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The study on a stochastic system with non-Gaussian noise and Gaussian colored noise

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ABSTRACT

In this paper, a stochastic system with correlation between non-Gaussian noise and Gaussian colored noise is investigated. We carry out the functional methods to derive the approximate Fokker–Planck equation, and the expressions of stationary probability density function and mean first-passage time are presented. Also we explore the effects of correlation between non-Gaussian and Gaussian noise for the mean first-passage time.

1. Introduction

In the past years, the effects of correlations between additive and multiplicative noise have been widely studied. The steady-state and transient properties of the bistable systems with correlated Gaussian noise have been discussed by many authors [1–10,20–23], and most of the previous studies have taken the assumption that the noise is Gaussian. However, Gaussian distributions are not appropriate in some practical cases. Many experimental evidences, particularly in sensory and biological systems [11], indicate that the study of non-Gaussian noises is necessary [12–16]. The stochastic resonance induced by non-Gaussian colored noise has been examined [15]. The effective Markovian Fokker–Planck equation for the stochastic system driven by non-Gaussian noise has been obtained by using a path integral approach [13]. The fact that the stationary probability density and the mean first-passage time can be influenced by correlation intensity and correlation time has been shown for the stochastic system with coupling between non-Gaussian and Gaussian white noise [12].

The paper is organized as follows: Section 2 is to consider a stochastic system with correlation between non-Gaussian noise and Gaussian colored noise, and to derive the approximate Fokker–Planck equation. In Section 3 we take a special system as an example to obtain the stationary probability density function and the mean first-passage time, and the effect of correlation between non-Gaussian and Gaussian colored noises for the mean first-passage time is presented. Section 4 gives our conclusions to close this paper.

2. Approximative Fokker–Planck equation

Consider the Langevin equation with cross-correlated non-Gaussian and Gaussian colored noises

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x) + g_1(x)\eta(t) + g_2(x)\xi(t), \tag{1}$$

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where $\eta(t)$ is a non-Gaussian noise

$$\frac{\mathrm{d}\eta(t)}{\mathrm{d}t} = -\frac{1}{\tau_1}\frac{\mathrm{d}}{\mathrm{d}\eta}V_q(\eta) + \frac{1}{\tau_1}\varepsilon(t), \tag{2}$$

where $\varepsilon(t)$ is a Gaussian white noise of zero mean and correlation $\langle\varepsilon(t)\varepsilon(s)\rangle = 2D_1\delta(t-s)$,$V_q(\eta)$ is given by

$$V_q(\eta) = \frac{D_1}{\tau_1(q-1)}\ln\left[1 + \frac{\tau_1}{D_1}(q-1)\frac{\eta^2}{2}\right]. \tag{3}$$

q is related to Tsallis entropy [17,18], $\xi(t)$ is a Gaussian colored noise which can be described as

$$\frac{\mathrm{d}\xi}{\mathrm{d}t} = -\frac{1}{\tau_2}\xi + \frac{1}{\tau_2}\Gamma(t), \tag{4}$$

with

$$\langle\Gamma(t)\Gamma(s)\rangle = 2D_2\delta(t-s), \tag{5}$$

$$\langle\Gamma(t)\varepsilon(s)\rangle = \langle\Gamma(s)\varepsilon(t)\rangle = 2\lambda\sqrt{D_1D_2}\delta(t-s). \tag{6}$$

The stationary probability distribution of Eq. (2) is given by [13]

$$P_q(\eta) \propto \left[1 + (q-1)\left(\frac{\tau_1}{2D_1}\right)\eta^2\right]_+^{-\frac{1}{q-1}}. \tag{7}$$

