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Universality of measurements on quantum markets

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Abstract

Two of the authors have recently discussed financial markets operated by quantum computers—quantum market games. These "new markets" cannot by themselves create opportunity of making extraordinary profits or multiplying goods, but they may cause the dynamism of transaction which would result in more effective markets and capital flow into hands of the most efficient traders. Here we focus upon the problem of universality of measurement in quantum market games offering a possible method of implementation if the necessary technologies would be available. It can be also used to analyse material commitments that elude description in orthodox game-theoretic terms.

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1. Introduction

Quantum market games [1,2] have a unique bootstrap supporting the use of quantum formalism to describe them—one of the interpretations of quantum mechanics based on the Fokker Wheeler Feynman direct interaction approach [3,4] refers to market transactions [5]. On the other hand, quantum theory implies properties of players' strategies that assuredly form new standards of market liquidity. Quantum strategies cannot be copied nor destroyed what is guaranteed by the *no-cloning* and *no-deleting theorems* [7] but they can be identified in a non-destructive way (for example, with a test making use of the *controlled-swap* gate that is used in the quantum fingerprinting [6]). In addition, they can be shared in a perfect, requiring no regulations way among players–shareholders (for example in such a way that any group of *k* shareholders can adopt the strategy and no smaller group of shareholders can make profit on this strategy [8]). These features are very promising and initiate research projects that aims at implementation of quantum games in various fields [9]. Optimal management of such quantum strategies requires an appropriate portfolio theory [10]. This should not be regarded as a disadvantage as risk is associated even with classical arbitrage transactions [11] and there is a constant need for an appropriate theory to manage the risk associated with any activity.

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Currently, quantum theory is the only one that promises this degree of perfection therefore there is only a faint possibility that quantum game theory might be overvalued. Contemporary markets undergoing a process of globalization would intensify such evolution. Markets exploring quantum phenomena regardless of such details as whether its "quantumness" would derive from instruments or human mind properties, would offer effectiveness impossible in classical markets and therefore would replace them sooner or later [12,13]. In the present paper, we show how quantum market games can possibly be implemented with help of set of universal primitives. Existence of such universal primitives is essential for the quantum market idea ever coming true. We focus here on the very core of a quantum market (*the quantum board* or *the universum of the game*) although the main technological progress in the field of quantum information concerns communication scenarios and their security that constitute sort of peripheral devices from our point of view. Commercial quantum information processing instruments have already appeared on the market and our analysis reveals possible new niches to be found when the appropriate technologies would be available. To set the stage, we first, in Sections 2 and 3, describe main ideas behind quantum market games and analyse simple two-qubits strategies. In Section 4, we discuss main features of measurements of quantum tactics. Then, in Section 5, we show how to adopt Perdrix formalism to prove the universality property of quantum market games. It follows that reusable quantum hardware for quantum markets is feasible. Finally, in Section 4, we discuss a possible implementation of the game of 20 questions.

2. Quantum market games

Quantum game theory investigates conflict situations involving quantum phenomena. Therefore it exploits formalism of quantum theory. Strategies are vectors (called states) in some Hilbert space and can be interpreted as superpositions of trading decisions. Tactics and moves are performed by unitary transformations on states. The idea behind using quantum games is to explore the possibility of forming linear combination of amplitudes that are complex Hilbert space vectors whose squared absolute values give probabilities of players actions. Description of complex quantum games with unlimited number of players or non-constant pay-offs is an open problem [14,15]. There are several possible ways of accomplishing this task. We have proposed a generalization of market games to the quantum domain in Ref. [1]. If a game allows a great number of players in it is useful to consider it as a two-players game: the kth trader against the rest of the world (RW). Any concrete algorithm $\mathscr{A}$ should allow for an effective strategies of the RW type. Let the real variable q

$$q:=\ln c-E(\ln c)$$

denotes the logarithm of the price at which the kth player can buy the asset $\mathfrak{G}$ shifted so that its expectation value in the state $|\psi\rangle_k$ vanishes. The expectation value of x is denoted by $E(x)$. The representation of prices by their logarithms is very convenient and often used in financial mathematics.[1] We follow this convention. The variable p

