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Planar Dirac fermions in long-range-correlated random vector potential

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Abstract

We study the behavior of two-dimensional Dirac fermions in the presence of a static long-range-correlated random vector potential. By applying an exact path integral representation for the propagator of a spinor particle we obtain asymptotics of the gauge invariant spectral function and the correlation function of the local density of states, both in the ballistic regime of sufficiently high energies. We also discuss localization properties of the random Dirac wave functions in the complementary zero energy limit and the putative localization scenario.

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1. Introduction

In recent years, the standard theory of electron localization in two dimensions has been extended to the situations where, instead of potential disorder, the fermions are subject to a random vector potential (often referred to as random magnetic field or RMF) whose Gaussian spatial fluctuations are described by a pairwise correlator

$$\langle a_i(\vec{q})a_j(\vec{q}\,')\rangle = w_{ij}(\vec{q})\delta(\vec{q}+\vec{q}\,'), \tag{1}$$

where $w_{ij}(\vec{q}) = (\alpha/|\vec{q}|^\eta)(\delta_{ij} - \frac{q_i q_j}{q^2})$.

This problem was first encountered and then extensively studied in the context of the compressible quantum Hall effect at even denominator filling fractions. In order to describe the ostensibly Fermi liquid-like properties of these strongly correlated electronic states, a new kind of quasiparticles (“composite fermions”) which would

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effectively "see" an ordinary potential disorder as a long-range-correlated RMF described by Eq. (1) with $\eta = 2$ was introduced in [1,2].

Shortly thereafter, a (pseudo)relativistic version of the short-range-correlated, $\eta = 0$, RMF problem had been proposed as the means of describing the integer quantum Hall plateau transitions [3] where it was later identified with the continuum limit of some network models. The intrinsic scale invariance of the $\eta = 0$ problem prompted the use of the powerful machinery of two-dimensional conformal field theory in most of the subsequent work on this topic.

Recently, a novel (this time, long-range-correlated, $\eta = 2$) variant of the relativistic RMF problem has emerged in the theory of localization of the Dirac-like nodal quasiparticles in the mixed state of planar d-wave superconductors.

In the presence of vortices of the d-wave superconducting order parameter, not only do the nodal quasiparticles' energies get semiclassically Doppler shifted due to the circulating supercurrent [4], but also their wave functions acquire intrinsically quantum mechanical Bohm–Aharonov (BA) phases.

In order to account for both effects on equal footing, a singular gauge transformation similar to that of Ref. [2] was implemented in Ref. [5]. In the case of a disordered vortex array, which corresponds to the experimentally well-documented vortex line liquid phase where the vortices are believed to be randomly pinned by columnar or other strong defects, the transformation of Ref. [5] converts the nodal quasiparticles into auxiliary neutral Dirac fermions. The latter, in addition to a random scalar potential accounting for the Doppler shift, are subject to an effective RMF with zero mean described by Eq. (1) where $\eta = 2$ and α is proportional to the areal density of vortices [6–9].

Moreover, a formally similar RMF problem has emerged in the theory of the pseudogap phase of the cuprates described as a plasma of thermally excited vortex–antivortex pairs, the parameter α being proportional to temperature [10,11]. Furthermore, one encounters yet another formal analog of this problem when analyzing the effect of topological structural defects (dislocations) on the Dirac-like electronic excitations with the momenta near conical (K-)points in the hexagonal Brillouin zone of a graphite layer [12].

However, despite its occurrence in all of the above physical situations, the $\eta = 2$ RMF problem for massless Dirac fermions has, thus far, remained unsolved. In fact, this problem does not appear to be readily amenable to any of the methods applied to its previously studied $\eta = 2$ non-relativistic [1,2] and $\eta = 0$ Dirac [3] counterparts.

In the present Letter, we fill in this gap by developing a novel non-perturbative path integral approach to the single-fermion action

$$\mathrm{S} = \int \mathrm{d}\vec{r} \int \mathrm{d}t\, \bar{\psi}(t,\vec{r})\big[i\hat{\gamma}_0\partial_t - \hat{\vec{\gamma}}\big(\vec{\nabla} - \vec{a}(\vec{r})\big)\big]\psi(t,\vec{r}), \tag{2}$$

where the correlation properties of the static random vector potential $\vec{a}(\vec{r})$ are governed by Eq. (1), and the $\hat{\gamma}_\mu$-matrices obey the algebra $\{\hat{\gamma}_\mu, \hat{\gamma}_\nu\} = \vec{1}\delta_{\mu\nu}$ (hereafter, we put the speed of the Dirac fermions equal to unity).

