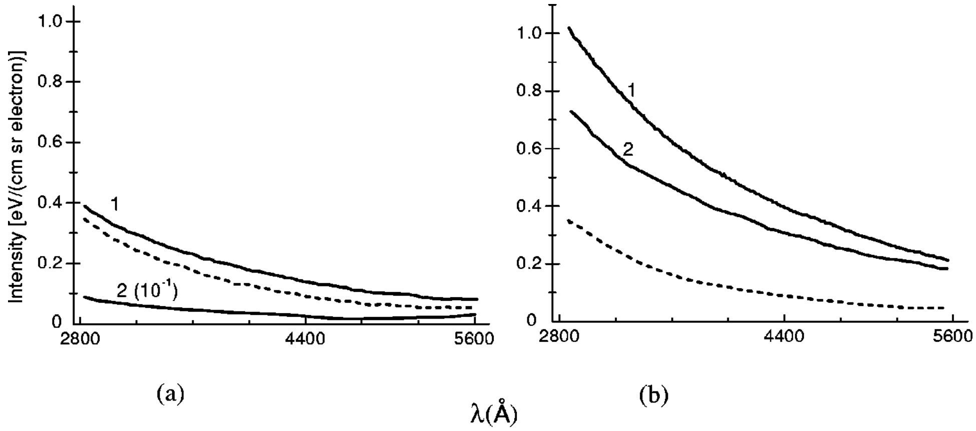


FIG. 2. Spectral distribution of radiation. $\psi=15°$, $\theta=10°$, $f_0=320$ Å; (a) $l/f_0=1000$, (b) $l/f_0=100$. Ordinates of curves 2 in (a) are ten times magnified.

known expressions (see, e.g., Ref. [1]). The perpendicular component under observation in the plane of incidence in the latter case is known to be zero.

Our obtained results show that even a small roughness of interface creates nonzero perpendicular component of radiation. As the roughness height $f_0$ grows and the ratio $l/f_0$ falls, depolarization is observed, whose extent is increased for smaller radiation angles. Thus for the values $f_0=320$ Å, $l/f_0=100$, $\theta=10°$, $\psi=15°$, and $\lambda=4400$ Å, the perpendicular component reaches nearly 75% of the parallel component. The latter in its turn is four times larger than for transition radiation from a planar surface. Note also that at large radiation angles the curves 1 lie below the dotted curves.

In the case of normal incidence of a charged particle ($\psi=0°$), the transition radiation at the angle $\theta=0°$ is completely unpolarized. Radiation pattern in this case is symmetric relative to the normal to average plane of interface, i.e., to the $z$ axis, like in the case of planar surface. However, when the incidence angle $\psi$ is nonzero, the symmetry against $z$ axis is violated.

Note also that parallel component of the radiation smoothly falls off with increase of the wavelength $\lambda$ (as in the case of planar surface). Meanwhile for the perpendicular component this fall of intensity is not always smooth.

Almost the same behavior is obtained when the rough interface is described by a random function. The only difference with the regular case is that perpendicular component here always decreases smoothly with growth of $\lambda$.

Thus it may be concluded that roughness of the interface essentially affects the transition radiation.

## VI. DISCUSSION

It is worth discussing the state of the theory and experiment in this area, in order to understand more clearly the ways of their possible development.

The author's theory is based on a method known in optics, but is applied to radiation of charged particles. The key result is that this radiation is formed not only by longitudinal coherence length (which is well known in the theory of transition radiation), but also by transversal distance.

The expressions presented in this paper clearly establish the relations between the interface parameters and radiation characteristics.

This paper claims to be a significant development of paper [7] where dielectric characteristics of two media differ insignificantly. Meanwhile, here these media are taken arbitrary, which makes possible experimental verification of the paper results.