$$p:=E(\ln c)-\ln c$$

describes the situation of a player who is supplying the asset $\mathfrak{G}$ at the price c according to his strategy $|\psi\rangle_k$. Supplying $\mathfrak{G}$ can be regarded as demanding \$ at the price c^{-1} in the $1\mathfrak{G}$ units and both definitions are equivalent. Note that we have defined q and p so that they do not depend on possible choices of the units for $\mathfrak{G}$ and \$. For simplicity we will use such units that $E(\ln c)=0$. Strategies $|\psi\rangle_k$ belong to Hilbert spaces H_k. The initial state of the game $|\Psi\rangle_{in}:=\sum_k|\psi\rangle_k$ is a vector in the direct sum of Hilbert spaces of all players. Following Refs. [12,13] we define canonically conjugate Hermitian operators of demand $\mathscr{Q}_k$ and supply $\mathscr{P}_k$ for each Hilbert space H_k. They are analogous to the quantum position and momentum operators. The capital flows resulting from an ensemble of simultaneous transactions correspond to the physical process of measurement. A transaction consists in a transition from the state of traders strategies $|\Psi\rangle_{in}$ to the one describing the capital flow state $|\Psi\rangle_{out}:=\mathscr{T}_\sigma|\Psi\rangle_{in}$, where $\mathscr{T}_\sigma:=\sum_{k_d}|q\rangle_{k_d k_d}\langle q|+\sum_{k_s}|p\rangle_{k_s k_s}\langle p|$ is the projective operator defined by the division σ of the set of traders $\{k\}$ into two separate subsets $\{k\}=\{k_d\}\cup\{k_s\}$, the ones who are buying at the

[1] For example, discount factors become additive, etc.

price $e^{q_{k_d}}$ and the ones who are selling at the price $e^{-p_{k_s}}$ in the round of the transaction in question. Note that the operator $\mathscr{T}_\sigma$ is the identity only in trivial cases because all possible strategies are scarcely in use at the same moment. The game consist in an implementation of an effective algorithm $\mathscr{A}$ whose role is to determine the division σ of the market, the set of price parameters $\{q_{k_d}, p_{k_s}\}$ and the values of capital flows. The later are settled by the distribution

$$\int_{-\infty}^{\ln c} \frac{|\langle q|\psi\rangle_k|^2}{{}_k\langle\psi|\psi\rangle_k}\, \mathrm{d}q,$$

which is interpreted as the probability that trader $|\psi\rangle_k$ is willing to buy the asset $\mathfrak{G}$ at the price c or lower [17]. In an analogous way the distribution

$$\int_{-\infty}^{\ln 1/c} \frac{|\langle p|\psi\rangle_k|^2}{{}_k\langle\psi|\psi\rangle_k}\, \mathrm{d}p$$

gives the probability of selling $\mathfrak{G}$ by trader $|\psi\rangle_k$ at the price c or greater. These probabilities are in fact conditional because they describe the situation after the division σ is completed. If one considers the RW strategy it make sense to declare its simultaneous demand and supply states because for one player RW is a buyer and for another it is a seller. To describe such situation it is convenient to use the Wigner formalism.[2] The pseudo-probability $W(p,q)\mathrm{d}p\,\mathrm{d}q$ on the phase space $\{(p,q)\}$ known as the Wigner function is given by

$$W(p,q) := h_E^{-1} \int_{-\infty}^{\infty} e^{i\hbar_E^{-1}px} \frac{\langle q + x/2|\psi\rangle\langle\psi|q - x/2\rangle}{\langle\psi|\psi\rangle}\, \mathrm{d}x.$$

This measure is not positive definite except for the cases presented below. In the general case the pseudo-probability density of RW is a countable linear combination of Wigner functions, $\rho(p,q) = \sum_n w_n W_n(p,q)$, $w_n \geqslant 0$, $\sum_n w_n = 1$. The diagrams of the integrals of the RW pseudo-probabilities (see Ref. [17])

$$F_d(\ln c) := \int_{-\infty}^{\ln c} \rho(p = const., q)\, \mathrm{d}q$$

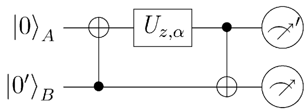
(RW bids selling at e^{-p}) and

$$F_s(\ln c) := \int_{-\infty}^{\ln 1/c} \rho(p, q = const.)\, \mathrm{d}p$$

(RW bids buying at e^q) against the argument $\ln c$ may be interpreted as the dominant supply and demand curves in the Cournot convention, respectively [17]. Note, that due to the lack of positive definiteness of ρ, F_d and F_s may not be monotonic functions. Textbooks on economics give examples of such departures from the law of supply and the law of demand (Giffen assets).