2. Path integral representation for gauge-invariant propagator

In order to gain a preliminary insight into the problem we first attempt to apply the customary self-consistent Born approximation (SCBA) to the conventional (retarded) fermion propagator $\widehat{\mathrm{G}}^R(t,\vec{r}) = \Theta(t)\langle\psi(t,\vec{r})\bar{\psi}(0,\vec{0})\rangle$. A closed SCBA equation can be obtained for the fermion self-energy which is defined via the Fourier transform $\widehat{\mathrm{G}}^R(\epsilon,\vec{p}) = [\epsilon\hat{\gamma}_0 - \vec{p}\hat{\vec{\gamma}} + \widehat{\Sigma}^R(\epsilon,\vec{p})]^{-1}$

$$\widehat{\Sigma}^R(\epsilon,\vec{p}) = \int \frac{\mathrm{d}\vec{q}}{(2\pi)^2} w_{ij}(\vec{q})\hat{\gamma}_i \widehat{\mathrm{G}}^R(\epsilon,\vec{p}+\vec{q})\hat{\gamma}_j. \tag{3}$$

Although one can readily solve Eq. (3) for any $\eta < 2$, including the previously studied case of $\eta = 0$ [3], at $\eta = 2$ the analysis is impeded by the fact that this equation now exhibits an infrared divergence with the size L of the system ($\widehat{\Sigma} \propto \hat{\gamma}_0\sqrt{\ln L}$), thus rendering the SCBA inapplicable.

This intrinsic divergence remains for all ϵ and $\vec{p}$, even if one proceeds beyond the SCBA by replacing (3) with the equation $\widehat{\Sigma} = n\widehat{\mathrm{T}} * \widehat{\mathrm{G}}$, where n stands for the density of Poissonian-distributed localized RMF sources (hereafter, motivated by the discussion of the mixed state of d-wave superconductors, we refer to them as "vortices"). Although the exact T-matrix given by the expression $\widehat{\mathrm{T}} = \hat{w} * (\hat{I} - \widehat{\mathrm{G}} * \hat{w})^{-1}$ does account for all the events of multiple scattering by the same vortex, the above infrared divergence cannot be made disappear, for it is directly related to the non-gauge invariant nature of $\widehat{\mathrm{G}}^R(t, \vec{r})$.

This problem does not arise, however, when calculating such observables such as the gauge-invariant Green function of the nodal quasiparticles introduced in the context of the d-wave superconductors [13–15]

$$\widehat{\mathrm{G}}^R_{\mathrm{inv}}(t, \vec{r}) = \Theta(t)\left\langle \psi(t, \vec{r}) \exp\left(-i \int_{\Gamma} \vec{a}(\vec{r}\,')\mathrm{d}\vec{r}\,'\right) \bar{\psi}(0, \vec{0}) \right\rangle, \tag{4}$$

which is manifestly invariant under arbitrary time-independent gauge transformations for any choice of the contour Γ.

In Refs. [13–15], where the amplitude (4) was discussed in the case of a dynamic vector potential $\vec{a}(t, \vec{r})$ (the latter being a part of the fictitious time-dependent gauge field representing either spin [13] or pairing [14,15] fluctuations), the contour Γ was chosen as a straight line between the end points $(0, \vec{0})$ and $(t, \vec{r})$.

It must be noted, however, that in the Lorentz-invariant situation considered in Refs. [13–15] the amplitude (4) with the straight-line contour Γ appears to exhibit an unphysical power-law behavior characterized by a negative (instead of a positive, as one would have expected on the general physical grounds and as Refs. [13–15] erroneously claimed to be the case) anomalous exponent [20].

Although, thus far, no physically sound alternative to the heuristic form of Eq. (4) conjectured in Refs. [13–15] has been found, it turns out that in the non-Lorentz-invariant case of a static vector potential $\vec{a}(\vec{r})$ the situation is rather different.