It is obvious that for $q = 1$, η become a Gaussian colored noise and for $q \neq 1$, η is a non-Gaussian noise. In the region $|q-1| \ll 1$, the term η^2 can be replaced by its expectation value, Eq. (2) becomes [11,16,24]

$$\frac{\mathrm{d}\eta(t)}{\mathrm{d}t} = -\frac{1}{\tau_{\mathit{eff}}}\eta + \frac{1}{\tau_{\mathit{eff}}}\varepsilon(t), \tag{8}$$

and

$$\langle\varepsilon(t)\varepsilon(s)\rangle = 2D_{\mathit{eff}}\delta(t-s), \tag{9}$$

$$\langle\Gamma(t)\varepsilon(s)\rangle = \langle\Gamma(s)\varepsilon(t)\rangle = 2\lambda\sqrt{D_{\mathit{eff}}D_2}\delta(t-s), \tag{10}$$

where

$$\tau_{\mathit{eff}} = \frac{2(2-q)}{5-3q}\tau_1, \qquad D_{\mathit{eff}} = \left(\frac{2(2-q)}{5-3q}\right)^2 D. \tag{11}$$

For the initial condition that at time $t = 0$, the random variables ξ and η have the values ξ_0 and η_0 respectively, and the solutions of Eqs. (4) and (8) can be written as

$$\xi(t) = \xi_0\exp\left(-\frac{t}{\tau_2}\right) + \frac{1}{\tau_2}\int_0^t \exp\left(-\frac{t-t'}{\tau_2}\right)\Gamma(t')\mathrm{d}t', \tag{12}$$

$$\eta(t) = \eta_0\exp\left(-\frac{t}{\tau_{\mathit{eff}}}\right) + \frac{1}{\tau_{\mathit{eff}}}\int_0^t \exp\left(-\frac{t-t'}{\tau_{\mathit{eff}}}\right)\varepsilon(t')\mathrm{d}t'. \tag{13}$$

We therefore have

$$\begin{aligned}\langle\xi(t)\eta(s)\rangle &= \xi_0\eta_0\exp\left(-\frac{t}{\tau_2} - \frac{s}{\tau_{\mathit{eff}}}\right) + \frac{2\lambda\sqrt{D_{\mathit{eff}}D_2}}{\tau_{\mathit{eff}}\tau_2}\int_0^{\min(t,s)}\exp\left(-\frac{t}{\tau_2} - \frac{s}{\tau_{\mathit{eff}}} + \left(\frac{1}{\tau_2} + \frac{1}{\tau_{\mathit{eff}}}\right)t'\right)\mathrm{d}t' \\ &= \xi_0\eta_0\exp\left(-\frac{t}{\tau_2} - \frac{s}{\tau_{\mathit{eff}}}\right) + \frac{2\lambda\sqrt{D_{\mathit{eff}}D_2}}{\tau_{\mathit{eff}}+\tau_2}\exp\left(-\frac{t}{\tau_2} - \frac{s}{\tau_{\mathit{eff}}}\right)\star\left(\exp\left(-\frac{|t-s|}{\tau}\right) - 1\right),\end{aligned} \tag{14}$$

where $\tau = \tau_{\mathit{eff}}$ for $t < s$ and $\tau = \tau_2$ for $t \geq s$.
For large t, s, the cross-correlation function is independent of the initial value and is only a function of the time difference $t - s$, namely,

$$\langle\xi(t)\eta(s)\rangle = \frac{2\lambda\sqrt{D_{\mathit{eff}}D_2}}{\tau_{\mathit{eff}}+\tau_2}\exp\left(-\frac{|t-s|}{\tau}\right). \tag{15}$$

According to the stochastic Liouville equation,the probability density $P(x,t)$ obeys

$$\begin{aligned}\frac{\partial}{\partial t}P(x,t) &= \frac{\partial}{\partial t}\langle\delta(x(t)-x)\rangle \\ &= -\frac{\partial}{\partial x}f(x)P(x,t)-\frac{\partial}{\partial x}g_1(x)\langle\eta(t)\delta(x(t)-x)\rangle-\frac{\partial}{\partial x}g_2(x)\langle\xi(t)\delta(x(t)-x)\rangle.\end{aligned}\tag{16}$$