3. A two-qubits agent's strategy

The bewildering phenomenon of quantum dense coding [18] enables us sending two classical bit of information by exchanging one qubit. This can be presented in the game-like setting as follows. Suppose we intend to send the information from A to B. Then the circuit (we use the standard graphical representation of quantum controlled not (Cnot) gate, bit swap and measurement, see Ref. [24])



$$Cnot(U_{z,\alpha} \otimes I) Swap\ Cnot\ Swap |0\rangle_A |0'\rangle_B$$
$$= \cos(\alpha)|0'\rangle_A|0\rangle_B + i\,\sin(\alpha)(E_z(X)|0'\rangle_A|I\rangle_B + E_z(X')|I'\rangle_A|0\rangle_B + E_z(X'')|I'\rangle_A|I\rangle_B)$$

[2]Actually, this approach consists in allowing pseudo-probabilities into consideration. From the physical point of view this is questionable but for our aims it is useful, cf. the discussion of the Giffen paradox [16].

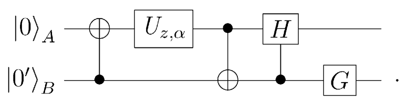
where

$$X := \sigma_x, \quad X' := \sigma_z = HXH, \quad X'' := \sigma_y = \mathrm{i}XX'$$

are tactics[3] given by the Pauli matrices, describes such a process. The representation of tactics $U_{z,\alpha}$ in terms of the final strategy is secure against interception because the owner of the pair of qubits A and B can keep the information distinguishing these two qubits secret (one classical bit); without this information the interception of the pair of qubits A and B is insufficient for identification of the tactics $U_{z,\alpha}$. Such a method has previously been applied by Wiesner to construct quantum counterfeit-proof banknotes [20]. By adopting a tactics $U_{z,\alpha}$ that corresponds to one of four pairwise maximally distant pairs of antipodal points of the sphere S_3[4] the owner of qubit A is able to send two classical bits to the owner of qubit B while sending only one qubit. According to the analysis given in Ref. [21], the measurable qubits B and A can be interpreted as market polarizations of their owner (if $|0\rangle$-supply and if $|1\rangle$-demand) and therefore his/her inclination to buy at low or high prices what can easily be seen if we replace the meters with the controlled-Hadamard gates with control qubit B. In order to connect unequivocally any of the three conjugated bases [20] (or mutually unbiased [22]) with one of their three possible market functions (eigenvectors (fixed points) of X with supply inclination, eigenvectors of X' with demand inclination and eigenvectors of X'' with polarization) we should transform the strategy B (after the controlled-Hadamard gate!) with the involutive tactics

$$G := \tfrac{1}{\sqrt{2}}(X' + X'')$$

that transforms eigenvectors of X' into eigenvectors of X''. The consideration of the third conjugated basis is necessary to guarantee the security of the information a la Wiesner' banknotes (the information about the respective price carried by qubit A uses two conjugated bases—sets of fixed points of tactics X and X').



(1)

If there is no restriction on tactics $U_{z,\alpha}$ the agent is able to play more effectively by adopting superpositions of previously allowed strategies. Collective tactics and strategies are also possible. Elsewhere, we have shown that agents can enter into alliances that can be implemented via *the controlled-NOT* gates (implemented as gates between qubit strategies). These gates are universal. Obviously, from the technical point of view, quantum markets can have various different properties. The polarization qubit is redundant in two-sided auctions but in bargaining games [21] another qubit is necessary to distinguish the agents who are bidding. Much more additional qubits are necessary if the corresponding supply and demand curves are continuous (floating point precision)—one qubit for each binary digit of the logarithm of price. However, these are theoretically unimportant details—all such forms of quantum markets can be implemented with the use of elementary market measurements alone what follows from the analysis by Nielsen [23,24], Raussendorf and Briegel [25], and Perdrix and Jorrand [26,27]. The rest of the paper is devoted to this problem.