Namely, in the case of interest, the most important features, such as the mean free path (see Eqs. (11), (12) below) which controls the exponential (rather than a power-law, as in the case of a dynamic RMF [13–15,20]) decay of Eq. (4) or the structure of the "near-shell" singularity of the fermion spectral function (see Eqs. (13), (14) below), remain robust against deformations of the contour Γ (apparently, in the static case the only potentially important is the spatial projection of Γ, whereas its pitch in the temporal direction is not).

Moreover, while being representative of the properties of a generic gauge-invariant one-fermion amplitude, the above choice of the contour Γ facilitates the calculation of Eq. (4) along the lines of the earlier analyses of the non-relativistic version of the $\eta = 2$ RMF problem [16–19].

Of course, the spinor nature of the amplitude (4) makes it impossible to apply the results of Refs. [16–19] directly. Therefore, we first utilize the (relatively unknown) path integral representation devised in [21] and, for a fixed RMF configuration $\{\vec{a}(\vec{r})\}$, cast the time Fourier transform of Eq. (4) in the form of a path integral over the spatial coordinate $\vec{r}(\tau)$ and an auxiliary vector-like variable $\vec{k}(\tau)$ parametrized by the proper time τ (not to be confused with the real time t!)

$$\widehat{\mathrm{G}}^R_{\mathrm{inv}}(\epsilon, \vec{r}|\vec{a}) = i \int_0^\infty \mathrm{d}\tau \int_{\vec{r}(0)=\vec{0}}^{\vec{r}(\tau)=\vec{r}} \mathrm{D}\vec{r}\,\mathrm{D}\vec{k} \exp\left(i\widehat{\mathrm{S}}_0(\tau) + i \int_0^\tau \vec{a}(\vec{r})\frac{\mathrm{d}\vec{r}}{\mathrm{d}\tau'}\,\mathrm{d}\tau' - i \int_\Gamma \vec{a}(\vec{r}\,')\,\mathrm{d}\vec{r}\,'\right), \tag{5}$$

where the first term in the exponent represents the (matrix-valued) free fermion action

$$\widehat{\mathrm{S}}_0(\tau) = \int_0^\tau \mathrm{d}\tau'\left[\epsilon\hat{\gamma}_0 + \vec{k}\left(\frac{\mathrm{d}\vec{r}}{\mathrm{d}\tau'} - \hat{\vec{\gamma}}\right)\right], \tag{6}$$

while the other two correspond to the RMF coupling term in Eq. (2) and the line integral in the exponential factor inserted into Eq. (4), respectively. In Eq. (5), the usual τ-ordering of the $\hat{\gamma}$-matrices must be performed according to the order of their appearance in the series expansion of the exponent. In contrast to various approximate (e.g., Bloch–Nordsieck) representations, the integration over $\vec{k}(\tau)$ allows one to exactly account for the spinor structure of the fermion propagator.

Averaging (5) over the Gaussian RMF introduces the exponential attenuation factor

$$W\big[\vec{r}(\tau)\big] = \exp\left[-\frac{1}{2}\int_0^\tau \mathrm{d}\tau_1 \int_0^\tau \mathrm{d}\tau_2 \int \frac{\mathrm{d}\vec{q}}{(2\pi)^2} e^{i\vec{q}(\vec{r}(\tau_1)-\vec{r}(\tau_2))} w_{ij}(\vec{q}) \frac{\mathrm{d}r_i}{\mathrm{d}\tau_1}\frac{\mathrm{d}r_j}{\mathrm{d}\tau_2}\right], \tag{7}$$

whose argument, in the case of $\eta = 2$, turns out to be proportional to the so-called Amperian area of a closed contour which is composed of a given trajectory $\vec{r}(\tau)$ and the straight-line segment traversed backwards (from $\vec{r}$ to $\vec{0}$) [16,17].