Using the fact $f(y)\frac{\partial}{\partial x}\delta(y-x)=\frac{\partial}{\partial x}[f(x)\delta(y-x)]$ and the Novikov theorem, we have

$$\begin{aligned}\langle\eta(t)\delta(x(t)-x)\rangle &= -\frac{\partial}{\partial x}\int_0^t\frac{D_{eff}}{\tau_{eff}}\exp\left(-\frac{t-s}{\tau_{eff}}\right)\left\langle\delta(x(t)-x)\frac{\delta x(t)}{\delta\eta(s)}\right\rangle\mathrm{d}s \\ &\quad -\frac{\partial}{\partial x}\int_0^t\frac{2\lambda\sqrt{D_2D_{eff}}}{\tau_{eff}+\tau_2}\exp\left(-\frac{t-s}{\tau_{eff}}\right)\left\langle\delta(x(t)-x)\frac{\delta x(t)}{\delta\xi(s)}\right\rangle\mathrm{d}s.\end{aligned}\tag{17}$$

The functional derivative $\frac{\delta x(t)}{\delta\eta(s)}$ satisfies

$$\frac{\delta x(t)}{\delta\eta(s)}=\theta(t-s)\left\{g_1(x(s))+\int_s^t[f'(x(t'))+g_1'(x(t'))\eta(t')+g_2'(x(t'))\xi(t')]\frac{\delta x(t')}{\delta\eta(s)}\mathrm{d}t'\right\}.\tag{18}$$

Here $\theta(t-s)$ is the unit step function. Its solution is

$$\frac{\delta x(t)}{\delta\eta(s)}=\theta(t-s)g_1(x(s))\exp\left\{\int_s^t[f'(x(t'))+g_1'(x(t'))\eta(t')+g_2'(x(t'))\xi(t')]\mathrm{d}t'\right\},\tag{19}$$

where $f'(x)$ denotes the first derivatives of f with respect to x. Since

$$\frac{\mathrm{d}}{\mathrm{d}t}g_1(x(t))=\frac{g_1'(x(t'))}{g_1(x(t'))}[f(x(t))+g_1(x(t))\eta(t)+g_2(x(t))\xi(t)]g_1(x(t)),\tag{20}$$

then the integral of Eq. (20) gives

$$g_1(x(s))=g_1(x(t))\exp\left\{-\int_s^t\frac{g_1'(x(t'))}{g_1(x(t'))}[f(x(t'))+g_1(x(t'))\eta(t')+g_2(x(t'))\xi(t')]\mathrm{d}t'\right\}.\tag{21}$$

Combining Eqs. (19) and (21),we get

$$\begin{aligned}\frac{\delta x(t)}{\delta\eta(s)} &= \theta(t-s)g_1(x(t))\exp\left\{\int_s^t\left[f'(x(t'))-\frac{g_1'(x(t'))}{g_1(x(t'))}f(x(t'))\right.\right. \\ &\quad \left.\left.+\ g_2'(x(t'))\xi(t')-\frac{g_1'(x(t'))}{g_1(x(t'))}g_2(x(t'))\xi(t')\right]\mathrm{d}t'\right\}.\end{aligned}\tag{22}$$

According to the Ansatz of Hanggi, we have the following approximation like Ref. [19]