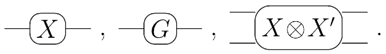
4. Measurements of tactics

A measurement of tactics consists in determination of the strategy or, more precisely, finding out which of its fixed points we have to deal with. If the tactics being measured changes the corresponding strategy, then non-demolition measurements reduce the strategy to one of its fixed points and the respective transition amplitudes are given by coordinates of the strategy in the fixed point basis (Born rule). As we will show, measurements of the tactics X, G and $X \otimes X'$ suffice to implement quantum market games. According to the

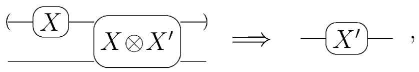
[3] We call any unitary transformation that changes agent's (player's) strategy a tactics. We follow the notation introduced in Ref. [19]: $SU(2) \ni \mathcal{U}_{z,\alpha} = \mathrm{e}^{i\alpha\vec{\sigma}\cdot E_z(\vec{\sigma})} = I\cos\alpha + i\vec{\sigma}\cdot E_z(\vec{\sigma})\sin\alpha$, where the vector $E_z(\vec{\sigma}) = \langle z|\vec{\sigma}|z\rangle/\langle z|z\rangle$ represents the expectation value of the vector of Pauli matrices $\vec{\sigma} := (\sigma_1, \sigma_2, \sigma_3)$ for a given strategy $|z\rangle$. The family $\{|z\rangle\}, z \in \overline{\mathbb{C}}$ of complex vectors (states) $|z\rangle := |0\rangle + z|I\rangle$ $(|\pm\infty\rangle := |I\rangle)$ represents all trader's strategies in the linear subspace spanned by the vectors $|0\rangle$ and $|I\rangle$.

[4] $U = a_0 I + \mathrm{i}\sum_k a_k \sigma_k$, where $a_0 = \cos\alpha$, $a_k = n_k \sin\alpha$ and $\sum_\mu (a_\mu)^2 = 1$. The corresponding tactics are $\pm I \pm \sigma_k$, where the antipodal points have different signs but represent equivalent tactics.

Qcircuit.tex standard macros [28], we will denote the corresponding measuring gates as (rounded off shape is used to distinguish measuring gates):


(2)

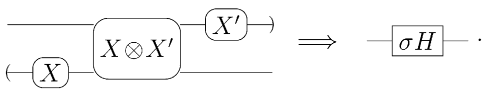
Note that measurement of the tactics $X \otimes X'$ provides us with information whether the two strategies agree or disagree on the price but reveals no information on the level of the price in question. To get information about the prices we have to measure $X \otimes I$ and $I \otimes X'$, respectively. Note that the measurement of X' can be implicitly accomplished by measurement of X and subsequently $X \otimes X'$. This is shown graphically by (see Ref. [26] for details):



where the parentheses are used to denote auxiliary qubits. In the following paragraphs, we will analyse q-circuits with various number of auxiliary qubits that would allow for implementation of tactics via measurement only—the approach proposed by Perdix [26].

5. Universality of measurements: implementing tactics via measurements

Teleportation and measurement form surprisingly powerful tools in implementation of tactics. The method used by Perdrix and Jorrand [26,27] to analyse the problem of universality in quantum computation can be easily adopted to the situation we are considering. Following Ref. [26], we begin by showing how a strategy encoded in one qubit can be transferred to another (from the upper one to the lower one in the figure below) and how it changes with a sequence of tactics σH, where σ is one of the Pauli matrices (including the identity matrix):


(3)

Assuming that the input qubit is in the state:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle,$$

after measuring $\mathbb{I} \otimes X$ (with classical outcome $j = \pm 1$) we obtain:

$$|\psi_1\rangle = |\psi\rangle \otimes X'^{(1-j)/2}\frac{|0\rangle + |1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}(\mathbb{I} \otimes X'^{(1-j)/2})(\alpha|00\rangle + \alpha|01\rangle + \beta|10\rangle + \beta|11\rangle).$$