Experimental study of the issues raised in this paper is still unsatisfactory. Known are experiments [5,6] analyzing the radiation created by penetration of electrons into various metals. In these experiments roughness of the interface is shown to affect the intensity and polarization of the transition radiation. However, the roughness density and structure were not taken into account in these experiments. The work [6] dealt merely with a matte-finished surface (where the size of inhomogeneities is of order of the wavelengths in the analyzed range of spectrum, while their density provides visual diffusiveness of the surface).

Both determination of radiation characteristics which surface roughness parameters are given and solution of the inverse problem require establishment of the functional relation between the interface and radiation characteristics. In all mentioned experiments no such dependence was found, probably due to small size of inhomogeneities making difficult their proper description. This fact to some extent holds back the development of theory.

Today only semiqualitative comparison of experiment and theory has been carried out, since the character of interface was not taken into account. Numerical analysis has been performed by the author, using the data of experiment [6].

In view of the total lack of information about the state of the target surface, we have made an attempt to correlate our model results, at least approximately, with experiment. Although some agreement exists, full interpretation of the effects is possible only after detailed specification of the surface roughness in experiments (height distribution and characteristic size of inhomogeneities).

Finally we observe that the obtained results may find application in diagnostics of targets, especially important in analysis of nanostructures. Remember that resolution power in optics is limited by diffraction. Electrons are also known to behave like waves, with wavelength $\lambda = h/mv$; $m = m_0/\sqrt{1-\beta^2}$, where $m_0$ is the electron rest mass and $h = 6.62\times 10^{-27}$erg sec. However, due to small value of $h$, the diffraction of charged particles with energy easily reached in practice will be determined by wavelengths much shorter than for visible light. Therefore the diffraction of charged particles is more weakly expressed.

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## APPENDIX: RESULTS EXPRESSED THROUGH THE FRESNEL COEFFICIENTS

Final results of the paper [see, e.g., Eq. (14)] include expressions (11) for the fields $E_{q'1,2}$ and $E_{x1,2}$ of transition radiation in the case of non-normal incidence on a planar interface between media with arbitrary dielectric constants.

Below, these fields are expressed through the well-known Fresnel coefficients of reflection ($r^{||}_{1,2}$ and $r^{\perp}_{1,2}$) and refraction ($t^{||}_{1,2}$, $t^{\perp}_{1,2}$):

$$r^{||}_{1,2} = \frac{\varepsilon_{2,1}\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2} - \varepsilon_{1,2}\sqrt{\varepsilon_{2,1} - \varepsilon_{1,2}\sin^2\theta'_{1,2}}}{\varepsilon_{2,1}\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2} + \varepsilon_{1,2}\sqrt{\varepsilon_{2,1} - \varepsilon_{1,2}\sin^2\theta'_{1,2}}},$$

$$r^{\perp}_{1,2} = \frac{\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2} - \sqrt{\varepsilon_{2,1} - \varepsilon_{1,2}\sin^2\theta'_{1,2}}}{\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2} + \sqrt{\varepsilon_{2,1} - \varepsilon_{1,2}\sin^2\theta'_{1,2}}},$$

$$(1 + r^{||}_{1,2})\sqrt{\frac{\varepsilon_{1,2}}{\varepsilon_{2,1}}} = t^{||}_{1,2}, \quad 1 + r^{\perp}_{1,2} = t^{\perp}_{1,2}. \tag{A1}$$