3. Gauge-invariant propagator in the ballistic regime

While being strictly positive, the Amperian area coincides with the absolute value of the regular (algebraic) one only for non-self-intersecting closed contours, which, strictly speaking, precludes Eq. (7) from being interpreted as an additional term in the effective Dirac action. Nevertheless, this becomes possible for the trajectories whose projections onto the straight path Γ are single-valued, whence (7) reduces to

$$W\big[\vec{r}(\tau)\big] \approx \exp\left(-\frac{\alpha}{2}\int_0^r |x_\perp|\,\mathrm{d}x_\parallel\right), \tag{8}$$

where the parallel and transverse components of the vector $\vec{r}(\tau) = (x_\parallel, x_\perp)$ are defined with respect to the direction of the fermion's propagation.

The relativistic kinematics of the Dirac fermions makes the present situation markedly different from the previously studied non-relativistic version of the $\eta = 2$ RMF problem [16–19]. As opposed to the interpretation of the non-relativistic analog of $\delta \mathrm{S} = i \ln W[\vec{r}(\tau)]$ as an effective linear confining potential [16–19], Eq. (8) can be best thought of as an (imaginary) position-dependent mass whose linear increase with $|x_\perp|$ restrains the Dirac fermion's motion in the direction perpendicular to its "classical trajectory" $\vec{r}_0(\tau) = \tau(\vec{r}/r)$, the latter representing the saddle point of the path integral (5).

Comparing the saddle-point action $\mathrm{S}_0 \sim \epsilon\tau$ to the characteristic RMF correction $\delta\mathrm{S} \sim (\alpha\epsilon/p)^{1/2}\tau$, we find that the condition $\mathrm{S}_0 \gg \delta\mathrm{S}$, under which the path integral (5) is dominated by the trajectories close to $\vec{r}_0(\tau)$, is satisfied in the "ballistic" regime of high energies and large momenta ($\epsilon p \gg \alpha$).

In this regime, one can integrate over the momentum $\vec{p}(\tau)$, thereby effectively squaring the Dirac operator (for details, see [20,21]), as well as the longitudinal spatial $x_\parallel(\tau)$ component of the coordinate vector $\vec{r}(\tau)$, thus reducing (5) to the one-dimensional path integral over $x_\perp(\tau)$.

As in the non-relativistic $\eta = 2$ RMF problem [16–19], the computation of the remaining path integral amounts to finding the resolvent of the second order ordinary differential equation

$$\big[\big(p_\parallel^2 - \epsilon^2\big) - \partial_\perp^2 + (\alpha\epsilon/p_\parallel)^2 x_\perp^2 + i\alpha\,\mathrm{sign}\,x_\perp\big] g\big(\epsilon, p_\parallel\big|x_\perp, x'_\perp\big) = \delta\big(x_\perp - x'_\perp\big), \tag{9}$$

whose exact form can be readily obtained in terms of the Wronskian of a pair of linearly independent solutions of the corresponding homogeneous equation

$$g(\epsilon, p_\parallel|0,0) = \sqrt{\frac{ip_\parallel}{\alpha|\epsilon|}}\left[\sum_{\lambda=\pm}\frac{\Gamma(z_\lambda + 1/2)}{\Gamma(z_\lambda)}\right]^{-1}, \tag{10}$$

where $z_\lambda = ip_\parallel(\epsilon - \lambda p_\parallel)(\epsilon + \lambda p_\parallel + i\alpha/p_\parallel)/4\epsilon\alpha$ and $\Gamma(z)$ is the standard Γ-function.

The asymptotical large-distance behavior of the gauge-invariant propagator is controlled by the closest to the real $p_\parallel$-axis zero of the denominator in Eq. (10). Applying to $g(\epsilon, p_\parallel|0,0)$ the one-dimensional Fourier transformation with respect to $p_\parallel$ we arrive at the expression

$$\widehat{\mathrm{G}}^R_{\mathrm{inv}}(\epsilon, \vec{r}) \sim \left(\epsilon\hat{\gamma}_0 - \frac{\vec{r}}{r^2}\hat{\vec{\gamma}}\right)\frac{e^{-r/l}}{\sqrt{|\epsilon|l}}\sin\bigl(|\epsilon|r + \phi_1\bigr), \tag{11}$$

where $\phi_1 \sim 1$, which decays exponentially at $r \to \infty$ with the characteristic length scale

$$l \approx \frac{8|\epsilon|}{\pi\alpha}. \tag{12}$$

The latter can be naturally identified with the RMF mean free path which, according to Eq. (12), decreases as the fermion energy gets lower, contrary to the situation for $\eta = 0$ [3] (but qualitatively similar to that found in the non-relativistic variant of the $\eta = 2$ problem where $l \sim |\epsilon|^{1/6}\alpha^{-2/3}$ [16,17,19]).