Measurement of $X \otimes X'$ with outcome $k = \pm 1$ sets our qubits in state:

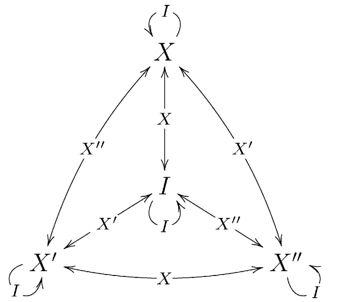
$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(\mathbb{I} \otimes X^{(1-k)/2}X'^{(1-j)/2})[(\alpha + \beta)(|00\rangle + |10\rangle) + (\alpha - \beta)(|01\rangle - |11\rangle)].$$

The final measurement $X' \otimes \mathbb{I}$ with outcome $l = \pm 1$ gives us the final state:

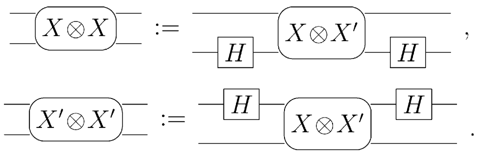
$$\begin{aligned}|\psi_3\rangle &= [X^{(1-l)/2} \otimes X^{(1-k)/2}X'^{(1-j)/2}HX^{(1-l)/2}][|0\rangle \otimes (\alpha|0\rangle + \beta|1\rangle)] \\ &= X^{(1-l)/2}|0\rangle \otimes X^{(1-k)/2}X'^{(1-j\cdot l)/2}H|\psi\rangle,\end{aligned}$$

and the equivalence of the circuits given above is proved.

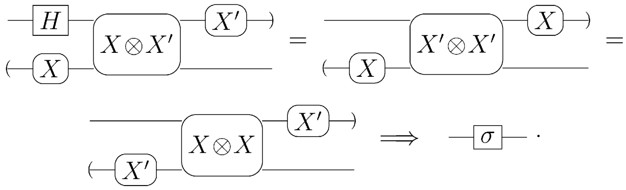
Thus, the strategy encoded in the upper state is transferred from the lower qubit and changed with the tactics σH, where $\sigma = X^{(1-k)/2}X'^{(1-j\cdot l)/2}$. It is evident that the same tactics is adopted when we switch the supply measurements with the demand ones ($X \leftrightarrow X'$). Simple calculation shows that the composite tactics $H\sigma H$ and $\sigma_i\sigma_k$ reduce to some Pauli (matrix) tactics. Therefore, an even sequence of tactics (3) can be perceived as the Markov process over vertices of the graph:



It follows that any Pauli tactics can be implemented as an even number of tactics-measurements (3) by identifying it with some final vertex of random walk on this graph. Although the probability of drawing out the final vertex at the first step is $\frac{1}{4}$, the probability of staying in the "labyrinth" exponentially decreases to zero. Having a method of implementation of Pauli tactics, allows us to modify the tactics (3) so that to implement the tactics H—the fundamental operation of switching the supply representation with the demand representation. It can be also applied to measure compliance with tactics representing the same side of the market (direct measurement is not possible because the agents cannot make the deal):



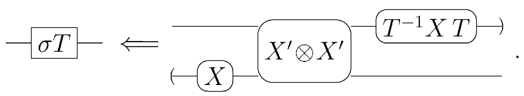
In addition, this would allow for interpretation via measurement of random Pauli tactics σ because of the involutiveness of H the gate (3) can be transformed to


(4)

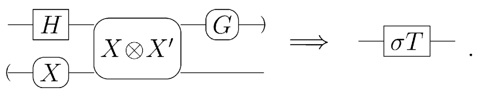
The gate (4) can be used to implement the phase-shift tactics:

$$T := \begin{pmatrix} 1 & 0 \\ 0 & \dfrac{1+\mathrm{i}}{\sqrt{2}} \end{pmatrix}.$$

T commutes with X', hence:

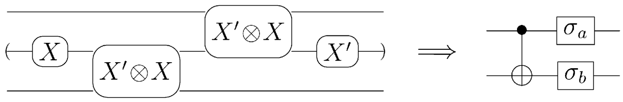


Elementary calculations demonstrate that $T^{-1}XT = (X - X'')/\sqrt{2}$ and $H(X - X'')/\sqrt{2}H = G$, therefore:



We have seen earlier that it is possible to remove the superfluous Pauli operators, cf. (4). To end the proof of universality of the set of gates (2) we have to show how to implement the alliance *Cnot* (note that $\{H, T, Cnot\}$

a set of universal gates [29]). This gate can be implemented as the circuit (as before, the gate is constructed up to a Pauli tactics) [26]:



(5)

The explicit calculations are as follows:[5]

Let us assume the input qubits are the first and the second, in state:

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

with the third used as an auxiliary one. States $|\pm\rangle$ are defined as follows:

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle).$$

The first measurement $\mathbb{I} \otimes \mathbb{I} \otimes X$ gives us the state below, depending on classical outcome $j = \pm 1$:[6]

$$\begin{aligned} |\psi_1\rangle &= |\psi\rangle \otimes X'^{(1-j)/2}|+\rangle \\ &= (\mathbb{I} \otimes \mathbb{I} \otimes X'^{(1-j)/2})(\alpha|00+\rangle + \beta|01+\rangle + \gamma|10+\rangle + \delta|11+\rangle). \end{aligned}$$

After $\mathbb{I} \otimes X \otimes X'$ with outcome $k = \pm 1$ we obtain:

$$\begin{aligned} |\psi_2\rangle &= \frac{1}{2}[\mathbb{I} \otimes X'^{(1-k)/2} \otimes X'^{(1-j)/2}]\{(\alpha+\beta)(|000\rangle + |010\rangle) + (\alpha-\beta)(|001\rangle - |011\rangle) \\ &\quad + (\gamma+\delta)(|100\rangle + |110\rangle) + (\gamma-\delta)(|101\rangle - |111\rangle)\} \\ &= \frac{1}{\sqrt{2}}[\mathbb{I} \otimes X'^{(1-k)/2} \otimes X'^{(1-j)/2}]\{(\alpha+\beta)|0+0\rangle + (\alpha-\beta)|0-1\rangle \\ &\quad + (\gamma+\delta)|1+0\rangle + (\gamma-\delta)|1-1\rangle\}. \end{aligned}$$

Next measurement $X' \otimes \mathbb{I} \otimes X$ with outcome $l = \pm 1$ results in state:

$$|\psi_3\rangle = [\mathbb{I} \otimes X'^{(1-k)/2} X^{(1-l\cdot j)/2} \otimes X'^{(1-l)/2}][\alpha|00+\rangle + \beta|01+\rangle + \delta|10-\rangle + \gamma|11-\rangle].$$

After the final measurement of $\mathbb{I} \otimes \mathbb{I} \otimes X'$ with eigenvalues $m = \pm 1$ we get:

$$|\psi_3\rangle = [X'^{(1-k)/2} \otimes X^{(1-l\cdot j)/2} \otimes X'^{(1-l)/2}][\alpha|00+\rangle + \beta|01+\rangle + \delta|10-\rangle + \gamma|11-\rangle].$$

After the final measurement of $\mathbb{I} \otimes \mathbb{I} \otimes X'$ with eigenvalues $m = \pm 1$ we get:

$$\begin{aligned} |\psi_4\rangle &= [X'^{(1-m\cdot k)/2} \otimes X^{(1-l\cdot j)/2} \otimes X^{(1-m)/2}][(\alpha|00\rangle + \beta|01\rangle + \delta|10\rangle + \gamma|11\rangle) \otimes |0\rangle] \\ &= [X'^{(1-m\cdot k)/2} \otimes X^{(1-l\cdot j)/2} \otimes X^{(1-m)/2}][CNot|\psi \otimes |0\rangle]. \end{aligned}$$

It follows that the above circuits are equivalent.