$$\begin{aligned}\langle\eta(t)\delta(x(t)-x)\rangle &\approx -\frac{\partial}{\partial x}g_1(x)\int_0^t\frac{D_{eff}}{\tau_{eff}}\exp\left(-\frac{t-s}{\tau_{eff}}\right)\left\langle\delta(x(t)-x)*\exp(f'(x_s)-\frac{g_1'(x_s)}{g_1(x_s)}f(x_s))(t-s)\right\rangle\mathrm{d}s \\ &\quad -\frac{\partial}{\partial x}g_2(x)\int_0^t\frac{2\lambda\sqrt{D_2D_{eff}}}{\tau_{eff}+\tau_2}\exp\left(-\frac{t-s}{\tau_{eff}}\right)\left\langle\delta(x(t)-x)*\exp\left(f'(x_s)-\frac{g_2'(x_s)}{g_2(x_s)}f(x_s)\right)(t-s)\right\rangle\mathrm{d}s \\ &= -\frac{D_{eff}}{1-\tau_{eff}\left[f'(x_s)-\frac{g_1'(x_s)}{g_1(x_s)}f(x_s)\right]}\frac{\partial}{\partial x}g_1(x)P(x,t) \\ &\quad -\frac{2\lambda\sqrt{D_2D_{eff}}\tau_{eff}}{(\tau_{eff}+\tau_2)\left(1-\tau_{eff}\left(f'(x_s)-\frac{g_2'(x_s)}{g_2(x_s)}f(x_s)\right)\right)}\frac{\partial}{\partial x}g_2(x)P(x,t),\end{aligned}\tag{23}$$

where x_s is the steady-state value of deterministic system.
Similarly, we have

$$\begin{aligned}\langle\xi(t)\delta(x(t)-x)\rangle &\approx -\frac{\partial}{\partial x}g_2(x)\int_0^t\frac{D_2}{\tau_2}\exp\left(-\frac{t-s}{\tau_2}\right)\left\langle\delta(x(t)-x)*\exp\left(f'(x_s)-\frac{g_2'(x_s)}{g_2(x_s)}f(x_s)\right)(t-s)\right\rangle\mathrm{d}s \\ &\quad -\frac{\partial}{\partial x}g_1(x)\int_0^t\frac{2\lambda\sqrt{D_2D_{eff}}}{\tau_{eff}+\tau_2}\exp\left(-\frac{t-s}{\tau_2}\right)\left\langle\delta(x(t)-x)*\exp\left(f'(x_s)-\frac{g_1'(x_s)}{g_1(x_s)}f(x_s)\right)(t-s)\right\rangle\mathrm{d}s\end{aligned}$$

$$= -\frac{D_2}{1-\tau_2\left[f'(x_s)-\frac{g_2'(x_s)}{g_2(x_s)}f(x_s)\right]}\frac{\partial}{\partial x}g_2(x)P(x,t)$$
$$-\frac{2\lambda\sqrt{D_2D_{eff}}\tau_2}{(\tau_{eff}+\tau_2)\left(1-\tau_2\left(f'(x_s)-\frac{g_2'(x_s)}{g_2(x_s)}f(x_s)\right)\right)}\frac{\partial}{\partial x}g_1(x)P(x,t). \quad (24)$$

Inserting Eqs. (23) and (24) into Eq. (16), the approximate Fokker–Planck equation is obtained

$$\frac{\partial}{\partial t}P(x,t) = -\frac{\partial}{\partial x}f(x)P(x,t)+\frac{D_{eff}}{1-\tau_{eff}\left[f'(x_s)-\frac{g_1'(x_s)}{g_1(x_s)}f(x_s)\right]}\frac{\partial}{\partial x}g_1(x)\frac{\partial}{\partial x}g_1(x)P(x,t)$$
$$+\frac{2\lambda\sqrt{D_2D_{eff}}\tau_{eff}}{(\tau_{eff}+\tau_2)\left(1-\tau_{eff}\left(f'(x_s)-\frac{g_2'(x_s)}{g_2(x_s)}f(x_s)\right)\right)}\frac{\partial}{\partial x}g_1(x)\frac{\partial}{\partial x}g_2(x)P(x,t)$$
$$+\frac{D_2}{1-\tau_2\left[f'(x_s)-\frac{g_2'(x_s)}{g_2(x_s)}f(x_s)\right]}\frac{\partial}{\partial x}g_2(x)\frac{\partial}{\partial x}g_2(x)P(x,t)$$
$$+\frac{2\lambda\sqrt{D_2D_{eff}}\tau_2}{(\tau_{eff}+\tau_2)\left(1-\tau_2\left(f'(x_s)-\frac{g_2'(x_s)}{g_2(x_s)}f(x_s)\right)\right)}\frac{\partial}{\partial x}g_2(x)\frac{\partial}{\partial x}g_1(x)P(x,t). \quad (25)$$