The "near-maximum" ($|\epsilon^2 - p^2| < \alpha$) behavior of the electron spectral function is determined by the two-dimensional Fourier transform of $\mathrm{Im}\,\widehat{\mathrm{G}}^R_{\mathrm{inv}}(\epsilon, \vec{r})$ which can be obtained by convoluting Eq. (10) with the kernel $(p^2 - q^2)^{-3/2}$, thus resulting in the expression

$$\mathrm{Im}\,\widehat{\mathrm{G}}^R_{\mathrm{inv}}(\epsilon, \vec{p}) \approx \sqrt{\frac{\epsilon}{l}}\,\frac{\epsilon\hat{\gamma}_0 - \vec{p}\hat{\vec{\gamma}}}{[(\epsilon^2 - p^2)^2 + (\epsilon/l)^2]^{3/4}}, \tag{13}$$

which explicitly demonstrates that the bare pole gets replaced by branch cuts of the function $z_\lambda^{3/2}$ (hence, the emergence of the exponent $3/4$ in (13)).

At larger deviations from the maximum ($\alpha \ll |\epsilon^2 - p^2| < \epsilon^2$) the spectral function is determined by the large-z_λ asymptotics of the Γ-functions in Eq. (10). In this limit, the spectral function resembles the Lorentzian

$$\mathrm{Im}\,\widehat{\mathrm{G}}^R_{\mathrm{inv}}(\epsilon, \vec{p}) \propto \alpha\frac{\epsilon\hat{\gamma}_0 - \vec{p}\,\hat{\vec{\gamma}}}{(\epsilon^2 - p^2)^2 + \alpha^2}, \tag{14}$$

although, under a closer inspection, the shape of $\mathrm{Im}\,\widehat{\mathrm{G}}^R_{\mathrm{inv}}(\epsilon, \vec{p})$ turns out to be asymmetrical as a result of the Lorentz invariance's being broken by the static disorder. Moreover, the product of its maximum height and width reveals a marked departure from the Lorentzian value ($\approx \pi/4$ instead of 1).

4. Averaged products of Green functions

Our eikonal-type approach also enables one to study other gauge-invariant physical observables, such as a correlation function of the local density of states (DOS) which is related to the average $\langle\widehat{\mathrm{G}}^R\widehat{\mathrm{G}}^R\rangle$. Owing to its gauge invariant nature, the latter is free of infrared divergencies and can be cast in the form of a two-particle path integral

$$\bigl\langle\widehat{\mathrm{G}}^R(\epsilon, \vec{r})\widehat{\mathrm{G}}^R(\epsilon, -\vec{r})\bigr\rangle = \int_0^\infty \mathrm{d}\tau_1 \int_0^\infty \mathrm{d}\tau_2 \int_0^{\vec{r}} \mathrm{D}\vec{r}_1\,\mathrm{D}\vec{k}_1 \int_0^{\vec{r}} \mathrm{D}\vec{r}_2\,\mathrm{D}\vec{k}_2\, e^{i\widehat{\mathrm{S}}_0(\tau_1)} e^{i\widehat{\mathrm{S}}_0(\tau_2)} \prod_{i,j=1,2} W\bigl(\vec{r}_i(\tau_1) - \vec{r}_j(\tau_2)\bigr), \tag{15}$$

where the product of the W-factors yields the exponent of the Amperian area of the closed contour formed by a pair of trajectories $\vec{r}_1(\tau)$ and $\vec{r}_2(\tau)$.

We mention, in passing, that the average (15) equals that composed of a pair of gauge-invariant amplitudes ($\langle\widehat{\mathrm{G}}^R\widehat{\mathrm{G}}^R\rangle = \langle\widehat{\mathrm{G}}^R_{\mathrm{inv}}\widehat{\mathrm{G}}^R_{\mathrm{inv}}\rangle$), since, regardless of the choice of Γ in Eq. (4), the phase factors from the two functions $\widehat{\mathrm{G}}^R_{\mathrm{inv}}$ exactly cancel against each other.