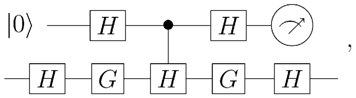
The measurement of the tactics G performed within the quantum market game frame of reference causes interpretative problems[7] that can be resolved if we replace the measurement of G with *controlled H* gate (cf. (1)) and the measurement of entanglement for another pair of conjugated bases $X' \otimes X''$ in the set of universal primitives. Owing to the fact that $G = HGHGH$, the measurement of G can be implemented

[5] The tactics H transforms the demand picture to the supply picture ($X \leftrightarrow X'$), which results in a switch from control qubit to the controlled qubit.

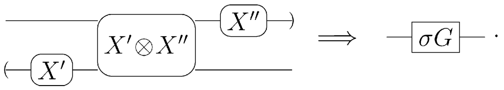
[6] Note that the second and the third qubit appear in reversed order in figure (5).

[7] Eigenvectors of G lack any rational interpretation in terms of strategies in the quantum market games model. Nevertheless, the tactics G can be interpreted as a transformation between various bases spanned by vectors representing market strategies. Besides, it can be implemented as sequence of measurement of strategies ($X', \otimes X''$ and X'') as explained in the text.

in the following way [24]:



where the tactics G is obtained from (3) by the cyclic replacement $X \to X'$, $X' \to X''$ i $X'' \to X$:



In fact, the universality property has any set of primitive that contains the *controlled H* gate and measurements $X^k, X^p \otimes X^q, X^r \otimes X^s$, where $p \neq q$, $r \neq s$ and $p \neq r$—this can be easily checked [26]. It follows that to implement a quantum market[8] it suffices to have, beside possibility of measuring strategy-qubits and control of the supply-demand context, a direct method measuring entanglement of a pair of qubits in conjugated bases.

6. Quantum intelligence à la 20 questions

Let us recall the anecdote popularized by John Archibald Wheeler [30]. The plot concerns the game of 20 questions: the player has to guess an unknown word by asking up to 20 questions (the answers could be only yes or no and are always true). In the version presented by Wheeler, the answers are given by a "quantum agent" who attempts to assign the task the highest level of difficulty without breaking the rules. In the light of the previous discussion, any quantum algorithm (including classical algorithms as a special cases) can be implemented as a sequence of appropriately constructed questions-measurements. The results of the measurements (i.e., answers) that are not satisfactory cause further "interrogation" about selected elementary ingredients of the reality (qubits). If quantum intelligence (QI) is perceived in such a way (as quantum game) then it can be simulated by a deterministic automaton that follows a chain of test bits built on a quantum tenor [31]. The automaton completes the chain with afore prepared additional questions at any time that an unexpected answer is produced. Although the results of the test will be random (and actually meaningless—they are instrumental), the kind and the topology of tests that examine various layers multi-qubit reality and the working scheme of the automaton are fixed prior to the test. The remarkability of performance of such an automaton in a game against Nature is by the final measurement that could reveal knowledge that is out of reach of classical information processing, cf. the already known Grover and Shor quantum algorithms and the Elitzur–Vaidman bomb tester. Needless to say, such an implementation of a game against quantum Nature leaves some room for perfection. The tactics *CNot* and H belong to the normalizer of the n-qubit Pauli group G_n [24], hence their adoption allows to restrict oneself to single corrections of "errors" made by Nature that precede the final measurement. It is worth noting that a variant of implementation of the tactics T makes it possible to postpone the correction provided the respective measurements methods concern the current state of the cumulated errors [32]. Therefore in this setting of the game some answers given by Nature, though being instrumental, have a significance because of the influence of the following tests. There is no need for the final error correction—a modification of the measuring method is sufficient. In that way the course of game is fast and the length of the game is not a random variable. This example shows that in some sense the randomness in game against quantum Nature can result from awkwardness of agents and erroneous misinterpretation of answers that are purely instrumental. If only one error (lie) in the two-person framework are allowed fast quantum algorithms for solving the problem exist (Ulams's problem) [33].

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[8] In fact, any finite-dimensional quantum computational system can be implemented in that way [26].