where the approximation is valid under the following conditions:

$$1-\tau_2\left[f'(x_s)-\frac{g_2'(x_s)}{g_2(x_s)}f(x_s)\right]>0$$
$$1-\tau_{eff}\left[f'(x_s)-\frac{g_2'(x_s)}{g_2(x_s)}f(x_s)\right]>0.$$

3. Case study: Symmetric bistable system

As a special case, we consider the bistable system

$$\frac{dx}{dt} = x - x^3 + x\eta(t) + \xi(t). \quad (26)$$

3.1. Steady-state distribution

The approximate Fokker–Planck equation corresponding to Eq. (26) can be written as

$$\frac{\partial}{\partial t}P(x,t) = -\frac{\partial}{\partial x}A(x)P(x,t)+\frac{\partial^2}{\partial x^2}B(x)P(x,t), \quad (27)$$

where

$$A(x) = x - x^3 + \frac{D_{eff}}{1+2\tau_{eff}}x + \frac{2\lambda\sqrt{D_2D_{eff}}\tau_{eff}}{(\tau_2+\tau_{eff})(1+2\tau_{eff})},$$
$$B(x) = \frac{D_{eff}}{1+2\tau_{eff}}x^2 + \left\{\frac{2\lambda\sqrt{D_2D_{eff}}\tau_{eff}}{(\tau_2+\tau_{eff})(1+2\tau_{eff})}+\frac{2\lambda\sqrt{D_2D_{eff}}\tau_2}{(\tau_2+\tau_{eff})(1+2\tau_2)}\right\}x + \frac{D_2}{1+2\tau_2}. \quad (28)$$

Then the steady-state probability distribution function $P_{st}(x)$ can be obtained as

(a) when $\lambda^2 > (\tau_2+\tau_{eff})^2(1+2\tau_{eff})(1+2\tau_2)/(\tau_2+\tau_{eff}+4\tau_2\tau_{eff})^2$,

$$P_{st}(x) = \frac{N_1}{B(x)}\exp\left\{-\frac{1+2\tau_{eff}}{D_{eff}}\left[\frac{1}{2}x^2 - cx + \frac{c^2+a-e}{2}\ln|x^2+cx+e|\right.\right.$$
$$\left.\left.-\frac{c(c^2+a)-3ce-2b}{4\sqrt{\frac{c^2}{4}-e}}\ln\frac{|x+\frac{c}{2}-\sqrt{\frac{c^2}{4}-e}|}{|x+\frac{c}{2}+\sqrt{\frac{c^2}{4}-e}|}\right]\right\}; \quad (29)$$