As a result, we obtain

$$\begin{aligned}
E_{q'1,2} = {} & \frac{ie\beta_z}{2\pi^2 c}\cos\theta'_{1,2}\Bigg\{\frac{\sin\theta'_{1,2}}{1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2} \pm \beta_z\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2}} + \frac{r^{||}_{1,2}\sin\theta'_{1,2}}{1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2} \mp \beta_z\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2}} \\
& - \frac{t^{||}_{1,2}\sin\theta'_{2,1}}{1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2} \pm \beta_z\sqrt{\varepsilon_{2,1}}\cos\theta'_{2,1}} \mp \frac{\beta_x\cos\theta'_{x1,2}}{\beta_z\cos\theta'_{1,2}}\frac{\varepsilon_{1,2}}{\varepsilon_{2,1}}\Bigg[\frac{\sin\theta'_{1,2}}{1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2} \pm \beta_z\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2}} \\
& \times\left(1 \mp \frac{2\beta_z\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2}\left(\frac{\varepsilon_{2,1}}{\varepsilon_{1,2}} - 1\right)}{1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2} \mp \beta_z\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2}}\right) + \frac{r^{||}_{1,2}\sin\theta'_{1,2}}{1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2} \mp \beta_z\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2}} \\
& - \frac{\frac{\varepsilon_{2,1}}{\varepsilon_{1,2}}t^{||}_{1,2}\sin\theta'_{2,1}}{1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2} \pm \beta_z\sqrt{\varepsilon_{2,1}}\cos\theta'_{2,1}}\Bigg]\Bigg\},
\end{aligned}$$

$$\begin{aligned}
E_{x1,2} = {} & -\frac{ie\beta_x}{2\pi^2 c}\left(\frac{1}{1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2} \pm \beta_z\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2}} + \frac{r^{\perp}_{1,2}}{1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2} \mp \beta_z\sqrt{\varepsilon_{1,2}}\cos\theta'_{1,2}}\right. \\
& \left. - \frac{t^{\perp}_{1,2}}{1-\beta_x\sqrt{\varepsilon_{1,2}}\cos\theta'_{x1,2} \pm \beta_z\sqrt{\varepsilon_{2,1}}\cos\theta'_{2,1}}\right).
\end{aligned} \tag{A2}$$

Here the terms without Fresnel coefficients correspond to the wave generated by the charge itself. Terms including $r^{||}_{1,2}$ and $r^{\perp}_{1,2}$ describe the wave reflected from the interface, while those including $t^{||}_{1,2}$ and $t^{\perp}_{1,2}$ the wave created by a charge either before transition and propagated forward, or after transition and propagated backward. In the case of normal incidence, one obtains the known expressions (2.45e) and (2.45f) of Ref. [2].

Substitution of expressions (A2) in the final results of this paper gives the expressions for transition radiation on rough interfaces, including the direct radiation from the particle itself, and terms due to reflection of this radiation from, and transition through, the surface.

In particular we also express the relations (15) through the following Fresnel coefficients:

$$\frac{dI^{||}_{\omega,\mathbf{k}}}{d\omega d\Omega}=\frac{e^2\beta^2}{16\pi^2 c}\frac{1}{|1-\beta\cos\theta|^2\cos^2\theta}\sum_s\left|L_s(\cos\theta'+\cos\theta)(1-\beta\cos\theta')\right.$$
$$\left.\times\cos\varphi'\left(\frac{\sin\theta'}{1-\beta\cos\theta'}+\frac{r_2^{||}\sin\theta'}{1+\beta\cos\theta'}-\frac{t_2^{||}\sin\theta_1'}{1-\beta\sqrt{\varepsilon}\cos\theta_1'}\right)\right|^2,$$

$$\frac{dI^{\perp}_{\omega,\mathbf{k}}}{d\omega d\Omega}=\frac{e^2\beta^2}{16\pi^2 c}\frac{1}{|1-\beta\cos\theta|^2}\sum_s\left|L_s(1-\beta\cos\theta')\left[(\cos\theta'+\cos\theta)\sin\varphi'-\left(s\frac{\lambda}{l_y}\right)\frac{\sin\theta'}{\cos\theta'}\right]\right.$$
$$\left.\times\left(\frac{\sin\theta'}{1-\beta\cos\theta'}+\frac{r_2^{||}\sin\theta'}{1+\beta\cos\theta'}-\frac{t_2^{||}\sin\theta_1'}{1-\beta\sqrt{\varepsilon}\cos\theta_1'}\right)\right|^2. \tag{A3}$$


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