References

[1] E.W. Piotrowski, J. Sładkowski, Quantum market games, Physica A 312 (2002) 208.
[2] E.W. Piotrowski, J. Sładkowski, Quantum game theory in finance, Quant. Finance 4 (2004) 1.
[3] J.A. Wheeler, R.P. Feynman, Interaction with the absorber as the mechanism of radiation, Rev. Mod. Phys. 17 (1945) 157.
[4] J.A. Wheeler, R.P. Feynman, Rev. Mod. Phys. 21 (1949) 425.
[5] J.G. Cramer, The transactional interpretation of quantum mechanics, Rev. Mod. Phys. 58 (1986) 647.
[6] H. Buhrman, R. Cleve, J. Watrous, R. de Wolf, Quantum finger-printing, Phys. Rev. Lett. 87 (2001) 167902.
[7] M. Horodecki, R. Horodecki, A. Sen De, U. Sen, No-deleting and no-cloning principles as consequences of conservation of quantum information; quant-ph/0306044.
[8] R. Cleve, D. Gottesman, H.-K. Lo, How to share a quantum secret, Phys. Rev. Lett. 82 (1999) 648.
[9] N. Patel, States of play, Nature 445 (2007) 44.
[10] S.M. Maurer, T. Hogg, B.A. Huberman, Portfolios of quantum algorithms, Phys. Rev. Lett. 87 (2001) 257901.
[11] E.W. Piotrowski, J. Sładkowski, Arbitrage risk induced by transaction costs, Physica A 331 (2004) 233.
[12] E.W. Piotrowski, J. Sładkowski, Trading by quantum rules—quantum anthropic principle, Int. J. Theoretical Phys. 42 (2003) 1101.
[13] E.W. Piotrowski, J. Sładkowski, Quantum-like approach to financial risk: quantum anthropic principle, Acta Phys. Pol. B 32 (2001) 3873.
[14] A.P. Flitney, D. Abbott, An introduction to quantum game theory, Fluctuation Noise Lett. 2 (2002) R175.
[15] E.W. Piotrowski, J. Sładkowski, The next stage: quantum game theory, in: Mathematical Physics Research at the Cutting Edge, Nova Science Publishers, New York, 2003.
[16] J. Sładkowski, Giffen paradoxes in quantum market games, Physica A 324 (2003) 234.
[17] E.W. Piotrowski, J. Sładkowski, The merchandising mathematician model, Physica A 318 (2003) 496.
[18] C.H. Bennett, S.J. Wiesner, Communication via one- and two-particle operators on EPR states, Phys. Rev. Lett. 69 (1992).
[19] E.W. Piotrowski, J. Sładkowski, Quantum computer: an appliance for playing market games, Int. J. Quantum Inf. 2 (2004) 495.
[20] S. Wiesner, Conjugate coding, SIGACT News 15/1 (1983) 78, ⟨http://kh.bu.edu/qcl/pdf/wiesners198316024137.pdf⟩.
[21] E.W. Piotrowski, J. Sładkowski, Quantum bargaining games, Physica A 308 (2002) 391.
[22] W.K. Wootters, Quantum measurements and finite geometry; quant-ph/0406032.
[23] M.A. Nielsen, Computable functions, quantum measurements, and quantum dynamics, Phys. Rev. Lett. 79 (1997) 2915.
[24] M.A. Nielsen, I.L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, Cambridge, 2000.
[25] R. Raussendorf, H.J. Briegel, Quantum computing via measurements only, Phys. Rev. Lett. 86 (2000) 5188.
[26] S. Perdrix, State transfer instead of teleportation in measurement-based quantum computation, Int. J. Quantum Inf. 3 (2005) 219.
[27] P. Jorrand, S. Perdrix, Unifying quantum computation with projective measurements only and one-way quantum computation; quant-ph/0404125.
[28] B. Eastin, S.T. Flammia, Q-circuit tutorial, quant-ph/0406003.
[29] Barenco, et al., Elementary gates for quantum computation, Phys. Rev. A 52 (1995) 3457.
[30] P.C.W. Davies, J.R. Brown (Eds.), The Ghost in the Atom: A Discussion of the Mysteries of Quantum Physics, Cambridge University Press, Cambridge, 1993.
[31] D. Deutsch, The Fabric of Reality, Penguin, London, 1998.
[32] P. Jorrand, S. Perdrix, Non-probabilistic termination of measurement-based quantum computation; quant-ph/0311142.
[33] S. Mancini, Using quantum mechanics to cope with liars; quant-ph/0508156.