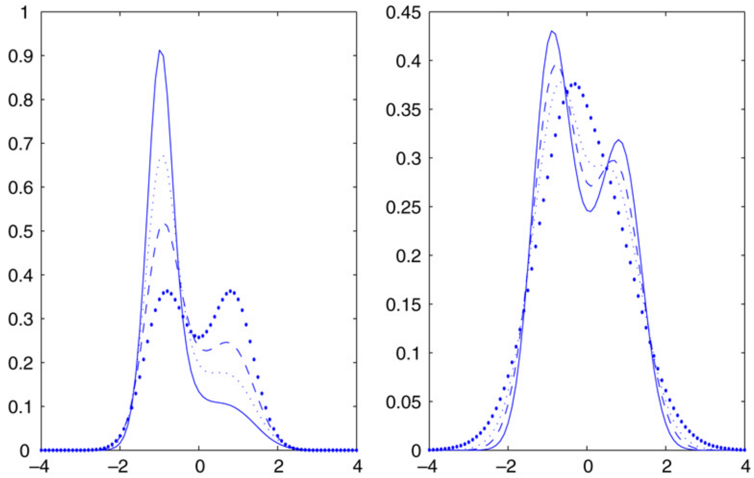

Fig. 1. The stationary probability distribution function. $P_{st}(x)$. (a) $D_1 = 0.5, D_2 = 0.5, \tau_1 = 0.1, \tau_2 = 0.5, q = 0.7$ are fixed. λ takes 0 (point line), 0.3 (dashed line), 0.5 (dotted line), 0.7 (solid line). (b) $D_1 = 0.3, D_2 = 0.3, \tau_1 = 0.01, \tau_2 = 0.1, \lambda = 0.1$ are fixed. q takes 1.0 (solid line), 1.3 (dashed line), 1.4 (dotted line), 1.5 (point line).

(b) when $\lambda^2 < (\tau_2 + \tau_{eff})^2(1 + 2\tau_{eff})(1 + 2\tau_2)/(\tau_2 + \tau_{eff} + 4\tau_2\tau_{eff})^2$,

$$P_{st}(x) = \frac{N_2}{B(x)} \exp\left\{ -\frac{1+2\tau_{eff}}{D_{eff}} \left[\frac{1}{2}x^2 - cx + \frac{c^2+a-e}{2}\ln|x^2+cx+e| - \frac{c(c^2+a)-3ce-2b}{2\sqrt{e-\frac{c^2}{4}}} Arctg\left(\frac{1}{\sqrt{e-\frac{c^2}{4}}}\left(x+\frac{c}{2}\right)\right)\right]\right\}; \quad (30)$$

(c) when $\lambda^2 = (\tau_2 + \tau_{eff})^2(1 + 2\tau_{eff})(1 + 2\tau_2)/(\tau_2 + \tau_{eff} + 4\tau_2\tau_{eff})^2$,

$$P_{st}(x) = \frac{N_3}{B(x)} \exp\left\{ -\frac{1+2\tau_{eff}}{D_{eff}} \left[\frac{1}{2}x^2 - cx + \frac{c^2+a-e}{2}\ln|x^2+cx+e| + (c(c^2+a)-3ce-2b)\frac{1}{2x+c}\right]\right\}; \quad (31)$$

where $a = -1 - \frac{D_{eff}}{1+2\tau_{eff}}, b = -\frac{2\lambda\sqrt{D_2 D_{eff}}\tau_{eff}}{(\tau_2+\tau_{eff})(1+2\tau_{eff})}, c = \frac{2\lambda\sqrt{D_2 D_{eff}}}{\tau_2+\tau_{eff}}(\frac{\tau_{eff}}{D_{eff}} + \frac{\tau_2(1+2\tau_{eff})}{D_{eff}(1+2\tau_2)}), e = \frac{D_2(1+2\tau_{eff})}{D_{eff}(1+2\tau_2)}$, N_1, N_2, N_3 are normalization constants.

The critical curve separating the bimodal and unimodal regions is

$$\frac{\lambda^2 D_2 D_{eff}\tau_2^2}{(\tau_2+\tau_{eff})^2(1+2\tau_2)^2} + \frac{1}{27}\left(\frac{2D_{eff}}{1+2\tau_{eff}} - 1\right)^3 = 0. \quad (32)$$

The approximate steady-state distribution function $P_{st}(x)$ is plotted in Fig. 1. It can be seen from Fig. 1(a) that when parameters $D_1 = 0.5, D_2 = 0.5, \tau_1 = 0.1, \tau_2 = 0.5, q = 0.7$ are fixed, the curves of SPD changed from the bimodal to unimodal with the λ increasing from 0 to 0.7, which means that the noise-induced transition phenomenon occurs. The same phenomena would happen when $D_1 = 0.3, D_2 = 0.3, \tau_1 = 0.01, \tau_2 = 0.1, \lambda = 0.1$ are fixed and q changed from 1 to 1.5, see Fig. 1(b).