In the ballistic regime, the integral (15) receives its main contribution from the pairs of trajectories with single-valued projections to the direction of the vector $\vec{r}$. Upon separating the coordinate variables onto the "center of mass" and the relative motion ($\vec{r}^{\pm} = \vec{r}_1 \pm \vec{r}_2$, $\vec{k}^{\pm} = \vec{k}_1 \pm \vec{k}_2$) parametrized by $\tau^{\pm} = \tau_1 \pm \tau_2$ and integrating over all the variables but $x_{\perp}^{-}(\tau^{+})$, one finds that the average (15) can be again related to the solution of Eq. (9), albeit with some extra powers of two stemming from the Jacobian of the above transformation. As a result, the variance of the local DOS ($\delta\nu(\epsilon, \vec{r}) = \nu(\epsilon, \vec{r}) - \langle\nu(\epsilon)\rangle$) decays with distance as

$$\left\langle \delta\nu\left(\epsilon, \vec{r}\right)\delta\nu\left(\epsilon, \vec{0}\right)\right\rangle \propto \frac{|\epsilon| e^{-2r/l}}{\sqrt{rl}} \sin\left(2|\epsilon| r + \phi_2\right), \tag{16}$$

where $\phi_2 \sim 1$.

Notably, the average of the product of two Green functions (16) does not amount to the product of the two averages (each of which is given by Eq. (11)), thereby indicating the presence of non-trivial vertex corrections.

Nevertheless, the exponential decay of Eq. (16) is controlled by a length scale equal to the half of the mean free path (12), which corroborates our conclusion that (alongside a number of other robust features such as the "near-shell" asymptotic behavior (13)) Eq. (12) is, in fact, independent of the choice of the contour Γ in Eq. (4).

In a similar manner, one finds the average $\langle \widehat{\mathrm{G}}^{A}\widehat{\mathrm{G}}^{R}\rangle = \langle \widehat{\mathrm{G}}^{A}_{\mathrm{inv}}\widehat{\mathrm{G}}^{R}_{\mathrm{inv}}\rangle \propto (\epsilon^2/lr)^{1/2} e^{-2r/l}$ which can be used for computing such transport coefficients as optical conductivity $\sigma(\omega, T)$. However, such a result would be limited to the regime of high frequencies or temperatures $\max(\omega, T) \gg \alpha^{1/2}$ where other mechanisms of scattering, which may exist in the known realizations of the RMF problem (see Refs. [6–12]), might become important.

5. Localization of zero-energy modes

At low energies and small momenta $\epsilon p < \alpha$ our approach ceases to be valid, and, for instance, the low-energy behavior of the DOS cannot be readily inferred from the above discussion. In order to shed some light on the effect of RMF scattering on the low-energy part of the Dirac spectrum we look into the properties of the zero-energy states. Quite remarkably, the latter exist for an arbitrary configuration of the non-uniform magnetic field, thanks to the exact cancellation between the orbital and the Zeeman terms in the total energy of a spinor particle with the gyromagnetic ratio equal two.

Parametrizing an arbitrary (up to a gauge transformation) RMF configuration in terms of a scalar function $a_i(\vec{r}) = \epsilon_{ij}\partial_j \Phi(\vec{r})$ one can write a pair of independent zero-energy wave functions (there are no other states with $\epsilon = 0$ as long as the total RMF flux vanishes) as follows

$$\Psi_{\pm}\left(\vec{r}\right) \propto \left(\vec{1} \pm \hat{\gamma}_0\right) \begin{pmatrix} e^{\Phi(\vec{r})} \\ e^{-\Phi(\vec{r})} \end{pmatrix}. \tag{17}$$

In a finite-size system, the degree of the wave functions' localization (or a lack thereof) can be inferred from the set of inverse participation ratios

$$P_n = \left\langle \frac{\int |\Psi(\vec{r})|^{2n}\, \mathrm{d}\vec{r}}{L^2 (\int |\Psi(\vec{r}')|^2\, \mathrm{d}\vec{r}')^n} \right\rangle. \tag{18}$$