3.2. Mean first-passage time

The approximate expression of the MFPT for a particle to reach the final state $x_2 = 1$ from the initial state $x_1 = -1$ is

$$T(x_1 \to x_2) = \int_{-1}^{1} \frac{dx}{B(x)P_{st}(x)} \int_{-\infty}^{x} P_{st}(y)dy. \quad (33)$$

Fig. 2(a) shows that the MFPT as a function of D_1 is increased when λ increased. The curve is changed from monotonically decreasing function to a function which has a peak. Fig. 2(b) shows that the peak of the MFPT moves to the right when q decreased.

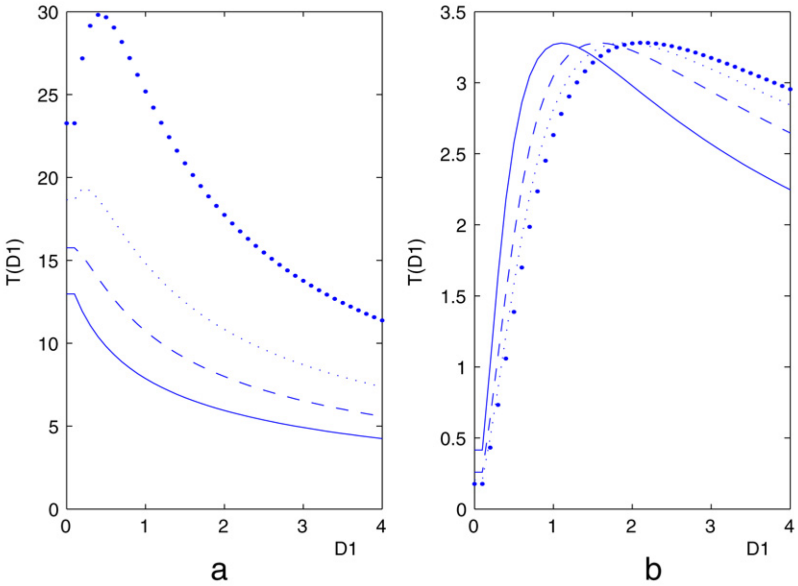

Fig. 2. The mean first-passage time as a function of parameter D_1. (a) $D_2 = 0.5$, $\tau_1 = 0.1$, $\tau_2 = 0.5$, $q = 0.7$ are fixed, λ takes 0 (solid line), 0.3 (dashed line), 0.5 (dotted line), 0.7 (point line). (b) $D_2 = 0.5$, $\tau_1 = 0.01$, $\tau_2 = 0.05$, $\lambda, = 0.3$ are fixed, q takes 0.5 (point line), 0.75 (dotted line), 1 (dashed line), 1.25 (solid line).

3.3. Numerical simulations

To verify the validity of our analytical result, it is necessary to perform numerical simulation. Like [12], second-order Runge–Kutta algorithm with a time step of $\Delta t = 10^{-3}$ is adopted. For the general problem

$$\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= f(x) + g_1(x)\eta(t) + g_2(x)\xi(t), \\
\frac{\mathrm{d}\eta(t)}{\mathrm{d}t} &= \rho(\eta(t)) + \frac{1}{\tau_1}\varepsilon(t), \\
\frac{\mathrm{d}\xi}{\mathrm{d}t} &= -\frac{1}{\tau_2}\xi + \frac{1}{\tau_2}\Gamma(t),
\end{aligned} \tag{34}$$

the algorithm is as follows:

$$\begin{aligned}
x(t+\Delta t) &= x(t) + \frac{\Delta t}{2}[f(x) + f(x_1) + g_1(x)\eta + g_1(x_1)\eta_1 + g_2(x)\xi + g_2(x_1)\xi_1], \\
x_1 &= x(t) + f(x)\Delta t + g_1(x)\eta\Delta t + g_2(x)\xi\Delta t, \\
\eta(t+\Delta t) &= \eta(t) + \frac{\Delta t}{2}[\rho(\eta) + \rho(\eta_1)] + \frac{R_1}{\tau_1}, \\
\eta_1 &= \eta(t) + \rho(\eta)\Delta t + \frac{R_1'}{\tau}, \\
\xi_1 &= \xi(t) - \frac{\xi}{\tau_2}\Delta t + \frac{R_2'}{\tau_2}, \\
\xi(t+\Delta t) &= \xi(t) + \frac{\Delta t}{2}\left[-\frac{\xi}{\tau_2} - \frac{\xi_1}{\tau_2}\right] + \frac{R_2}{\tau_2},
\end{aligned} \tag{35}$$

where

$$\begin{aligned}
R_1' &= R_1 = \sqrt{2D_1\Delta t}Z_1, \\
R_2' &= R_2 = \sqrt{2D_2\Delta t}Z_1.
\end{aligned} \tag{36}$$

The random number Z_1, Z_2 can be generated as follows:

$$\begin{aligned}
Z_1 &= w_1 \\
Z_2 &= \lambda w_1 + (1-\lambda^2)^{\frac{1}{2}} w_2.
\end{aligned} \tag{37}$$

Here w_1, w_2 are independent Gaussian random numbers.

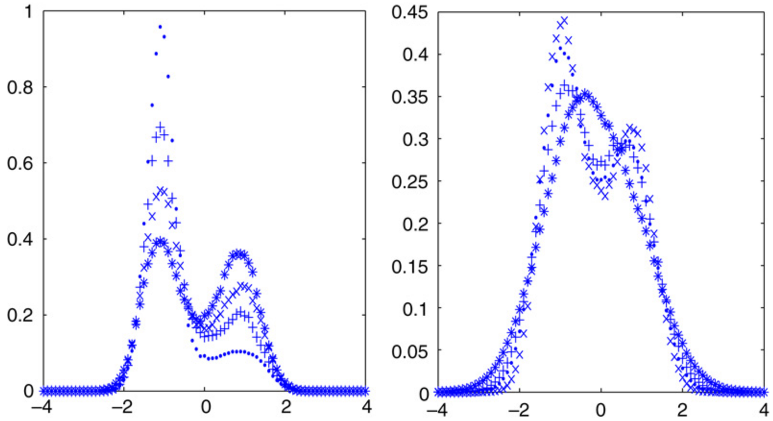

Fig. 3. The stationary probability distribution function. $P_{st}(x)$. (a) $D_1 = 0.5$, $D_2 = 0.5$, $\tau_1 = 0.1$, $\tau_2 = 0.5$, $q = 0.7$ are fixed. λ takes 0 ($*$), 0.3 ($\times$), 0.5 ($+$), 0.7 ($\cdot$). (b) $D_1 = 0.3$, $D_2 = 0.3$, $\tau_1 = 0.01$, $\tau_2 = 0.1$, $\lambda = 0.1$ are fixed. q takes 1.0 ($\times$), 1.3 ($\cdot$), 1.4 ($+$), 1.5 ($*$).

The numerical results of the steady-state distribution function are plotted in Fig. 3. The same parameters are used in Figs. 1 and 3. It can be seen that the analytical results are consistent with the numerical computations.

4. Conclusions

The approximate Fokker–Planck equation and the mean first-passage time are considered for a stochastic system with correlation between non-Gaussian noise and Gaussian colored noise. Then we examine the effects of parameters λ and q for the steady-state probability distribution function and mean first-passage time. Numerical simulation is performed to check the validity of analytical results. It is shown that the analytical results are consistent with the numerical computations.

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