Performing the Gaussian average over the disorder field $\Phi(\vec{r})$ with the weight $P[\Phi(\vec{r})] \propto \exp(-\int \mathrm{d}\vec{r}\, (\vec{\nabla}^2\Phi)^2/2\alpha)$ we obtain

$$P_n \propto \frac{\alpha^{n-1}}{L^2}, \tag{19}$$

which is suggestive of strong localization of the zero-energy states, the "localization length" being of order $\xi \sim \alpha^{-1/2}$. This conclusion is in stark contrast with the $\eta = 0$ case [3] where the prelocalized zero-energy wave functions exhibit a multifractal spectrum of anomalous dimensions $P_n \propto L^{-an+bn^2}$. Conceivably, as the energy decreases past $\alpha^{1/2}$, the elastic mean free path (given by Eq. (12) at high energies) saturates at $l \sim \xi$, followed by the onset of localization at still lower energies.

In the $\eta = 0$ case, it was recently argued [22,23] that the corresponding localization scenario belongs to the so-called "C" universality class which encompasses the d-wave superconductors with a strong inter-node scattering in the absence of time reversal symmetry [24,25]. It was also conjectured in Refs. [22,23] that the low-energy properties of the system are described by the non-linear σ-model (NLσM) on the coset space $\mathrm{OSp}(2|2)/\mathrm{GL}(1|1)$.

In order to extend this conjecture to the Dirac $\eta = 2$ RMF problem one would have to derive an effective NLσM of the appropriate symmetry by carefully separating between the massless and massive modes of the (this time, non-local) Hubbard–Stratonovich disorder fields, by analogy with the procedure carried out in the non-relativistic $\eta = 2$ RMF problem [26].

There is a good reason to believe that the resulting localization scenario falls into the already existing classification of the d-wave universality classes chartered in Refs. [24,25]. However, given the notorious sensitivity of the d-wave localization patterns not only to the type (potential vs. magnetic vs. extended defects, etc.) but even to the strength (Gaussian vs. unitary) of disorder, such important details as the asymptotical form of the low-energy DOS ($\langle \nu(\epsilon) \rangle \propto \epsilon^2$ for a generic class "C" system [27]) or a possible crossover from "C" to another, e.g., the so-called "A", universality class at intermediate energies due to the predominantly small-angle nature of the BA scattering, remain to be worked out.

6. Summary

To conclude, in the present Letter we investigated the properties of the two-dimensional Dirac fermions subject to a long-range-correlated random vector potential. In the ballistic regime of large quasiparticle energies, we obtained the asymptotics of a representative gauge-invariant fermion Green function and the DOS correlation function. In the complementary low-energy regime, we found a signature of strong localization of the zero-energy states by computing their inverse participation ratios.

Thus far, it has been rather difficult to find a conclusive experimental evidence of the quasiparticle localization in the high-T_c cuprates. However, theoretical predictions based on the results of this Letter which pertain to the more experimentally accessible ballistic regime may be possible to test with a number of standard probes (thermal transport, ARPES, tunneling, specific heat, NMR) performed in the vortex line liquid phase of the superconducting cuprates [28].

To this end, it is worth mentioning that the results of a recent numerical analysis of the low-field thermal quasiparticle conductivity in the vortex line liquid state of the d-wave superconductors [29] appear to be in good agreement with the analytical result based on the analog of Eq. (12) [28].

Moreover, the results of this Letter can be used to study the effects of both thermal [10,11] and quantum [13–15] phase fluctuations on the spectrum of the nodal quasiparticles in the pseudogap phase of the cuprates as well as that of randomly distributed dislocations on the Dirac-like electronic excitations in graphite [12].

In particular, when applied to the theories of the pseudogap phase [13–15] where the nodal fermions are scattered by the quasi-static (thermal) gauge fluctuations which represent either the effect of a disordered flux-phase [13] or that of thermally excited vortices [14,15], the results of this Letter imply that the gauge-invariant propagator should exhibit an exponential, rather than a power-law, decay, contrary to the conclusions drawn in Refs. [13–15].

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